

SOLUTION & ANSWER FOR AIEEE-2009 VERSION – A

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART A – PHYSICS

1. Statement 1 : For a charged particle moving from Point P to Point Q -----

Ans : Statement – 1 is true, Statement – 2 is true; Statement – 2 is the correct explanation of Statement - 1

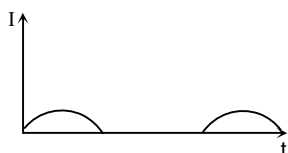
2. The above is a plot of binding energy per nucleon E_b , against the nuclear mass M -----

Ans : (i) and (iv)

Sol: Heavy nuclide disintegrates to lighter ones by releasing energy. Lighter nuclei combine to form releasing energy.

3. A p – n junction (D) shown in the figure can act --

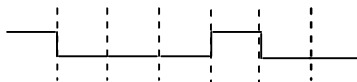
Ans :



Sol: Diode acts as a half wave rectifier without filter.

4. The logic circuit shown below has the input waveforms 'A' -----

Ans :



Sol: $\overline{A+B} = A.B = \text{AND gate}$

5. If x , v and a denote the displacement, the velocity and the acceleration -----

Ans : $\frac{aT}{x}$

Sol: $\frac{aT}{x} = \frac{-A\omega^2 \sin \omega t \cdot T}{A \sin \omega t} = -\omega^2 T = \text{constant}$

6. In an optics experiment, with the position of the object -----

Ans : (2f, 2f)

Sol: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow u = v = 2f$

7. A thin uniform rod of length ' ℓ ' and mass m is swinging freely ----

Ans : $\frac{1}{6} \frac{\ell^2 \omega^2}{g}$

Sol: $E = \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{m \ell^2}{3} \omega^2 = mgh$
 $\Rightarrow h = \frac{\ell^2 \omega^2}{6g}$

8. Let $P(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a -----

Ans : $\frac{Qr_1^2}{4\pi\epsilon_0 R^4}$

Sol: $q = \int_0^{r_1} 4\pi r^2 dr \frac{Q}{\pi R^4} r = \frac{Qr_1^4}{R^4}$
 $E = \frac{Qr_1^4}{R^4} \cdot \frac{1}{4\pi\epsilon_0 r_1^2} = \frac{Qr_1^2}{4\pi\epsilon_0 R^4}$

9. The transition from the state $n = 4$ to $n = 3$ in a ---

Ans : $5 \rightarrow 4$

Sol: Transition $n_x \rightarrow n_{x-1}$ is minimum for larger x .

10. One kg of a diatomic gas is at a pressure of -----

Ans : $5 \times 10^4 \text{ J}$

Sol: $PV = nRT$

$$P \frac{m}{\rho} = nRT$$

$$V = \frac{5}{2} nRT = \frac{5}{2} \rho \frac{m}{V} = 5 \times 10^4 \text{ J}$$

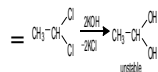
11. Statement 1 : The temperature dependence of resistance is usually -----

Ans : Statement – 1 is false, Statement – 2 is true.

12. The magnitude of the magnetic field (B) due to the loop -----

$$\text{Ans : } \frac{\mu_0 I (b-a)}{24ab}$$

$$\text{Sol: } B = \frac{\mu_0 I}{2} \left(\frac{30}{360} \right) \left(\frac{1}{a} - \frac{1}{b} \right)$$



13. Due to the presence of the current I_1 at the -----
--

Ans : The forces on AD and BC are zero.

Sol: \vec{B} vector is parallel to each current element along AD and BC
 $I d\vec{\ell} \times \vec{B} = 0$

14. A mixture of light, consisting of wavelength 590 nm and an unknown

Ans : 442.5 nm

$$\begin{aligned} \text{Sol: } 3\beta_{\text{known}} &= 4\beta_{\text{unknown}} \\ \therefore 3\lambda_{\text{known}} &= 4\lambda_{\text{unknown}} \\ \lambda_{\text{unknown}} &= \frac{3}{4}\lambda_{\text{known}} = 442.5 \text{ nm} \end{aligned}$$

15. Two points P and Q are maintained at the potentials -----

Ans : $2.24 \times 10^{-16} \text{ J}$

$$\begin{aligned} \text{Sol: } W &= q\Delta V \\ &= -1.6 \times 10^{-19} \times (-14) \times 100 \\ &= 2.24 \times 10^{-16} \text{ J} \end{aligned}$$

16. The surface of a metal is illuminated with the light -----

Ans : 1.41 eV

$$\text{Sol: } h\nu = \frac{1240}{400} = 3.1 \text{ eV}$$

$$KE_{\text{max}} = 1.68 \text{ eV}$$

$$\therefore \phi = 3.1 - 1.68 = 1.41 \text{ eV}$$

17. A particle has an initial velocity of -----

Ans : $7\sqrt{2}$ units

$$\begin{aligned} \text{Sol: } \vec{v} &= \vec{u} + \vec{at} \\ &= (3\hat{i} + 4\hat{j}) + 10(0.4\hat{i} + 0.3\hat{j}) \\ |\vec{v}| &= |(7\hat{i} + 7\hat{j})| \end{aligned}$$

