Important Questions 2010
Class-XII (Maths)

Q. 1. In a group there are 2 men and 3 women. 3 persons are selected at random from the group. Find the probability that 1 man and 2 women or 2 men and 1 woman are selected.

Q. 2. Bag *A* contains 6 white and 7 black balls, and another bag *B* contains 4 white and 5 black balls. One ball is drawn from the bag *A* and without noticing its colour, is put in the second bag B. A ball is then drawn from the second bag *B*. Find the probability that the ball drawn is white in colour.

Q. 3. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both balls are red or both are black?

Q. 4. From a well shuffled pack of 52 cards, 3 cards are drawn one-by-one with replacement. Find the probability distribution of number of queens.

Q. 5. A fair die is tossed twice.If the number appearing on the top is less than 3,it is a success.Find the probability distribution of successes.

Q. 6. Evaluate the following limits:-

i.
$$\lim_{x\to\frac{\pi}{6}}\left[\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}\right]$$

ii.
$$\lim_{x\to 0}\left[\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right]$$

iii.
$$\lim_{x\to 0}\frac{\log\left(1+x^3\right)}{\sin^3 2x}$$

iv.
$$\lim_{x\to\pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}$$

v.
$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{x-\frac{\pi}{4}}$$

Q. 7. If f(x)= $\begin{cases} 3ax+b; \text{ if } x>1 \\ 11; \text{ if } x=1 \\ 5ax-2b; \text{ if } x<1 \end{cases}$ is continuous at x=1, find the value of a and b.

Q. 8. Discuss the continuity of f(x)=$|x|+|x-1|$ at x=0 and x=1.

Q. 9. Differentiate by first principle:-

i. $\tan\sqrt{x+4}$

ii. $\dfrac{\sin x}{x+4}$

Q. 10.

$$Prove: \frac{d}{dx}[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}]=\sqrt{a^2-x^2}$$

Q. 11.

$$If\ x=a(\theta-\sin\theta)\ and\ y=a(1-\cos\theta),\ find\ \frac{d^2y}{dx^2}\ at\ \theta=\frac{\pi}{2}$$

Q. 12. Differentiate w.r.t. 'x':-

i. $(\log x)^x+(x)^{\log x}$

ii. $\cot^{-1}\sqrt{\dfrac{1-\sin x}{1+\sin x}}$

Q. 13. A balloon which always remains spherical is being inflated by pumping in gas at the rate of 900 cm3 / sec. Find the rate at which the radius of the balloon is increasing when the radius of the balloon is 15 cm.

Q. 14. At what points on the curve $x^2+y^2-2x-4y+1=0$, is the tangent parallel to y-axis?

Q. 15. Find the equations of the tangent and the normal to the curve $y=x^3+2x+6$ at the point whose x-coordinate is 3.

Maths Algebra

One marks questions

Q. 1. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$; write the value of x and y.

Q. 2. If A is a square matrix, such that $|A|=2$, write the value of $|AA^T|$.

Q. 3. Write the value of x for which

Q. 4. Find x and y , if

$x+y=\begin{bmatrix}5&2\\0&9\end{bmatrix}$ and $x-y=\begin{bmatrix}3&6\\0&-1\end{bmatrix}$

Q. 5. Using determinants find the value of λ so that the points $(\lambda,7),(1,-5) and (-4,5)$ are collinear.

4 Marks Questions

Q. 6. If

$A=\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$, $A^2-4A+7I=0$. Hence, find A^{-1}

Q. 7. Prove that

$$\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)^3$$

Q. 8. If

$A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, show by induction that $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$

Q. 9. Prove that

$$\begin{vmatrix}1+a^2-b^2 & 2ab & -2b\\ 2ab & 1-a^2+b^2 & 2a\\ 2b & -2a & 1-a^2-b^2\end{vmatrix}=(1+a^2+b^2)$$

