

**Important Questions 2010**  
**Class-XII- Maths**  
**Complex Nos. & Quadratic Equations**

**Q.1.** Show that:

(a)  $1 + i^2 + i^4 + i^6 = 0$

(b)  $(1 + i)^8 (1 + 1/i)^8 = 256$

**Q.2.** Prove that:  $(1+i)^4 \left(1 + \frac{1}{i}\right)^4 = 16$

**Q.3.** Simplify:

(a)  $4\sqrt{-9} + 5\sqrt{25} - 3\sqrt{-9}$

(b)  $\sqrt{-25} + 3\sqrt{-36} + \sqrt{-49} - \sqrt{-225}$

**Q.4.** Express each one of the following in the standard form  $a + ib$ .

(i)  $\frac{5+4i}{4+5i}$  (ii)  $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$  (iii)  $\frac{1}{1 - \cos \theta + 2i \sin \theta}$

**Q.5.** Express  $\frac{1}{1 - \cos \theta + 2i \sin \theta}$  in the form  $A + iB$

**Q.6.** Represent  $\frac{(\cos x + i \sin x)(\cos y + i \sin y)}{(\cos u + i)(1 + i \tan u)}$  in the form  $A + iB$ .

**Q.7.** Express the following expression in the form of  $(a + ib)$ :

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i) - (\sqrt{3}-i\sqrt{2})}$$

**Q.8.** Prove that the following complex numbers are purely real:

(i)  $\left(\frac{2+3i}{3+4i}\right) \left(\frac{2-3i}{3-4i}\right)$

(ii)  $\left(\frac{3+2i}{2-3i}\right) + \left(\frac{3-2i}{2+3i}\right)$

**Q.9.** Reduce  $\left(\frac{1}{1-2i} + \frac{3}{1+i}\right) \left(\frac{3+4i}{2-4i}\right)$  to the standard form.

**Q.10.** Express the following complex numbers in the standard form. Also, find their conjugate:

(i)  $\frac{(1+i)^2}{3-i}$  (ii)  $\frac{(2+3i)^2}{2-i}$

**Q.11.** Find the conjugates of the following complex numbers:

(i)  $\frac{1}{3+5i}$  (ii)  $\frac{(3-i)^2}{2+i}$  (iii)  $\frac{(1+i)(2+i)}{3+1}$

**Q.12.** Write the following complex numbers in the polar form:

(i)  $-3\sqrt{2} + 3\sqrt{2}i$  (ii)  $-1 - i$

**Q.13.** Find the modulus and principal argument of  $-2i$ .

**Q.14.** Find the modulus of  $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ .

**Q.15.** Find the modulus and argument of the following complex numbers and hence express each of them in the polar form:

(i)  $\frac{1+2i}{1-3i}$  (ii)  $\frac{1-3i}{1+2i}$

**Q.16.** Find conjugate and modulus of  $\frac{(1+i)^{2n+1}}{(1-i)^{2n-1}}$

**Q.17.** Find the conjugate of  $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

**Q.18.** Find the conjugate of  $(3-2t)(2+3t) / (1+2t)(2-t)$

**Q.19.** Show that if  $\left| \frac{z-3i}{z+3i} \right| = 1$ , then  $z$  is a real number

**Q.20.** If  $Z$  is any non-zero complex number, prove that the multiplicative inverse of  $z$  is  $\frac{\bar{z}}{|z|^2}$ . Hence

find the multiplicative inverse of  $4 - \sqrt{-9}$ .