18. A motor cycle starts from rest and accelerates ---

Ans : 98 m

$$\text{Sol: } f = f_0 \left(\frac{330 - u_f}{330} \right) = 0.94 f_0$$

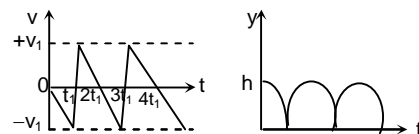
$$330 - u_f = 0.94 \times 330$$

$$u_f = 19.8 \text{ m/s}$$

$$\therefore S = \frac{1}{2} at^2 = 98 \text{ m}$$

19. Consider a rubber ball freely falling from a height -----

Ans :

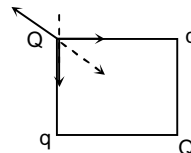


Sol: Velocity reverses instantaneously; downward direction of velocity needs to be treated as negative according to sign conservation.

20. A charge Q is placed at each of the opposite corners of a -----

Ans : $-2\sqrt{2}$

Sol:



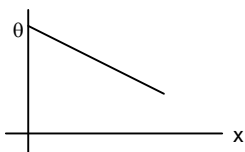
Magnitudes of forces

$$\frac{1}{K} \frac{Q^2}{(\sqrt{2}a)^2} = \frac{1}{K} \frac{Qq}{a^2} \cdot 2 \cdot \frac{1}{\sqrt{2}}$$

$$\frac{Q}{q} = -2\sqrt{2}$$

21. A long metallic bar is carrying heat from one of its ends to the other -----

Ans :



Sol: $\Delta Q = -KA \frac{\Delta T}{\Delta X}$
 $\Delta Q \text{ constant} \Rightarrow \frac{\Delta T}{\Delta X} \text{ constant.}$

22. A transparent solid cylindrical rod has a refractive -----

Ans : $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Sol: $\sin(90 - r) = \frac{1}{\mu} = \frac{\sqrt{3}}{2}$

$\cos r = \frac{\sqrt{3}}{2} \Rightarrow r = 30^\circ$

$\frac{\sin i}{\sin r} = \mu = \frac{2}{\sqrt{3}}$

$\sin i = \frac{1}{\sqrt{3}}$

$i = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

23. Three sound waves of equal amplitudes have frequencies -----

Ans : 1

Sol: Resultant frequency of $(\nu - 1)$ and $(\nu + 1)$ with ν cannot produce any beat. Beat can produce either with ν and $(\nu + 1)$ or ν and $(\nu - 1)$ where the beat frequency is 1.

24. The height at which the acceleration due to gravity becomes -----

Ans : 2 R

Sol: $g = \frac{GM}{R^2}$

$\frac{g}{9} = \frac{GM}{(R+h)^2} \Rightarrow \frac{(R+h)^2}{R^2} = 9$

$h = 2 R$

25. Two wires are made of the same material and have the same volume -----

Ans : 9 F

Sol: $\frac{\frac{F}{A}}{\frac{\Delta \ell}{\ell}} = Y \Rightarrow F = YA \frac{(\Delta \ell)}{\ell}$

$V_1 = A\ell ; V_2 = 3A \cdot \frac{\ell}{3} = V_1$

$\frac{\frac{F'}{3A}}{\frac{\Delta \ell}{\ell/3}} = Y \Rightarrow F' = Y \cdot 3A \frac{(\Delta \ell)}{\ell} \cdot 3 = 9 F$

26. In an experiment the angles are required to be measured using an -----

Ans : One minute

Sol: $\frac{1 \text{ m.s.d}}{N} = \frac{1/2}{30} = \frac{1}{60}^\circ$
 $= 1'$

27. An inductor of inductance $L = 400 \text{ mH}$ and resistors -----

Ans : $12 e^{-5t} \text{ V}$

Sol: $I = \frac{L}{R} = 0.2$

$E = E_0 e^{-t/\tau} = 12 e^{-5t}$

R_1 has no role

28. Assuming the gas to be ideal the work done -----

Ans : 400 R

Sol: $V_1 = \frac{nRT_1}{P}$

$V_2 = \frac{nRT_2}{P}$

$V_2 - V_1 = \frac{nR}{P} (T_2 - T_1)$

$P(V_2 - V_1) = nR(T_2 - T_1) = 400 R$

29. The work done on the gas in taking it from D to A is -----

Ans : -414 R

Sol: $V_A P_A = V_D P_D \Rightarrow \frac{V_A}{V_D} = \frac{P_D}{P_A} = \frac{1}{2}$

$W = nRT \ln\left(\frac{V_2}{V_1}\right) = 2R \cdot 300 \ln\left(\frac{1}{2}\right)$

$= -414 R$

30. The net work done on the gas in the cycle ----

Ans : 276 R

Sol: $W_{AB} = 400 \text{ R}$; $W_{CD} = -400 \text{ R}$
 $W_{DA} = -414 \text{ R}$;
 $W_{BC} = nRT \ln \frac{V_C}{V_B}$
 $= 2R \cdot 500 \cdot \ln 2$
 $= 693 \text{ R}$
 $W_{\text{total}} = (693 - 414) \text{ R}$
 $= 279 \text{ R}$

PART B – CHEMISTRY

31. Knowing that the Chemistry of lanthanoids is incorrect?

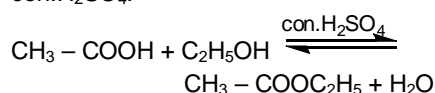
Ans: Ln(III) compounds are generally colourless.