6 Marks Questions

Q. 10. Using matrices, solve the following system of linear equation:

$2x-3y+5z=11, 3x+2y-4z=-5, x+y-2z=-3$ **or**

Using elementary transformation, find the inverse of the following matrix:

$$\begin{bmatrix}2 & -1 & 4\\ 4 & 0 & 2\\ 3 & -2 & 7\end{bmatrix}$$

Q. 11. For

$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, Find A^{-1}. Hence solve equations:

$x+2y+z=4, -x+y+z=0, x-3y+z=2$

Matrices & Determinants

Q. 1. For the value of x ,the following matrix is singular ? $\begin{bmatrix}5-x & x+1\\ 2 & 4\end{bmatrix}$

Q. 2. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A^T = I$, if the value of α is given by ?

Q. 3. If A and B are symmetric matrices of the same order, write whether $AB - BA$ is symmetric or skew symmetric.

Q. 4. Find the value of x if $\begin{vmatrix} 1 & 2 \\ 4 & 9 \end{vmatrix} = \begin{vmatrix} 3 & 3 \\ 6x & 4x \end{vmatrix}$

Q. 5. If

$A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$, find $(AB)^T$

Q. 6. If

$\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$ find x and y?

Q. 7. If

$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, then find K such that $A^2 - 8A + KI = 0$

Q. 8. Show that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

Q. 9. If A =

$\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, using P.M.I, show that An = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ $\forall n \in N$

Q. 10. Show that

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2.$$

Q. 11. Show that =

$$\begin{bmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{bmatrix} = 2xyz(x+y+z)^3$$

SECTION A

Q1. If A and B are non – singular matrices of the same order, then write the relationship between AdjAB, AdjA and AdjB.

Q2. If A, B, C are three non zero square matrices of same order, find the condition on A such that AB = AC then B = C.

Q3. Prove that modulus function is continuous everywhere.

Q4. Show that the function $f(x) = 2x^2 - 1$ is continuous at x = 3.

Q5. Find the derivative of y = sin(2x) w.r.t. x.

Q6. Find the derivative of y = loge sinx w.r.t. x

Q7. Find the principal value of $\cos^{-1}(-1)$.

Q8. Find the value of $\tan-1\{\tan(3\pi/4)\}$

Q9. Find the value of $\cos\{\cos^{-1}(1/4) + \sin^{-1}(1/4)\}$.

Q10. Evaluate : $\int (x-a)(x-b)(x-c)....(x-z)\,dx$

SECTION B

Q11. Prove that $(x/a)^n + (y/b)^n = 2$ touches the straight line x/a + y/b = 2 for all n €N, at the point (a,b).

Q12. Find the intervals in which the function $2x^3 + 9x^2 + 12x - 1$ are increasing or decreasing?
Or
Find the approximate value of √25.3(using differentials)

Q13. Prove that $\cos^{-1}(12/13) + \sin-1\,(3/5) = \sin^{-1}(56/65)$

Q14. Express $A = \begin{pmatrix} a & b \\ c & b \end{pmatrix}$ as a sum of symmetric and skew symmetric matrices.

Q15. For what value of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & if\ x \le 2 \\ ax + by & if\ 2 < x < 10 \\ 21 & if\ 10 \le x \end{cases}$$

$is\ continoous\ at\ x = 2\ and\ x = 10.$

Or
For what value of k such that the function defined by

$$f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & x \ne 5 \\ k & x = 5 \end{cases} \quad is\ continuous\ at\ x = 5$$

Q16. Show that all positive integral powers of a symmetric matrix are symmetric.

Q17. A trust fund Rs 30000 that must be invested in two types of different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs. 30000 among the two types of bonds. If the trust fund must obtain an annual total interest of

1. Rs 1800
2. Rs 2000

Q18. Find dy/dx if x √(y + 1) + y √(x + 1) = 0

Vector

Q. 1. Find the value of 'p' of the vectors $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}+p\hat{j}+3\hat{k}$ to be

i. parallel
ii. perpendicular.

Q. 2. Express the vector $3\hat{i}+2\hat{j}+9\hat{k}=\hat{i}+p\hat{j}+3\hat{k}$ as sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{i}+\hat{k}$and other is perpendicular to $\vec{b}$.

