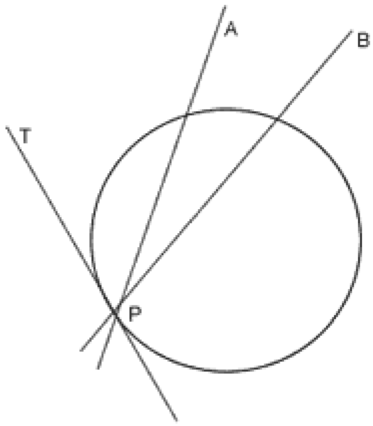
Chapter 10

Tangents to a circle

Tangent to a circle is a line which intersects the circle in exactly one point.



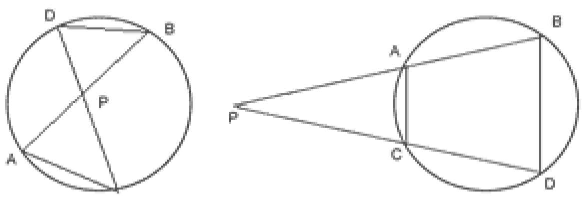
At a point of a circle there is one and only one tangent.

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

The lengths of tangents drawn from an external point to a circle are equal.

Centre of the circle lies on the bisector of the angle between the two tangents.

Theorem 1:- If two chords of a circle intersect inside or outside the circle, then the rectangle formed by the two parts of one chord is equal in area to the rectangle formed by the two parts of the other.



Given:- Two chords AB and CD of a circle such that they intersect each other at a point P lying inside in figure (i) or outside in figure (ii) of the circle.

To prove: - PA.PB = PC.PD

Construction:-AC and BD are joine P.

Proof:-

Case - (1) in figure (i) P lies inside the circle

In $\Delta^s PCA \, and \, PBD,$ we have

$\angle PCA = \angle PBD$ [Angles in jthe same segment]

$\angle APC = \angle BPD$ [vertically opposite angles]

$\therefore \, \Delta PCA \sim \Delta PBD$ (AA similarity)

Case- (2) In figure (ii) P lies outside the circle

$\angle PAC + \angle CAB = 180^0$ (limear pair)

and $\angle CAB + \angle PDB = 180^0$ (opposite angles of acyclicq and)

$\therefore \, \angle PAC = \angle PDB$

In $\Delta^s PCA \; and \; PBD$

$\angle PAC = \angle PDB$ [Proved above]

$\angle APC = \angle DPB$ [Common]

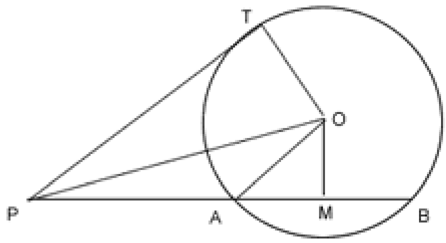
$\therefore \, \Delta PCA \sim \Delta PBD$ (AA similarity)

Hence, in either case,

$$\frac{PA}{PB} = \frac{PC}{PB}$$

Or, PA.PB = PC.PD

Theorem 2. If PAB is a secant to a circle intersecting it at A and B and PT is a tangent then PA.PB = PT^2.



Given: - PAB is secant intersecting the circle with centre O at A and B and a tangent PT at T.

To Prove: - PA.PB = PT^2

Construction: - $OM \perp AB$ is drawn OA, OP and OT are joined.

Proof: - PA = PM – AM

PB = PM + MB

$= PM = AM \qquad (\because\ AM = MB)$

$$\therefore\ PA.PB = (PM - AM).(PM + AM)$$
$$= PM^2 - AM^2$$

Also $OM \perp AB$

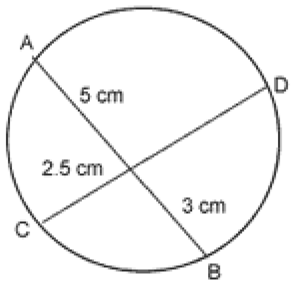
$\therefore$ $PM^2 = OP^2 - OM^2$ [Pythagoras theo.]

and $AM^2 = OA^2 - OM^2$ [Pythagoras theo.]

$$\therefore\ PA.PB = PM^2 - AM^2$$
$$= (OP^2 - OM^2) - (OA^2 - OM^2)$$
$$= OP^2 - OM^2 - OA^2 + OM^2$$
$$= OP^2 - OA^2$$
$$= OP^2 - OT^2 \qquad [\because OA = OT\ radii]$$

$\therefore$ PA.PB = PT^2 [Pythagoras theo.]