Sol: Except $\text{La}^{3+}, \text{Lu}^{3+}$, other lanthanoids exhibit colour in both solid state and in aqueous solution.

32. A liquid was mixed with ethanol and a drop of concentrated H_2SO_4 was added.....

Ans: CH_3COOH

Sol: Acetic acid forms ethyl acetate (ester) having fruity smell when heated with ethanol in presence of a drop of con. H_2SO_4 .



33. Arrange the carbanions, $(\text{CH}_3)_3\text{C}^-$, CCl_3^- , $(\text{CH}_3)_2\text{CH}^-$,

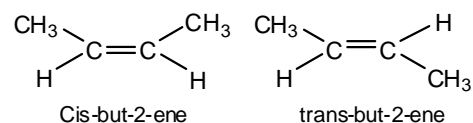
Ans: $\text{CCl}_3^- > \text{C}_6\text{H}_5\text{CH}_2^- > (\text{CH}_3)_2\text{CH}^- > (\text{CH}_3)_3\text{C}^-$

Sol: CCl_3^- is stable due to the $-I$ effect of three chlorine atoms. Further it is stabilised by the resonance due to the presence of d-orbitals on chlorine. $\text{C}_6\text{H}_5\text{CH}_2^-$ is stabilised by resonance. $(\text{CH}_3)_3\text{C}^-$ is the least stable ion due to the $+I$ effect of three methyl groups.

34. The alkene that exhibits geometrical

Ans: 2-butene

Sol:



35. In which of the following the sequence is not strictly.....

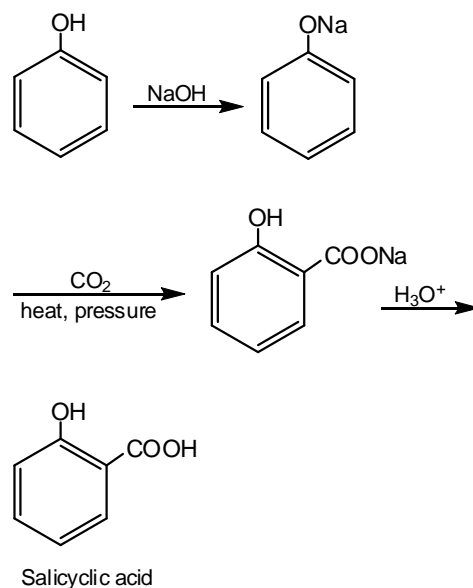
Ans: $\text{NH}_3 < \text{PH}_3 < \text{AsH}_3 < \text{SbH}_3$: increasing basic strength.

Sol: NH_3 is the most basic. Basic strength decreases from NH_3 to SbH_3 .

36. The major product obtained of phenol with sodium hydroxide

Ans: Salicylic acid

Sol:



It is Kolbe's reaction.

37. Which of the following is incorrect physisorptions?

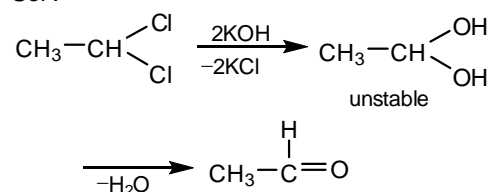
Ans: Enthalpy of adsorption ($\Delta H_{\text{adsorption}}$) is low and positive.

Sol: ΔH is negative for adsorption.

38. on heating with aqueous KOH, produces acetaldehyde?

Ans: CH_3CHCl_2

Sol :



39. In an atom, an electron is moving

Ans: $1.92 \times 10^{-3} \text{ m}$

$$\begin{aligned} \text{Sol : } \Delta x \text{ m} \Delta v &= \frac{h}{4\pi} \quad \Delta v = \frac{0.005}{100} \times 600 = 0.03 \\ \Delta x &= \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 0.03} \\ &= 1.92 \times 10^{-3} \text{ m} \end{aligned}$$

40. In a fuel cell methanol is used

Ans: 97%

$$\begin{aligned} \text{Sol : Efficiency} &= \frac{\Delta G}{\Delta H} \\ \Delta G &= (-394.4 \text{ kJ} + 2 \times -237.2 \text{ kJ}) \\ &\quad - (-166.2 \text{ kJ}) = -702.6 \text{ kJ} \\ \Delta H &= -726 \text{ kJ} \\ \text{Efficiency} &= \frac{-702.6}{-726} \times 100 = 97\% \end{aligned}$$

41. Two liquids X and Y form an ideal solution.

Ans: 400 and 600

$$\begin{aligned} \text{Sol : } \frac{1}{4}X + \frac{3}{4}Y &= 550 \\ \frac{1}{5}X + \frac{4}{5}Y &= 560 \\ \text{Solving X} &= 400 \text{ mm Hg} \\ Y &= 600 \text{ mmHg} \end{aligned}$$