OR

Prove that, for any two vectors $\vec{a}\ and\ \vec{b}$ $\left|\vec{a}*\vec{b}\right|^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2 - \left(\vec{a}.\vec{b}\right)^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$

Q. 3. If $3\hat{i}+2\hat{j}+9\hat{k}$ are unit vector such that $\hat{i}+p\hat{j}+3\hat{k}$ then find the value of $\vec{b}$

Q. 4. Find the diagonals of the parallelogram sides are the vectors $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}+p\hat{j}+3\hat{k}$ also find its area.

Q. 5. Find the value of $3\hat{i}+2\hat{j}+9\hat{k}$if vectors $\hat{i}+p\hat{j}+3\hat{k}$ and $\vec{b}$such that $3\hat{i}+\hat{k},\left|\vec{b}\right|=3$ and $\vec{a}.\vec{b} = 4$.

Area

Q. 1. Find the area of the region in the first quadrant enclosed by the x-axis, the line y=x and the circle $x^2+y^2=32$.

Q. 2. Find the area bounded by the curve $x^2=4y$and the straight line x= 4y-2.

Q. 3. Find the area of the region enclosed between the two circles $(x-1)^2+y^2=1$ and $x^2+y^2=1$.

OR

Find the area of the circle $x^2+y^2=16$ which is exterior to the parabola $y^2=6x$.

Q. 4. Using integration find the area of triangle ABC, whose vertices are A(2, 0) B(4, 5) and C(6, 3).

Q. 5. Find the area of the region bounded by x2=16y, y=1, y=4 and the y-axis in the first quadrant.

OR

Find the area :

$\{(x,y): 0 \le y \le x^2+3, 0 \le y \le 2x+3, 0 \le x \le 3\}$

Probability

Q. 1. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once.

Q. 2. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that at least one is a girl.

Q. 3. If A and B are two independent events, show that the probability of occurrence of at least one of A and B is given by : 1 – P(A').P(B')

Q. 4. Probability of solving specific problem independently by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, find the probability that the problem is solved.

Q. 5. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red.

Q. 6. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

Q. 7. In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers '**true**', if it falls tails, he answers '**false**'. Find the probability that he answers at least 12 questions correctly.

Q. 8. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

Q. 9. $If\ P(A)=3/8,\ P(B)\ 1/2\ and\ (A\cap B)\ 1/4,\ find\ P\left(\overline{A}/\overline{B}\right) and\ P\left(\overline{B}/\overline{A}\right)$

Q. 10. The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a trouser is 0.3 and the probability that he will buy a shirt given that he buys a trouser is 0.4. Find the probability that he will buy both a shirt and a trouser. Find also the probability that he will buy a trouser given that he buys a shirt.

Q. 11. A can solve 90% of the problem given in a book and B can solve 70%. What is the probability that at least one of them will solve the problem, selected at random from the book.

Q. 12. Three persons A, B, C throw a die in succession till one gets a 'six' and wins the game. Find their respective probabilities of winning, if A begins.

Q. 13. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Q. 14. An urn contains 4 white and 3 red balls. Find the probability distribution of the number of red balls in a random draw of three balls.

Q. 15. In a meeting 70% of the members favour a certain proposal,30% being opposed. A member is selected at random and let X = 0 if he opposed and X =1 if he is in favour. Find E(x) and Var(x).

Q. 16. Find the probability distribution of the number of doublets in 4 throws of a pair of dice.

Q. 17. In a hostel , 60 % of the students read Hindi news paper , 40 % read English news paper and 20 % read both Hindi and English news papers.A student is selected at random.

i. Find the probability that she reads neither Hindi nor English news papers.
ii. If she reads Hindi news paper,find the prob.that she reads English news paper,
iii. If she reads English news paper,find the prob.that she reads Hindi news paper.

Q. 18. A doctor is to visit a patient .From the past experiences , it is known that the probabilities that he will come by train , bus , scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ *and* $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ *and* $\frac{1}{12}$, if he comes by train , bus, and scooter respectively , but if he come by other means of transport , then he will not be late .When he arrives he is late .What is the probability that he comes by train ?