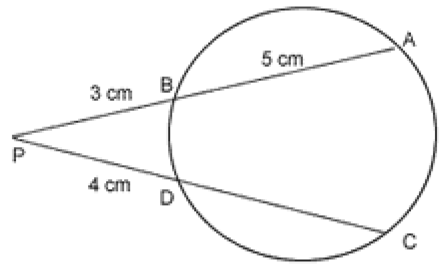
Example 1. In figure, chords AB and CD of the circle intersect at O. OA = 5cm, OB = 3cm and OC = 2.5cm. Find OD.



Solutions: - Chords AB and CD of the circle intersect at O

$$\therefore\ OA \times OB = OC \times OD$$
$$Or, \quad 5 \times 3 = 2.5 \times OD$$
$$Or, \quad OD = \frac{2 \times 3}{2.5} = 6cm$$

Example 2. In figure. Chords AB and CD intersect at P.



If AB = 5cm, PB = 3cm and PD = 4cm. Find the length of CD.

Solution:- PA = 5 + 3 = 8cm

PA X PB = PC X PD

Or, 8 X 3 = PC X 4

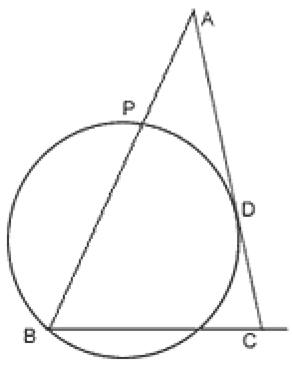
Or, PC = 8 X 3/4 = 6cm

$\therefore$ CD = PC - PD

= 6 - 4

= 2cm

Example 3. In the figure, ABC is an isosceles triangle in which AB = AC. A circle through B touches the side AC at D and intersect the side AB at P. If D is the midpoint of side AC, Then AB = 4AP.



Solution:- AP X AB = AD^2 = $(1/2AC)^2$

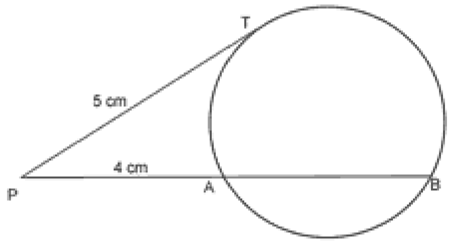
AP X AB = 1/4 AC^2 [AD = 1/2AC]

Or, 4 AP.AB = AC^2 [AC = AB]

Or, 4 AP.AB = AB^2

Or, 4 AP = AB

Example 4. In the figure. Find the value of AB Where PT = 5cm and PA = 4cm.



Solution:- PT^2 = PA X PB (Theory.2)