42. The half life period of a first order

Ans: 46.06 minutes

$$\begin{aligned} \text{Sol : } \lambda &= \frac{2.303}{t} \log \frac{N_0}{N_t} \\ \frac{0.693}{t_{1/2}} &= \frac{2.303}{t} \log \frac{100}{1} \\ \frac{0.693}{6.93} &= \frac{2.303}{t} \times 2 \\ t &= 46.06 \text{ minutes} \end{aligned}$$

43. Given:

$$E_{\text{Fe}^{3+}/\text{Fe}}^0 = -0.036 \text{ V,$$

Ans: 0.770 V

$$\begin{array}{l} \text{Sol :} \\ \text{Fe}^{3+} + 3e^- \rightarrow \text{Fe} \quad E^\circ(\text{V}) \quad nE^\circ(\text{V}) \\ \text{Fe}^{2+} + 2e^- \rightarrow \text{Fe} \quad -0.036 \quad -0.108 \dots (1) \\ \text{Fe}^{2+} + 2e^- \rightarrow \text{Fe} \quad -0.439 \quad -0.878 \dots (2) \\ (1) - (2) \rightarrow \\ \text{Fe}^{3+} + 1e^- \rightarrow \text{Fe}^{2+} \quad 0.77 \quad 0.77 \end{array}$$

44. On the basis of the following

Ans: -228.88 kJ

$$\begin{aligned} \text{Sol : Given } \text{H}_2\text{O}_{(l)} &\rightarrow \text{H}_{(aq)}^+ + \text{OH}_{(aq)}^- \\ \Delta H &= 57.32 \text{ kJ} \\ \therefore 57.32 \text{ kJ} &= \Delta_f H_{\text{OH}_{(aq)}^-}^0 - (-286.2 \text{ kJ}) \\ \Delta_f H_{\text{OH}_{(aq)}^-}^0 &= -228.88 \text{ kJ} \end{aligned}$$

45. Copper crystallises in fcc with a

Ans: 127 pm

$$\begin{aligned} \text{Sol : } \sqrt{2} a &= 4r \\ r &= \frac{\sqrt{2} \times 361 \text{ pm}}{4} = 127 \text{ pm} \end{aligned}$$

46. has an optical isomer?

Ans: $[\text{Co}(\text{en})_2(\text{NH}_3)_2]^{3+}$

Sol : Cis $[\text{Co}(\text{en})_2(\text{NH}_3)_2]^{3+}$ is optically active.

47. Solid $\text{Ba}(\text{NO}_3)_2$ is gradually dissolved

Ans: $5.1 \times 10^{-5} \text{ M}$

$$\begin{aligned} \text{Sol : } K_{\text{sp}}(\text{BaCO}_3) &= [\text{Ba}^{2+}] [\text{CO}_3^{2-}] = 5.1 \times 10^{-9} \\ \therefore [\text{Ba}^{2+}] &= \frac{5.1 \times 10^{-9}}{10^{-4}} \\ &= 5.1 \times 10^{-5} \text{ M} \end{aligned}$$

48. Xenon compounds is not feasible?

Ans: $\text{XeO}_3 + 6\text{HF} \rightarrow \text{XeF}_6 + 3\text{H}_2\text{O}$

Sol : XeO_3 reacts with aqueous alkali but not with acids.

49. Using MO theory shortest bond length?

Ans: O_2^{2+}

Sol : In O_2^{2+} the bond order is three, hence the shortest bond length.

50. In context with the transition elements, the following statements is incorrect?

Ans: In the highest oxidation states the transition metals show basic character and form cationic complexes.

Sol : In the highest oxidation state the transition metals show acidic character.

51. Calculate the wavelength

Ans: 0.40 nm

$$\text{Sol : } \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ Js}}{1.67 \times 10^{-27} \text{ kg} \times 10^3 \text{ ms}^{-1}} = 4 \times 10^{-10} \text{ m} = 0.4 \text{ nm}$$

52. A binary liquid solution is prepared by

Ans: The solution is non-ideal, showing +ve deviation from Raoult's Law.

Sol : n-heptane reduces the attraction between the ethanol molecules and there by mutually increases the vapourising tendency.

53. The number of stereoisomers possible

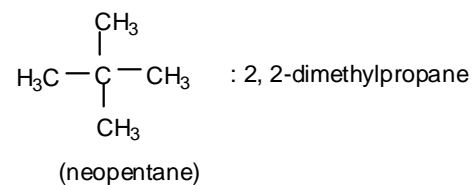
Ans: 4

Sol : The compound $CH_3-CH=CH-CH(OH)-Me$ can show geometrical as well as optical isomerism. Hence the cis and trans isomers can have the (+) and (-) forms. So the total number of stereoisomers is 4.

54. The IUPAC name

Ans: 2, 2-dimethylpropane

Sol :



55. correct order of ionic radius is:

Ans: $Na^+ > Li^+ > Mg^{2+} > Be^{2+}$

Sol : The ionic radii in pm are

$$\begin{array}{ll} Be^{2+} = 31 & Mg^{2+} = 72 \\ Li^+ = 76 & Na^+ = 102 \end{array}$$

56. The two functional groups present

Ans: >C=O and $-\text{OH}$

Sol : Carbohydrates are polyhydroxy carbonyl compounds.