Q. 19. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1,2,3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1,2,3 or 4 with the die

Linear Programming

Q. 1. One kind of cake requires 200gm of flour and 25gm of fat, and another kind of cake requires 100gm of flour and 50gm of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

Q. 2. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The Vitamin contents of one Kg food is given below :-

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3

Y	2	2	1

One kg.of food X costs Rs. 16 and one Kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet.

Q. 3. Maximise and Minimise:
Z = x + 2y
Subject to constraints $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0.$

Q. 4. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit.

Q. 5. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in following table :

Transportation Cost Per Quintal (in Rs.)

From / To	A	B
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum. What is the minimum cost.

Q. 6. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants.

Q. 7. Solve the following L.P.P graphically:

$Maximize : z = 60x + 15y$

$Subject\ contra\text{int}\ s$

$$x + y \leq 50$$
$$3x + y \leq 90, \quad x, y \geq 0$$

Q. 8. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5,760 to invest and has space for at most 20 items. A fan costs him Rs. 360 and a sewing machine Rs 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximise his profit.

Q. 9. If a young man drives his vehicle at 25 km/hr, he has to spend Rs. 2/km on petrol. If he drives it at a faster speed of 40km/hr, the petrol cost increases to Rs. 5/km. He has Rs. 100 to spend on petrol and travel within one hour. Express this as an L.P.P. and solve.

Q. 10. Determine graphically the minimum value of the objective function
Z = – 50*x* + 20*y*
subject to the constraints:
$2x - y \geq -5$

$3x + y \geq 3$

$2x - 3y \leq 12$

$x \geq 0, y \geq 0$

Q. 11. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours ofcraftsman's time.

i. What number of rackets and bats must be made if the factory is to work at full capacity?
ii. If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

Q. 12. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000

Three Dimensional Geometry

Q. 1. Find the direction cosines of X, Y and Z-axis.

Q. 2. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$.

Q. 3. Find the value of p so that the lines,
$\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7\text{-}7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles.

Q. 4. Find the shortest distance between the lines whose vector equations are:-
$\vec{r} = (1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and
$\vec{r} = (s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$

Q. 5. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane 2x – 3y + 4z -6 = 0.

Q. 6. Find the vector equation of the plane passing through the intersection of planes
$\vec{r}.(\hat{i}+\hat{j}+\hat{k}) = 6$ and $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k}) = -5$ and the point(1,1,1)

Q. 7. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$

Q. 8. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then
$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$

Q. 9. Show that the angles between the diagonals of a cube is cos-1$\left(\frac{1}{3}\right)$.

Q. 10. Find the equation of the line passing through the point (-1, 3, -2) and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$

Q. 11. Find the foot of the perpendicular drawn from the point (0, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$, Also, find the length of the perpendicular.

Q. 12. Find the shortest distance between the following pairs of lines whose cartesian equations are : $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{3}=\frac{y-2}{1}$, z=2

Q. 13. A plane meets the coordinate axis in A, B, C such that the centroid of triangle ABC is the point (p,q,r). Show that the equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$.

Q. 14. Find the equation of the plane passing through the point (1, 1, -1) and perpendicular to the planes x + 2y + 3z – 7=0 and 2x – 3y + 4z = 0.

Q. 15. Find the distance between parallel planes, $\vec{r}.(2\hat{i}-3\hat{j}+6\hat{k})=5$ and $\vec{r}.(6\hat{i}-9\hat{j}+18\hat{k})+20=0$

Q. 16. Show that the lines :

$\vec{r}=\left(\hat{i}+\hat{j}-\hat{k}\right)+\lambda\left(3\hat{i}-\hat{j}\right)$ and $\vec{r}=\left(4\hat{i}-\hat{j}\right)+\mu\left(2\hat{i}+3\hat{k}\right)$ are coplanar. Also, find the plane containing these two lines.

