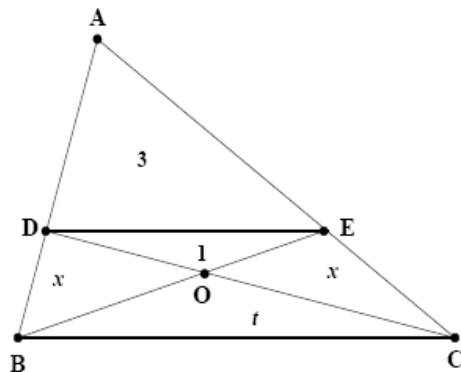


Q.1

Solution:



We denote the area of triangle PQR by $[PQR]$. We see that $[BOD]$ and $[COE]$ are equal. Let the common value be x , and let $[BOC] = t$. Using the fact that the ratio of areas of two triangles having equal altitudes is the same as the ratio of their respective bases, we obtain

$$\frac{x}{1} = \frac{BO}{OE} = \frac{t}{x}.$$

This gives $t = x^2$. Now ADE and ABC are similar so that

$$\frac{[ADE]}{[ABC]} = \frac{DE^2}{BC^2} = \frac{[ODE]}{[OBC]},$$

since ODE and OCB are also similar. This implies that

$$\frac{3}{4 + 2x + t} = \frac{1}{t},$$

which simplifies to $t = 2 + x$. Using $t = x^2$, we get a quadratic in x : $x^2 - x - 2 = 0$. Its solutions are $x = 2$ and $x = -1$. Since x cannot be negative, $x = 2$ and $t = 4$. Thus $[ABC] = 4 + 2x + t = 4 + 4 + 4 = 12$.

Q.2

Solution: Let t be the total number of CD's that Leela and Madan together sold. Then they obtain t^2 rupees together. Since Leela is the first one to take 10 rupees and also the last one to take 10 rupees, we must have

$$t^2 = 10(\text{an odd number}) + (\text{a number less than } 10).$$

Suppose $t = 10q + r$, where r is the remainder when t is divided by 10. Then $t^2 = 100q^2 + 20qr + r^2$. Comparing, we conclude that

$$r^2 = 10(\text{an odd number}) + (\text{a number less than } 10).$$

But we know that $0 \leq r < 10$. Taking $r = 0, 1, 2, \dots, 9$, we see that $r = 4$ or 6 (for other values of r , tens place in r^2 is even). But then $r^2 = 16$ or 36 . Hence the amount left for Madan at the end is 6 rupees.

Q.3

(a) Divide the $n+1$ numbers $1, 11, 111, \dots, 111 \dots 1$ (all having only 1 as digits) by n . Among the $n+1$ remainders so obtained, two must be equal as the possibilities for remainders are $0, 1, 2, \dots, n-1$ which are n in number. Thus there must be two numbers $x = 11 \dots 1$ and $y = 11 \dots 1$ having say j digits and k digits respectively which leave the same remainders after division by n . We may take $j < k$. Now we see that $y - x$ is divisible by n . But $y - x = 11 \dots 100 \dots 0$ where there are $k - j$ number of 1's and remaining zeros. Since n is coprime to 10, we see that n divides $m = 11 \dots 1$, a number having only 1's as its digits.

(b) If p/q is any rational number ($p > 0, q > 0$), then we may write $q = 2^r 5^s t$, where t is coprime to 10. Choose a number m having only 1's as its digits and is divisible by t . Consider $9m$, which has only 9 as its digits and is still divisible by t . Let $k = 9m/t$. We see that

$$qk = 9m2^r 5^s = (10^c - 1)2^r 5^s,$$

where c is the number of digits in m . Hence we can find d such that $qd = 10^b(10^c - 1)$ (multiply by a suitable power of 2 if $s > r$ and by a suitable power of 5 if $r > s$). Then

$$\frac{p}{q} = \frac{pd}{qd} = \frac{a}{10^b(10^c - 1)},$$

where $a = pd$.

Q. 4

Solution: For P: $I = I_R + I_V = V/R + V/R_V$

$$\begin{aligned} R &= \frac{V}{I} \left[\frac{R_V}{R_V - V/I} \right] \\ &= R_{\text{est}} \left[\frac{1}{1 - R_{\text{est}}/R_V} \right] \\ &\approx R_{\text{est}} \left[1 + R_{\text{est}}/R_V \right] \quad (\text{neglecting higher order terms in } R_{\text{est}}/R_V) \end{aligned}$$

$$\delta R_P = |R_{\text{est}} - R| = R_{\text{est}}^2/R_V \approx \frac{R_{\text{est}}^2}{R_V}$$