57. The bond dissociation energy of B - F in

Ans: Significant $p\pi-p\pi$ interaction between B and F in BF_3 whereas there is no possibility of such interaction between C and F in CF_4 .

Sol : $p\pi-p\pi$ back bonding in BF_3 makes B - F bonds stronger whereas in CF_4 no such back bonding is possible, as there is no vacant p-orbital in carbon.

58. In Cannizzaro reaction the slowest step is:

Ans: the transfer of hydride to the carbonyl group

Sol : The rate determining step in Cannizzaro reaction is the transfer of hydride ion from the anion formed by the addition of OH^- to the carbonyl group of the other aldehyde molecule.

59. represents linkage isomers?

Ans: $[Pd(PPh_3)_2(NCS)_2]$ and $[Pd(PPh_3)_2(SCN)_2]$

Sol : Linkage of the ambidentate ligand CNS is through two different sites in the complexes.

60. Buna – N synthetic rubber

Ans: $H_2C = CH - CN$ and
 $H_2C = CH - CH = CH_2$

Sol : Buna – N is a copolymer of 1, 3-butadiene and acrylonitrile ($CH_2 = CH - CN$)

PART C – MATHEMATICS

61. Let a, b, c be such that $b(a + c) \neq 0$

Ans: any odd integer

$$\text{Sol: } \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$

$$\Delta + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}$$

$$\Delta + (-1)^n \Delta = 0 \Rightarrow n \text{ is any odd integer}$$

62. If the mean deviation of the numbers 1, ...

Ans: 10.1

Sol: The mean of the given numbers is the middle term $1 + 50d$

$$\therefore \text{M.D} = \frac{2(d + 2d + \dots + 50d)}{101}$$

$$= \frac{2}{101} d(1 + 2 + \dots + 50)$$

$$= \frac{2}{101} d \frac{(50) \times (51)}{2} = \frac{2550}{101} d = 25.5d$$

$$d = 10.1$$

63. If the roots of the equation $bx^2 + cx + a = 0$

Ans: greater than $-4ab$

Sol: Roots of $bx^2 + cx + a = 0$ are imaginary
 $\Rightarrow c^2 - 4ab < 0$
 $\Rightarrow c^2 < 4ab$ — (1)
 Now the extremum value of $3b^2 x^2 + 6bcx + 2c^2$ is

$$\frac{-((6bc)^2 - 4 \times 3b^2 \times 2c^2)}{4 \times 3b^2}$$

$$= \frac{-12b^2 c^2}{12b^2} = -c^2 > -4ab \text{ by (1)}$$

64. Let A and B denote the statements

Ans: both a and b are true

$$\text{Sol: } \sum(\cos\alpha \cos\beta + \sin\alpha \sin\beta) = \frac{-3}{2}$$

$$\therefore \sum\cos^2\alpha + \sum\sin^2\alpha + 2\sum\cos\alpha \cos\beta + 2\sum\sin\alpha \sin\beta = 0$$

$$\text{i.e., } (\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$$

$$\text{i.e., } \cos\alpha + \cos\beta + \cos\gamma = 0 \text{ and } \sin\alpha + \sin\beta + \sin\gamma = 0$$

65. The lines $p(p^2 + 1)x - y + q = 0$

Ans: Exactly one value of p

Sol: Given two lines are parallel
 slope $m_1 = p(p^2 + 1)$, $m_2 = -(p^2 + 1)$
 Since $m_1 = m_2$ we get $p = -1$
 \therefore exactly one value of p

66. If A, B and C are three sets such that $A \cap B$

Ans: $B = C$

Sol: $A \cap B = A \cap C$ and $A \cup B = A \cup C \Rightarrow B = C$

67. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors

Ans:

$$\text{Sol: } 3p^2[u \ v \ w] - pq[v \ w \ u] - 2q^2[w \ v \ u] = 0$$

$$\Rightarrow 3p^2[u \ v \ w] - pq[u \ v \ w] + 2q^2[u \ v \ w] = 0$$

$$\Rightarrow [u \ v \ w] (3p^2 - pq + 2q^2) = 0$$

$$3p^2 - pq + 2q^2 = 0 (\because [u \ v \ w] \neq 0)$$

Not satisfying for any real values of p and q

68. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the ...