**Q.21.** Find the multiplicative inverse of the following complex numbers:

(i)  $1 - i$  (ii)  $(1+i\sqrt{3})^2$

**Q.22.** Find multiplicative inverse of  $2 + \sqrt{3}i$

**Q.23.** Find the square root of  $-15 - 8i$ .

**Q.24.** Find Modulus and Argument of :

(1)  $Z = -1 - \sqrt{3}i$  (2)  $Z = \frac{(1+i)^{13}}{(1-i)^8}$

**Q.25.** Find the value of  $x$  and  $y$  if

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

**Q.26.** If  $Z = -2 - \sqrt{-3}$ , find value of  $2Z^4 + 5Z^3 + 7Z^2 - Z + 41$

**Q.27.** If  $x+iy = \frac{a+ib}{a-ib}$ , prove that  $x^2 + y^2 = 1$ .

**Q.28.** If  $x+iy = \frac{\sqrt{1+i}}{\sqrt{1-i}}$ , prove that  $x^2 + y^2 = 1$

**Q.29.** If  $(x+iy)^3 = u+iv$ , then show that  $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

**Q.30.** If  $(\cos \theta - i \sin \theta)^2 = x-iy$ , prove that  $x^2 + y^2 = 1$

**Q.31.** Put the following in the form  $r(\cos q + i \sin q)$ , where  $r$  is a positive real number and  $-p < q \leq p$  :

$$\frac{(1+7i)}{(2-i)^2}$$

**Q.32.** Represent the complex number  $Z = 1 + i\sqrt{3}$  in the Polar form.

**Q.33.** Express the following number in polar form:  $\frac{1+2i}{1-3i}$

**Q.34.** Convert the complex number  $\frac{-16}{1+i\sqrt{3}}$  into polar form.

**Q.35.** Convert the complex number  $Z = \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$  in the Polar form.

**Q.36.** Convert the complex number  $-\sqrt{3}+i$  in the polar form and represent them in Argand plane.

**Q.37.** If  $\left(\frac{1+i}{1-i}\right)^m = 1$ , then find the least integral value of  $m$ .

**Q.38.** Let  $Z_1 = 2-i$ ,  $Z_2 = -2+i$ , find  $\operatorname{Re}\left(\frac{Z_1 Z_2}{\bar{Z}_1}\right)$ .

**Q.39.** If  $z_1$  and  $z_2$  are  $1-i$  and  $-2+4i$  find:  $\operatorname{Im}\left[\frac{z_1 z_2}{\bar{z}_1}\right]$

**Q.40 .** If  $Z_1 = 4+7i$ ,  $Z_2 = 1-i$ , find  $\operatorname{Im}(Z_1 Z_2)$

**Q.41.** Solve:  $2x^2 - (3+7i)x - (3-9i) = 0$

**Q.42.** Solve the equation:  $2z = |z| + 2i$ .

**Q.43 .** Solve :  $\sqrt{5}x^2 + x + \sqrt{5} = 0$ .

**Q.44.** Solve for  $x \in \mathbb{C}$

(i)  $x^2 + 4ix - 4 = 0$

(ii)  $-x^2 + x - 2 = 0$

**Q.45.** For any two complex numbers  $Z_1$  and  $Z_2$  Prove that:

$$\operatorname{Re}(Z_1 Z_2) = \operatorname{Re}(Z_1) \operatorname{Re}(Z_2) - \operatorname{Im}(Z_1) \operatorname{Im}(Z_2)$$

**Q.46.** If  $z$  is the complex number then prove that:

(a)  $z + \bar{z} = 2 \operatorname{Re}(z)$

(b)  $z - \bar{z} = 2i \operatorname{Im}(z)$

**Q.47.** Find the number of non - zero integral solutions of the equation  $|1 - t| \cdot x = 2x$

**Q.48.** If  $Z$  is a complex number such that  $\frac{z-1}{z+1}$  is purely imaginary, then prove that  $|z| = 1$ .

**Q.49.** Find the complex number,  $z$  satisfying the equations:

$$\frac{|z-4|}{|z-8|} = 1 \text{ and } \frac{|z-12|}{|z-8i|} = \frac{5}{3}$$

**Q.50.** Show that if  $\left| \frac{z-3i}{z+3i} \right| = 1$ , then  $z$  is a real number

**Q.51.** If  $a + ib = \frac{c+i}{c-i}$ , where  $a, b, c$  are real, prove that,  $a^2 + b^2 = 1$  and  $\frac{b}{a} = \frac{2c}{c^2 - 1}$ .

**Q.52.** If  $|z_1| = |z_2| = 1$ , prove that  $\left| \frac{1}{z_1} + \frac{1}{z_2} \right| = |z_1 + z_2|$

**Q.53.** Find the real value of  $\theta$  such that  $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$  is (i) purely real (ii) purely imaginary.

**Q.54.** If  $a$  and  $b$  are different complex numbers with  $|b| = 1$ , find  $\left| \frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right|$ .