Q. 17. Find the distance of the point (- 1, - 5, - 10) from the point of intersection of the line r = 2 i – j + 2 k + λ (3 i + 4 j + 2 k) and the plane r . (i – j + k) = 5

Q. 18.

If a line makes angles α, β, γ and δ with the diagonals of a cube

prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$

Q. 19. Find the distance between the point P(6,5,9) and the plane determined by the points, A (3,-1,2) , B (5,2,4) and C (-1,-1,6).

Q. 20.

Find the shortest distance between the lines

$\vec{r}=(1+2\lambda)\,i+(2+3\lambda)\,j+(3+4\lambda)k$ *and*

$\vec{r}=(2+3\mu)\,i+4(1+\mu)\,j+5(1+\mu)k$

Q. 21.

Find the equation of the line passing through the point (2,1,3) *and*

perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ *and* $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$

Vector Algebra

Q. 1. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units.

Q. 2. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively, form the vertices of a right angled triangle.

Q. 3. Find $\left|\vec{a} - \vec{b}\right|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a}\right| = 2$, $\left|\vec{b}\right| = 3$ and $\vec{a} \cdot \vec{b} = 4$.

Q. 4. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

Q. 5. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and acute angle θ with $\hat{k}$, then find θ and hence the components of $\vec{a}$.

Q. 6. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\left|\vec{a}\right| = 3$, $\left|\vec{b}\right| = 4$, $\left|\vec{c}\right| = 5$ and each one of them being perpendicular to the sum of other two, Find $\left|\vec{a} + \vec{b} + \vec{c}\right|$.

Q. 7. Find the value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$

Q. 8. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

Q. 9. If the sum of two unit vectors is a unit vector, Prove that the magnitude of their difference is $\sqrt{3}$.

Q. 10. If $\vec{a}$ and $\vec{b}$ are position vectors of points A and B respectively, then find the position vector of points of trisection of AB.

Q. 11. Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.

Q. 12. ABCD is a parallelogram. If the coordinates of A, B, C are (-2, -1), (3, 0) and (1, -2) respectively, Find the co-ordinate of D.

Q. 13. Show that the points A, B, C with position vectors $-2\vec{a} + 3\vec{b} + 5\vec{c}$, $\vec{a} + 2\vec{b} + 3\vec{c}$ and $7\vec{a} - \vec{c}$ respectively are collinear

Q. 14. If a vector makes α, β, γ with OX, OY and OZ respectively, prove that $\sin^2\alpha + \sin^2\beta + \sin^2 y = 2$

Q. 15. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove that $\sin\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} - \hat{b}\right|$.

Q. 16. If $\vec{a}+\vec{b}+\vec{c}=0,\ |\vec{a}|=3,\ |\vec{b}|=5,\ |\vec{c}|=7,$ find the angle between $\vec{a}$ and $\vec{b}$.

Q. 17. If $|\vec{a}|=2,\ |\vec{b}|=5$ and $|\vec{a}\times\vec{b}|=8,$ find $\vec{a}.\vec{b}$.

Q. 18. Consider two points P and Q with position vectors $\vec{OP}=3\vec{a}-2\vec{b}$ and $\vec{OQ}=\vec{a}+\vec{b}$. Find the position vector of a point R which divides the line joining P and Q in the ratio 2:1, (i) internally, and (ii) externally.

Q. 19. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a}=i+j+k, \vec{b}=i+2j+3k$

Q. 20. Show that the vectors 2 i – j + k , i– 3 j – 5 k and 3 i – 4 j – 4 k form the vertices of a right angled triangle

Q. 21.

Find a unit vector parallel to the vector $2\bar{a}-\bar{b}+3\bar{c}$ where $\bar{a}=i+j+k, \bar{b}=2i-j+3k$ and $\bar{c}=i-2j+k$

Differential Equations

Solve the following Differential Equations:

Q. 1. Determine order and degree (if defined) of differential equation $y''' + 2y'' + y' = 0$

Q. 2. $\sec^2x.\tan y\,dx + \sec^2y.\tan x\,dy = 0$.

Q. 3. $xdy - ydx = \sqrt{x^2+y^2}\,dx$

Q. 4. $2xy + y^2 - 2x^2\,dy/dx = 0$, $y = 2$ when $x = 1$.

Q. 5. $x \log x.\ dy/dx + y = (2/x)\log x$

Q. 6. $y\,dx + (x - y^2)\,dy = 0$

Q. 7. $(\tan^{-1}y - x)dy = (1 + y^2)\,dx$

Q. 8. $(1 + x^2)dy/dx + 2xy = \frac{1}{1+x^2}$;

Q. 9. $x^2dy + (xy + y^2)dx = 0$, $y = 1$ when $x = 1$

Q. 10. $dx/dy + y\cot x = 2x + x^2\cot x$, $y = 0$ when $x = \frac{\pi}{2}$

Q. 11. $(x + y)dy = dx$

Q. 12. $x(xdy - ydx) = ydx$, $y(1) = 1$.

Q. 13. $dy/dx = \cos(x + y) + \sin(x + y)$

Q. 14. $x\,dy/dx = y - x\tan(y/x)$.

Q. 15. $(x^2 + 1)\ dy/dx + 2xy = \sqrt{x^2+4}$

Q. 16. $(x^2 + y^2)dx + xy.dy = 0, \quad y(1) = 1$

Q. 17. $(x + y + 1)^2\ dy = dx, \qquad y(-1) = 0$

Q. 18. (xy2 + 2x)dx + (x2y + 2y)dy = 0

Q. 19. $dy/dx + \frac{y}{x} = \cos x + \frac{\sin x}{x}$

Q. 20. Solve the differential equation $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$

Q. 21. Solve the differential equation $x^2\ dy + y\ (x + y)\ dx = 0$, given that y = 1 when x = 1

Q. 22. Solve:

$$x\frac{dy}{dx} = y\ (\log y - \log x + 1)$$

Q. 23.

$$Solve\ \ x^2\frac{dy}{dx} = y(x+y)\ dx$$

Q. 24.

Find a particular solution satisfying the given condition

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2};\ y = 0\ when\ x = 1$$

Q. 25. Find the differential equation of all circles in the first quadrant which touch the co-ordinate axis.

Q. 26. Form the differential equation corresponding to $y^2 = m(a^2 - x^2)$ by eliminating parameters m and a.

Q. 27. Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of *x*-axis.

Q. 28. Form the differential equation of the family of circles having centre on *y*-axis and radius 3 units.

Application of Integrals

Q. 1. Find the area of the region in the first quadrant enclosed by the x-axis, the line y = x and the circle $x^2 + y^2 = 32$.

Q. 2. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Q. 3. Find the area of the region bounded by the parabola y = x2 and $y = |x|$.

Q. 4. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x=\frac{a}{\sqrt{2}}$.

Q. 5. Using integration, find the area of the region bounded by the triangle whose vertices are (1, 0), (2,2) and (3, 1).

Q. 6. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x=0, x=4, y=4 and y=0 into three equal parts.

Q. 7. Sketch the graph of $y=|x+3|$ and evaluate $\int_{-6}^{0}|x+3|dx$

Q. 8. Using the method of integration, find the area bounded by the curve $|x|+|y|=1$.

Q. 9. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$.

Q. 10. Using integration, find the area of the triangular region, the equations of whose sides are y=2x + 1, y=3x +1 and x = 4.

Q. 11. Find the area of the region $\{(x,y) : y^2 \le 8x,\ x^2+y^2 \le 9\}$

Q. 12. Find the area of the region between the circles $x^2 + y^2 = 4$ and $(x – 2)^2 + y^2 = 4$.

Q. 13. Find the area bounded by the ellipse $\frac{x^2}{y^2}+\frac{y^2}{b^2}=1$ and the co-ordinates x = ae and x = 0, where $b^2=a^2(1 – e^2)$ and e<1.

Q. 14. Find the area bounded by the curve $y^2 = 4a^2(x – 1)$ and the lines x = 1and y = 4a.

Q. 15. Using integration, find the area of the region bounded by the following curves, after making a rough sketch: $y = 1 + |x+1|,\ x=-3,\ x=3,\ y=0$.