5^2 = 4 X PB

PB = 25/4 = 6.25

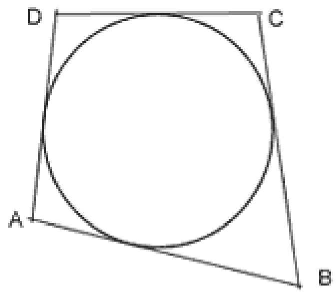
AB = PB - PA

AB = 6.25 - 4

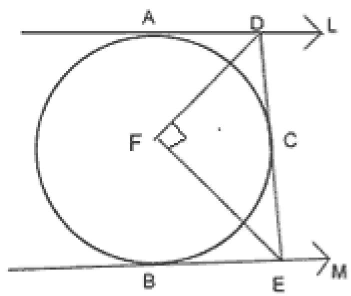
AB = 2.25 cm

Exercise - 20

1. In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6cm, BC = 7cm and CD = 4cm. Find AD.

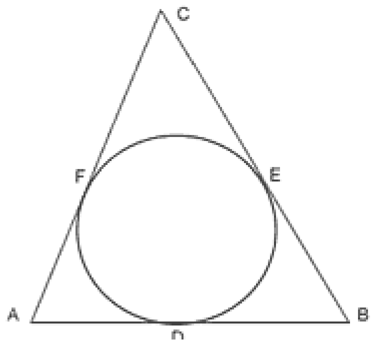


2. In figure. l and m are two parallel tangents at A and B. The tangent at C makes an intercept DE between the tangent l and m. Prove that $\angle DFE = 90^0$.



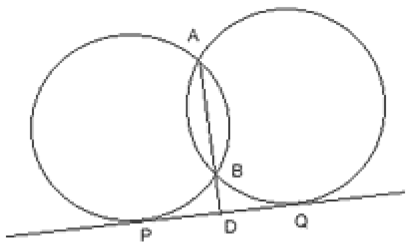
3. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

4. In figure, a circle is inscribed in a ΔABC having sides AB = 12 cm, BC = 8cm and AC = 10cm. Find AD, BE and CF.

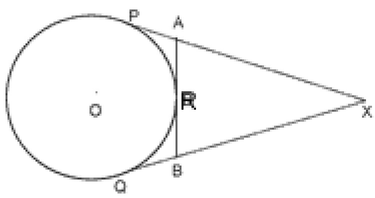


5. A circle is touching the side BC of a ΔABC at P and is touching AB and AC when produced at Q and R. Prove that AQ = ½ (Perimeter of ΔABC)

6. In figure. Two circles intersects each other at A and B. the common chord AB is produced to meet the common tangent PQ to the circle at D. Prove that DP = DQ.



7. In figure. XP and XQ are two tangents to a circle with centre O from a point X out side the circle. ARB is a tangent to the circle at R. prove that XA + AR = XB + BR.



8. A circle touches all the four sides a quadrilateral ABCD. Prove that the angles subtended at the centre of the circle by the opposite sides are supplementary.

9. If PA and PB are two tangents drawn from a point P to a circle with centre O touching it at A and B, prove that OP is the perpendicular perpendicular bisector of AB.

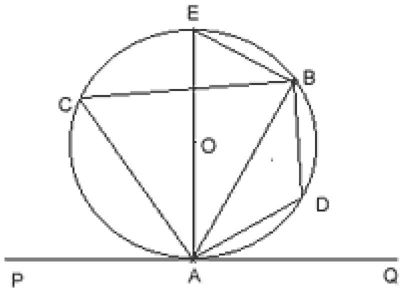
Answers

1. 3cm

4. AD = 7cm, BE = 5cm, CF = 3cm

Theorem 3. If a line touches a circle and from the point of contact a chord is drawn, the angle which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments.

Given:- PQ is a tangent to circle with centre O at a point A, AB is a chord and C, D are points in the two segments of the circle formed by the chord AB.



To Prove:- (i) $\angle BAQ = \angle ACB$

(ii) $\angle BAP = \angle ADB$

Construction:- A diameter AOE is drawn. BE is joined.

Proof: - In ΔABE

$\angle ABE = 90^0$ [Angle is a semicircle]

$\therefore \angle AEB + \angle BAE = 90^0$

$\angle BAE + \angle BAQ = \angle EAQ = 90^0$ $[EA \perp PQ]$

$\therefore AEB + \angle BAE = \angle BAE + \angle BAQ$

$\angle AEB = \angle BAQ$

From Theorem 3,

$\angle ACB = \angle AEB$

$\therefore \angle BAQ = \angle ACB$

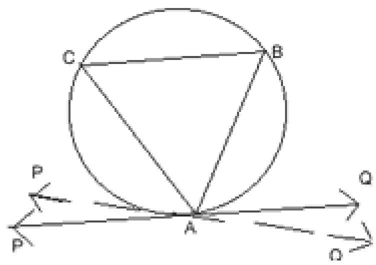
Again $\angle BAQ + \angle BAP = 180^0$ [Linear Pair]

and $\angle ACB + \angle ADB = 180^0$ [Opposite angles of a cyclicquad]