Ans: $(-6, 7)$

Sol: The normal to the plane is perpendicular to the line
 $\therefore 3 \times 1 + 3 \times -5 + 2x - \alpha = 0$
 $\Rightarrow \alpha = -6$
 $(2, 1, -2)$ lies in the plane
 $\therefore 2 + 3 \times 1 + 6 \times -2 + \beta = 0$

$$\Rightarrow \beta = 7$$

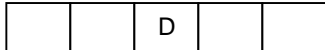
$$\therefore (\alpha, \beta) = (-6, 7)$$

69. From 6 different novels and 3 different

Ans: atleast 1000

Sol: 6 novels, 3 dictionaries

To select 4 novels and 1 dictionary. This can be done in ${}^6C_4 \times {}^3C_1$ ways = $15 \times 3 = 45$ ways



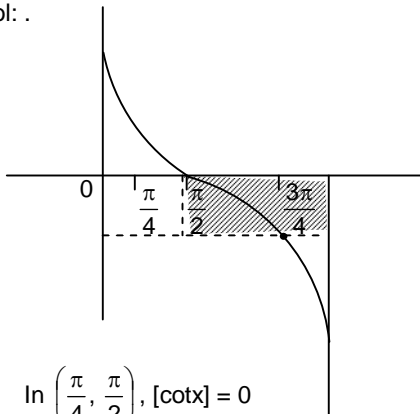
Dictionary is to be placed in the middle. 4 novels can be arranged in the 4 places in $4! = 24$ ways

\therefore Number of ways = $45 \times 24 = 1080$

70. $\int_0^{\pi} [\cot x] dx$, where $[\cdot]$ denotes the greatest ...

Ans: $-\frac{\pi}{2}$

Sol: .



$$\ln \left(\frac{\pi}{4}, \frac{\pi}{2} \right), [\cot x] = 0$$

$$\ln \left(\cot^{-1} 2, \frac{\pi}{4} \right), [\cot x] = 1$$

$$\ln \left[\frac{\pi}{2}, \frac{3\pi}{4} \right], [\cot x] = -1$$

$$\int_0^{\pi/4} [\cot x] dx = - \int_{\pi/2}^{\pi} [\cot x] dx + \int_{\pi/2}^{\pi} (-1) dx$$

$$\therefore \int_0^{\pi} [\cot x] dx = (-1) \left(\pi - \frac{\pi}{2} \right) = -\frac{\pi}{2}$$

71. For real x, let $f(x) = x^3 + 5x + 1$, then....

Ans: f is one-one and onto R

Sol: $f'(x) = 3x^2 + 5 > 0 \forall x \Rightarrow f(x)$ is increasing

Thus f is one-one

Also $f'(x) \neq 0$ for any real x. Thus f attains neither a maximum nor a minimum at any real points. That is f is an ever increasing function . Hence it is one-one and onto

72. In a binomial distribution $B \left(n, p = \frac{1}{4} \right)$, if

Ans: $\frac{1}{\log_{10} 4 - \log_{10} 3}$

Sol: $p = \frac{1}{4} \therefore q = \frac{3}{4}$

$$P(X = x) = {}^nC_x p^x q^{n-x}$$

Given that $1 - P(x=0) \geq \frac{9}{10}$

$$\therefore P(X=0) \leq 1 - \frac{9}{10} = \frac{1}{10}$$

$$\text{i.e., } {}^nC_0 p^0 q^{n-0} \leq \frac{1}{10}$$

$$\text{i.e., } \left(\frac{3}{4} \right)^n \leq \frac{1}{10}$$

$$\text{i.e., } n \log_{10} \frac{3}{4} \leq -1$$

$$\therefore n \leq \frac{-1}{\log_{10} \frac{3}{4}}$$

$$= \frac{1}{\log_{10} 4 - \log_{10} 3}$$

73. If P and Q are the points of intersection of

Ans: all except one value of p

Sol: Equation of circle passing through the intersection of the given circles is

$$\text{i.e., } x^2 + y^2 + 3x + 7y + 2p - 5 + \lambda(x + 5y + 2p - 5 + p^2) = 0$$

Given that it passes through (1, 1)

$$\text{i.e., } (7 + 2p) + \lambda(1 + p)^2 = 0$$

$$\Rightarrow \lambda = \frac{-(7 + 2p)}{(1 + p)^2}$$

i.e., λ does not exist at $p = -1$

\therefore all except one value of p

74. The projection of a vector on the three

Ans: $\left(\frac{6}{7}, \frac{-3}{7}, \frac{2}{7} \right)$

Sol: The given vector is $6i - 3j + 2k$. Its direction

$$\text{cosines are } \left(\frac{6}{\sqrt{6^2 + 3^2 + 2^2}}, \frac{-3}{\sqrt{6^2 + 3^2 + 2^2}}, \frac{2}{\sqrt{6^2 + 3^2 + 2^2}} \right)$$

$$= \left(\frac{6}{7}, \frac{-3}{7}, \frac{2}{7} \right)$$

75. If $\left| z - \frac{4}{z} \right| = 2$, then the maximum value

Ans: $\sqrt{5} + 1$

Sol: $\left| z + \frac{4}{z} \right| = 2 \Rightarrow |z| - \left| \frac{4}{z} \right| \leq 2$

$$\Rightarrow |z|^2 - 2|z| - 4 \leq 0$$

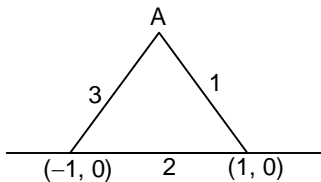
$$\Rightarrow |z| \in [0, 1 + \sqrt{5}]$$

\therefore Maximum value of $|z|$ is $1 + \sqrt{5}$

76. Three distinct points A, B and C are

Ans: $\left(\frac{5}{4}, 0 \right)$

Sol:



A, B, C are line on the ellipse with foci $(-1, 0)$ so the given centre of the $\triangle ABC$ is

$\left(\frac{5}{4}, 0 \right)$

77. The remainder left out when $8^{2n} - (62)^{2n+1} \dots$

Ans: 2

Sol: $8^{2n} - (62)^{2n+1} = (9-1)^{2n} - (63-1)^{2n+1}$

$$= [9^{2n} - {}^{2n}C_1 \cdot 9^{2n-1} + \dots + (-1)^{2n}] - [63^{2n+1} - {}^{(2n+1)}C_1 63^{2n} + \dots + (-1)^{2n+1}]$$

$$= [M(9) + 1] - [M(9) - 1]$$

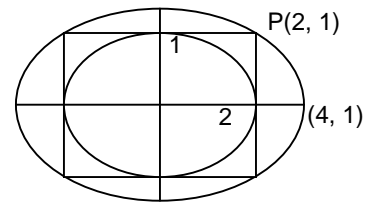
$$= M(9) + 2$$

\therefore The remainder when $8^{2n} - (62)^{2n+1}$ is divided by 9 is 2

78. The ellipse $x^2 + 4y^2 = 4$ is inscribed ...

Ans: $x^2 + 12y^2 = 16$

Sol:



Equation of required ellipse is $\frac{x^2}{4^2} + \frac{y^2}{b^2} = 1$

It passes through $(2, 1) \Rightarrow b = \frac{3}{4}$

\therefore Equation is $x^2 + 12y^2 = 16$

79. The sum to infinity of the series

Ans: 3

Sol: Put $S = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots$

$$\frac{1}{3}S = \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots$$

$$\frac{2}{3}S = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots$$

$$= \frac{2}{3} + \frac{4}{9} \cdot \frac{1}{1 - \frac{1}{3}}$$

$$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$S = 2$

\therefore required sum = $1 + 2 = 3$

80. The differential equation which represents the ...

Ans: $yy'' = (y')^2$

Sol: $y = c_1 e^{c_2 x} \Rightarrow \log y = \log c_1 + c_2 x$

$$\therefore \frac{1}{y} \cdot y' = c_2$$

Again differentiating

$$\frac{y \cdot y'' - (y') \cdot (y')}{(y)^2} = 0$$

i.e., $yy'' = (y')^2$

81. One ticket is selected at random from 50

Ans: $\frac{1}{14}$

Sol: Product is zero. Total number of selections are 00, 01, 02, ..., 10, 20, 30, 40. There are

14 cases out of which only 08 (sum of digits should be 8) is the favorable case.

$$\therefore \text{Required probability} = \frac{1}{14}$$

82. Let y be an implicit function of x defined ...

Ans: -1

Sol: Put $x^x = t$.

$$\text{Then } t^2 - 2t \cot y + \cot^2 y - \operatorname{cosec}^2 y = 0$$

$$\Rightarrow (t - \cot y)^2 = \operatorname{cosec}^2 y$$

$$\Rightarrow t - \cot y = \pm \operatorname{cosec} y$$

$$\Rightarrow t = \cot y \pm \operatorname{cosec} y$$

$$\Rightarrow x^x = \cot y \pm \operatorname{cosec} y$$

Differentiating, $x^x (1 + \log x)$

$$= (-\operatorname{cosec}^2 y \pm \operatorname{cosec} y \cot y) y'$$

$$= -\operatorname{cosec} y (\operatorname{cosec} y \pm \cot y) y'$$

$$= -\operatorname{cosec} y x^x y'$$

$$\Rightarrow y' = \frac{-(1 + \log x)}{\operatorname{cosec} y}$$

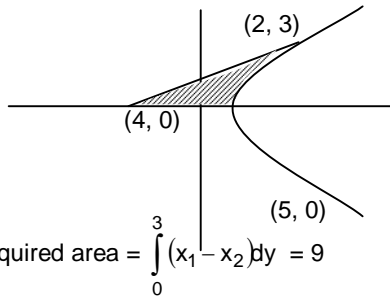
$$\text{When } x = 1, 1 - 2 \cot y - 1 = 0 \Rightarrow y = \frac{\pi}{2}$$

$$\therefore y'(1) = \frac{-(1 + \log 1)}{\operatorname{cosec} \frac{\pi}{2}} = -1$$

83. The area of the region bounded by the

Ans: 9

Sol:



$$\text{Required area} = \int_0^3 (x_1 - x_2) dy = 9$$

84. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such

Ans: $p(-1)$ is the minimum and $p(1)$ is the maximum of P

$$\text{Sol: } p(x) = x^4 + ax^3 + bx^2 + cx + d$$

$$p'(0) = 0 \Rightarrow (4x^3 + 3ax^2 + 2bx + c) \text{ at } x = 0 \text{ is zero}$$

$$\Rightarrow c = 0$$

$$\text{Given } p(-1) < p(1)$$

Suppose \exists a $S \in [-1, 1]$ where $p(x)$ is an extremum

$$\therefore p'(S) = 0 \Rightarrow S = 0 \quad (\because p'(x) = 0 \Rightarrow x = 0)$$