Q. 16. Draw a rough sketch of the curves y = sinx and y = cosx as x varies from o to $\frac{\pi}{2}$ and find the area of the region enclosed by them and x-axis.

Q. 17. Find the area lying above *x*-axis and included between the circle $x^2+ y^2 = 8x$ and the parabola $y^2 = 4x$.

Q. 18. Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

Q. 19. Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$.

Q. 20. Find the area of the region bounded by the parabolas $y^2 = 4$ ax and $x^2 = 4$ by

Integrals

Q. 1. $\int \frac{x^4+1}{x^2+1}dx$

Q. 2. $\int \frac{\sqrt{\tan x}}{\sin x . \cos x} dx$

Q. 3. $\int \frac{1}{1+\sqrt{x}}$ dx

Q. 4. $\int \frac{1}{\sqrt{x(1-2x)}} dx$

Q. 5. $\int \frac{\sin x}{\sin 3x} dx$

Q. 6. $\int \frac{dx}{(x^2+x)(x^2-1)}$

Q. 7. $\int \frac{\cos^2 x - \sin^2 x}{\sqrt{1+\cos 4x}} dx$

Q. 8. $\int e$ 3logx (x4+1)-1 dx

Q. 9. $\int x \sin x / 2 . \cos x / 2 . \cos x$ dx

Q. 10. $\int \cos mx . \cos nx$ dx

Q. 11. $\int \sin x \sqrt{1+\cos 2x}$ dx

Q. 12. $\int \left(e^x + \frac{1}{e^x}\right)^2 dx$

Q. 13. $\int \frac{\cos 4x - \cos 2x}{\sin 4x - \sin 2x} dx$

Q. 14. $\int e^x \sec x (1+\tan x) dx$

Q. 15. $\int \cos ec^3 x$ dx

Q. 16. $\int \tan x . \tan 2x . \tan 3x$ dx

Q. 17. $\int \frac{a}{b+ce^x}$ **dx**

Q. 18. $\int \log(x+\sqrt{a^2+x^2}) dx$

Q. 19. $\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx$

Q. 20. $\int \frac{\sin 2x}{\sin(x-\frac{\pi}{6}) . \sin(x+\frac{\pi}{6})} dx$

Q. 21. $\int \frac{(x-1)^2}{x^2+2x+2}dx$

Q. 22. $\int \sec^3 x.\tan x\, dx$

Q. 23. $\int \frac{1}{\sqrt{\sin^3 x.\sin(x+\alpha)}}dx$

Q. 24. $\int \frac{(x^4-x)^{1/4}}{x^5}dx$

Q. 25. $\int (\tan x + \cot x)^2 dx$

Q. 26. $\int \frac{x^2+1}{(x^2+2)(2x^2+1)}dx$

Q. 27. $\int \sin x.\sin 2x.\sin 3x\, dx$

Q. 28. $\int \frac{1+x+x^2}{x^2(1+x)}dx$

Q. 29. $\int x^2 \tan^{-1} dx$

Q. 30. $\int [\sin(\log x)+\cos(\log x)]dx$

Q. 31. $\int \frac{1}{(x+1)^2(x^2+1)}dx$

Q. 32. $\int \frac{\cos x - \sin x}{1+\sin 2x}dx$

Q. 33. $\int \frac{1}{\sqrt{a^2-b^2x^2}}dx$

Q. 34. $\int \frac{1}{\sqrt{(2-x)^2-1}}dx$

Q. 35. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$

Q. 36. $\int \frac{1}{1+x+x^2+x^3}dx$

Q. 37. $\int \frac{x}{3x^4-18x^2+11}dx$

Q. 38. $\int \frac{1}{1+x^4}dx$

Q. 39. $\int \frac{1}{\sqrt{(x-a)(x-b)}} dx$

Q. 40. $\int \left(\frac{x}{m} + \frac{m}{x} + x^m + m^x \right) dx$

Q. 41. $\int \frac{\sin 8x}{\sqrt{9 + \sin^4 4x}} dx$

Q. 42. $\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$

Q. 43. $\int \sqrt{x - x^2}\, dx$

Q. 44. $\int \frac{2 \sin 2x - \cos x}{6 - \cos^2 x - 4 \sin x} dx$