$\therefore \angle BAQ + \angle BAP = \angle ACB + \angle ADB$

$\angle BAP = \angle ADB \qquad [\because \angle BAQ = \angle ACB]$

Theorem 4. If a line is drawn through an end point of a chord of a circle so that the angle formed by it with the chord is equal to the angle subtend by chord in the alternate segment, then the line is a tangent to the circle.



Given:- A chord AB of a circle and a line PAQ. Such that $\angle BAQ = \angle ACB$ where c is any point in the alternate segment ACB.

To Prove:- PAQ is a tangent to the circle.

Construction:- Let PAQ is not a tangent then let us draw P' AQ' another tangent at A.

Proof: - AS P 'AQ' is tangent at A and AB is any chord

$\therefore \angle BAQ' = \angle ACB$ [theo.3]

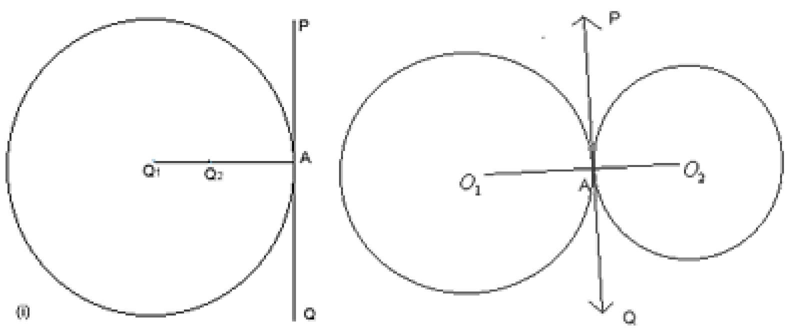
But $\therefore \angle BAQ' = \angle ACB$ (given)

$\therefore \angle BAQ' = \angle BAQ$

Hence AQ' and AQ are the same line i.e. P' AQ' and PAQ are the same line.

Hence PAQ is a tangent to the circle at A.

Theorem 5. If two circles touch each other internally or externally, the point of contact lie on the line joining their centres.



Given:- Two circles with centres O_1 and O_2 touch internally in figure (i) and externally in figure (ii) at A.

To prove: - The points O_1, O_2 and A lie on the same line.

Construction:- A common tangent PQ is drawn at A.

Proof: - In figure (i) $\angle PAQ_2 = \angle PAQ_1 = 90^0$ (PA is tangent to the two circles)

$\therefore$ O_1, O_2 and A are collinear.

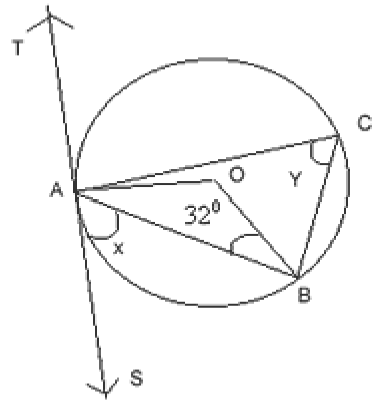
In figure (ii) $\angle PAO_1 = \angle PAO_2 = 90^0$ (PA is tangent to the circles)

$$\therefore \angle PAQ_1 + \angle PAQ_2 = 90^0 + 90^0$$
$$= 180^0$$

i.e. $\therefore \angle PAO_1 \text{ and } \angle PAO_2$ from a linear pair

$\therefore$ O_1, O_2 and A lie on the same line.

Example 5. In the given figure TAS is a tangent to the circle, with centre O, at the point A. If $\angle OBA = 32^0$, find the value of x and y.