$$\Rightarrow \text{only } 0 \text{ can be an extremum in } (-1, 1)$$

$$\text{But } p''(0) = (12x^2 + 6ax)_{x=0} = 0$$

$$\therefore x = 0 \text{ is not an extremum}$$

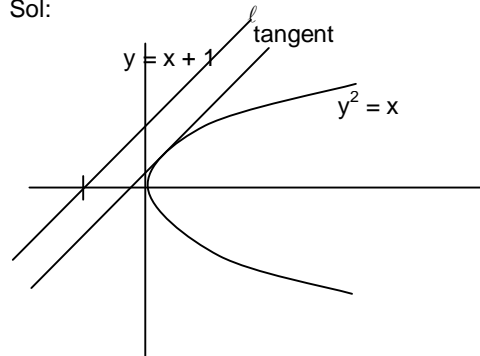
$\Rightarrow \exists$ no $x \in [-1, 1]$ where $p(x)$ is an extremum

Thus in $[-1, 1]$, $p(-1)$ is the minimum and $p(1)$ is the maximum

85. The shortest distance between the line ...

$$\text{Ans: } \frac{3\sqrt{2}}{8}$$

Sol:



Consider $x = y^2$. Slope of the curve = $\frac{1}{2y}$

Shortest distance = perpendicular distance
 \Rightarrow slope of tangent to the curve should be same as slope of line.

$$\therefore \frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$$

$$\Rightarrow x = \frac{1}{4}$$

Distance of $(\frac{1}{4}, \frac{1}{2})$ from line $x + 1 = y$ is

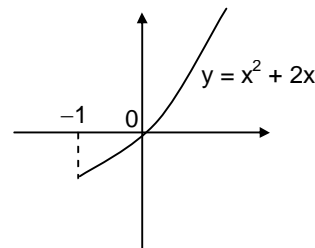
$$\frac{3\sqrt{2}}{8}$$

86. Let $f(x) = (x + 1)^2 - 1, x \geq -1$

Ans: Statement -1 is true, Statement -2 is true

Statement -2 is a correct explanation for Statement -1

Sol:



$$f(x) = (x + 1)^2 - 1, x \geq -1$$

$$f(x) = x^2 + 2x, x \geq -1$$

Obviously f is a bijection. (Refer graph)

\therefore Inverse of f exists

$$f^{-1}(x) = \sqrt{x + 1} - 1$$

$$\begin{aligned} \text{Let } f(x) &= f^{-1}(x) \\ x^2 + 2x &= \sqrt{x+1} - 1 \\ \therefore x^2 + 2x + 1 &= \sqrt{x+1} \\ \Rightarrow (x+1)^2 &= \sqrt{x+1} \end{aligned}$$

Possible only for $x = 0$ and $x = -1$
Statement 1 true and as shown above follows from statement 2

87. Let $f(x) = x|x|$ and $g(x) = \sin x$

Ans: Statement -1 is true, Statement 2 is false

$$\text{Sol: } f(x) = \begin{cases} x^2 & x > 0 \\ -x^2 & x < 0 \end{cases}$$

$$g(x) = \sin x$$

Since $|x|$ is not differentiable at $x = 0$

$g \circ f$ is not twice differentiable

\therefore statement 2 is false

But statement 1 is true

88. Statement – 1: The variance of the first n ...

Ans: Statement - 1 is true, Statement - 2 is false

Sol: Statement 2 is true (Direct result)

$$x_i = 2, 4, 6, \dots, 2n$$

$$\begin{aligned} \sigma^2 &= \frac{1}{n} \sum x_i^2 - \left(\frac{\sum x}{n} \right)^2 \\ &= \frac{2^2}{n} \sum n^2 - \frac{2^2}{n^2} (\sum n)^2 \\ &= \frac{2^2}{n} \left(\frac{n(n+1)(2n+1)}{6} - \frac{1}{n} \left(\frac{n(n+1)}{2} \right)^2 \right) \\ &= \frac{2^2}{n} \cdot \frac{n(n+1)}{2} \left(\frac{(2n+1)}{3} - \frac{(n+1)}{2} \right) \\ &= 2(n+1) \left(\frac{4n+2-3n-3}{6} \right) \end{aligned}$$

$$= \frac{n^2 - 1}{3}$$

\therefore Statement (1) is false

89. Statement – 1 : $\sim(p \leftrightarrow \sim q)$ is equivalent to ...

Ans: Statement -1 is true and Statement - 2 is false

Sol: Truth table for $\sim(p \leftrightarrow \sim q)$

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

\therefore Statement I is true and II is false

90. Let A be a 2×2 matrix

Ans: Statement - 1 is true, Statement - 2 is true;
Statement - 2 is not a correct explanation for Statement - 1

Sol: For a 2×2 matrix indeed $|\text{adj } A| = |A|$

Since $|\text{adj } A| = |A|^{n-1}$, where n is the order of the matrix

Statement 2 is true

In general, for a n^{th} order matrix, $\text{adj}(\text{adj } A) \neq A$.

But for a 2×2 matrix $\text{adj}(\text{adj } A) = A$

Statement 1 true

But Statement 1 does not follow from

Statement 2