Alternatively,

$$\begin{aligned} R_{\text{est}} &= \frac{V}{I} = \frac{R_V R}{R_V + R} \\ \delta R_P &= |R_{\text{est}} - R| = R \left[\frac{R_V}{R_V + R} - 1 \right] \approx \frac{R^2}{R_V} \end{aligned}$$

For Q: $V = I(R + R_A)$

$$R = V/I - R_A = R_{\text{est}} - R_A$$

$$\delta R_Q = |R_{\text{est}} - R| = R_A$$

$$\text{If } R = \sqrt{R_A R_V}, \text{ then } \delta R_P / \delta R_Q = R_{\text{est}}^2 / (R_A R_V) = R_{\text{est}}^2 / R^2 \approx 1$$

Q. 5

Solution: (a) Object is at $2f$, so the image is formed at the same distance from the lens (20 cm) to the right. (b) Since light has to retrace its path, the mirror should be placed so that the previous image is at its center of curvature. Thus the mirror must be placed 30 cm to the right of the lens. (c) For the plane mirror, reflection forms an image 40 cm to the right of the lens. Using the lens formula, we see that the final image is formed at a distance of $40/3$ cm to the left of the lens.

Q.6

Solution: (a) $V(\theta) = mgR(1 - \cos \theta)$, (b) $mgH - mgR(1 - \cos \theta)$, (c) For $H \ll R$ the body executes SHM with a time period of $\frac{1}{2\pi}\sqrt{\frac{R}{g}}$ - the time taken for it to travel from P to Q will be a quarter of this, i.e. $\frac{1}{8\pi}\sqrt{\frac{R}{g}}$. (d) At the lowest point, the speed is given by $\frac{1}{2}mv^2 = mgH$. So, $T - mg = \frac{mv^2}{R} = \frac{2mgH}{R}$, and thus $T = mg\left(1 + \frac{2H}{R}\right)$.

Q. 7

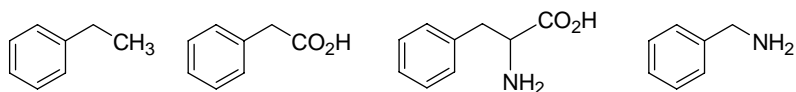
Answers:

- (a) a = 3, b = 8, c = 3, d = 2 and e = 4.
 - (b) f = 2, g = 1, h = 1.
 - (c) i = 2, j = 1, k = 1, l = 2
 - (d) 2.54 g of $I_2 = 1/100$ mole of I_2
 $= 2/100$ gm atom of Cu
 $\% \text{ Cu} = (2/100) \times (63.5/2) = 63.5\%$
-

Q. 8

Answers:

Bottle A = III, Bottle B = II, Bottle C = IV, Bottle D = I



Compound with the highest solubility in distilled water: IV

Q. 9

Answers:

(a) $2500 \times 4.184 \text{ kJ} = 10460 \text{ kJ}$

(b) 342 g of sucrose produces 5600 kJ of energy. To provide 10460 kJ we need $10460 \times \frac{342}{5600} \text{ g} = 638 \text{ g}$

$638 \text{ g} / 342 \text{ g} \times 12 \times 22.4 \text{ L} = 501 \text{ L}$

Q.10

Answers: (a) Difference in flower color is most likely due to environmental factors

(b) Perform cross breeding between the plants from Chandigarh and those from Shimla to find out whether we get any pink flower or flowers with any shade of color between pink and white in the F1 generation

(c) Grow the plants from Chandigarh in Shimla and check whether they still produce white flowers or bear pink flowers

Q.11

Answers:

(a) In experiment A, ethanol fermentation occurs producing CO_2 , turning lime water milky. Since acid is not produced the dye colour does not change.

In experiment B, lactic acid fermentation takes place, which produces acid but does not produce CO_2 . Hence dye colour changes to yellow but the lime water does not turn milky .

In experiment C, since the lime water turns milky, ethanol fermentation is occurring.

In addition, since removal of air did not affect the reaction, the fermentation is anaerobic and yeast must be the organism in the flask.

(b) In RBCs, lactic acid fermentation occurs.

Q. 12

Answers:

(a) The result of the radio-carbon dating was correct.

Reason: Vehicles running on the highway beside the house emitted carbon dioxide from the combustion of petrol or diesel, which are fossil fuels. The carbon in this carbon dioxide, coming from living material that has been converted into petroleum millions of years ago, would get assimilated into the tissues of the plant as it uses carbon dioxide from the surrounding atmosphere for photosynthesis. Therefore tissues of the plant, when used for radio-carbon dating, would show the age of the plant to be many thousands of years old.

(b) A simple experiment to test the validity of this explanation would be to collect seeds from the plant and grow them in a plot of land away from the highway or other sources of carbon dioxide coming from the burning of fossil fuels. Radio-carbon dating of plants growing from these seeds should show them as young plants.