Solution:- $In \ \Delta OBA, OA = OB \qquad (radii)$

$$\therefore \ \angle OAB = \angle OBA$$
$$= 32^0$$

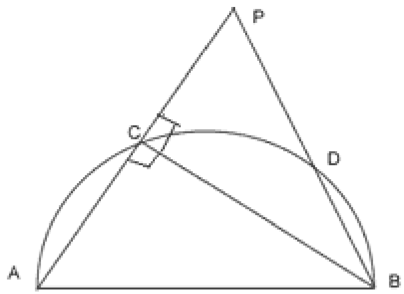
Now TAS is tangent at A

$$\therefore \ OA \perp TAS$$
$$\therefore \ \angle OAB + x = 90^0$$
$$Or, \ 32^0 + x = 90^0$$
$$\therefore \ x = 58^0$$

$\angle x = \angle y$ [Angles in the alternateseg]

$\angle x = \angle y = 58^0$

Example 6. In the given figure. $\angle C$ is right angle of ΔABC, . A semicircle is drawn on AB as diameter. P is any point on AC produced. When joined, BP meets the semi-circle in point D.
Prove that: $AB^2 = AC.AP + BD.BP$.



Solution:-

$$\angle ACB = 90^0$$
$$\therefore \ \angle BCP = 90^0$$
$$\therefore \ BC^2 = BP^2 - CP^2$$

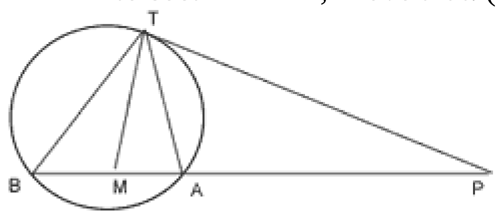
In ΔABC,

$$\begin{aligned}
AB^2 &= AC^2 + BC^2 \\
&= AC^2 + BP^2 - CP^2 \\
&= AC^2 - CP^2 + BP^2 \\
&= (AC - CP)\ (AC + CP) + BP^2 \\
&= (AC - CP).AP + BP^2 \\
&= AP.AC - AP.CP + BP^2 \\
&= AP.AC + BP^2 - AP.CP \\
&= AP.AC + BP^2 - BP.PD \qquad [AP.CP = BP.PD] \\
&= AP.AC + BP(BP - PD) \\
&= AP.AC + BP.BD
\end{aligned}$$
$$\therefore \ AB^2 = AP.AC + BP.BD$$

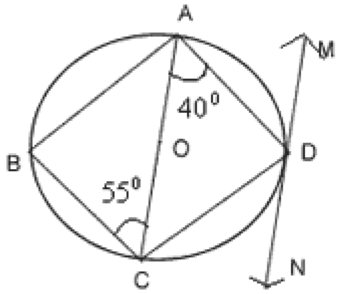
Exercise - 21

1. Two circles intersect at A and B. From a point P on one of these circles, two lines segments PAC and PBD are drawn intersecting the other circles at C and D respectively. Prove that CD is parallel to the tangent at P.

2. Two circles intersect in points P and Q. A secant passing through P intersects the circles at A an B respectively. Tangents to the circles at A and B intersects at T. Prove that A, Q, T and B are concyclic.
3. In the given figure. PT is a tangent and PAB is a secant to a circle. If the bisector of $\angle ATB$ intersect AB in M, Prove that: (i) $PMT = \angle PTM$ (ii) PT = PM



4. In the adjoining figure, ABCD is a cyclic quadrilateral. AC is a diameter of the circle. MN is tangent to the circle at D, $\angle CAD = 40^0$, $\angle ACB = 55^0$. Determine $\angle ADM$ and $\angle BAD$.



5. If ΔABC is isosceles with AB = AC, Prove that the tangent at A to the circumcircle of ΔABC is parallel to BC.
6. The diagonals of a parallelo gram ABCD intersect at E. Show that the circumcircles of ΔADE and ΔBCE touch each other at E.
7. A circle is drawn with diameter AB interacting the hypotenuse AC of right triangle ABC at the point P. Show that the tangent to the circle at P bisects the side BC.

Answers

4. 50^0, 75^0