Q. 45. $\int \frac{1 - 3x}{3x^2 + 4x + 2} dx$

Q. 46. $\int \frac{1}{\sin x . \cos^3 x} dx$

Q. 47. $\int \frac{1}{\sin^4 x + \cos^4 x} dx$

Q. 48. $\int \frac{3x + 1}{\sqrt{5 - 2x - x^2}} dx$

Q. 49. $\int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$

Q. 50. $\int \frac{(x-1)^2}{x^2 + 2x + 2} dx$

Q. 51. $\int \frac{\left(a^x + b^x\right)^2 dx}{a^x . b^x}$

Q. 52. $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x \mathrm{dx}}$

Q. 53. $\int \frac{1}{\cos 2x + 3 \sin^2 x} dx$

Q. 54. $\int \frac{1}{x^{1/2} + x^{1/3}} dx$

Q. 55. $\int \frac{1}{\sqrt{2x+3} + \sqrt{2x-3}} dx$

Q. 56. $\int \frac{1}{\sin x + \sqrt{3} \cos x} dx$

Q. 57. $\int \frac{1}{p+q\tan x}dx$

Q. 58. $\int \frac{\operatorname{cosec} x}{\log\tan x/2}dx$

Q. 59. $\int \log(1+x^2)dx$

Q. 60. $\int \frac{1}{1+\cot x}dx$

Q. 61. $\int \frac{\log(\log x)}{x}dx$

Q. 62. $\int \frac{x-3}{(x-1)^3}e^x dx$

Q. 63. $\int \left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$

Q. 64. $\int (x^3-1)^{1/5}.x^5 dx$

Q. 65. $\int \frac{\sqrt{1-\sin x}}{1+\cos x}e^{-x/2}dx$

Q. 67. $\int \frac{1}{\cos(x-a).\cos(x-b)}dx$

Q. 68. $\int \frac{\log x}{(1+\log x)^2}dx$

Q. 69. $\int \sin(\log x)dx$

Q. 69. $\int x^2 e^{x^3}\cos x^3 dx$

Q. 70. $\int \frac{(x-a)(x-b)}{(x-c)(x-d)}dx$

Q. 71. $\int x\sqrt{x^4+1}\ dx$

Q. 72. $\int (x+1)\sqrt{1-x-x^2}dx$

Q. 73. $\int \frac{x^3}{(x-1)(x-2)}dx$

Q. 74. $\int \frac{x^2+x+1}{(x-1)^3}$ dx

Q. 75. $\int 5^{5^{5^x}}.5^{5^x}.5^x$ dx

Q. 76. $\int \frac{\tan x+\tan^3 x}{1+\tan^3 x}dx$

Q. 77. $\int \frac{\sin x}{\sin 4x}dx$

Q. 78. $\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx$

Q. 79. $\int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx$

Q. 80. $\int \frac{1}{x(x^4+1)}dx$

Q. 81. $\int \frac{x^2+4}{x^4+16}dx$

Q. 82. $\int \sqrt{\tan x}dx$

Q. 83. $\int \frac{x^2}{x^2+6x+12}dx$

Q. 84. $\int \frac{x^2+1}{x^4+7x^2+1}dx$

Q. 85. $\int \frac{1}{(x-3)\sqrt{x+1}}dx$

Q. 86. $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}dx$

Q. 87. $\int \frac{x^4}{(x-1)(x^2+1)}dx$

Q. 88. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$

Q. 89. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$

Q. 90. $\int \sin^{3/2} x\cos^3 x\ dx$

Q. 91. $\int \frac{x^3+x}{x^4-9}dx$

Q. 92. $\int \frac{1}{1+x-x^2}dx$

Q. 93. $\int (\sin^{-1} x)^2 dx$

Q. 94. $\int x.\cos^3 x^2 \sin x^2 dx$

Q. 95. $\int \frac{\sqrt{5+\log\sqrt{x}}}{5x}dx$

Q. 96. $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$

Q. 97. $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}dx$

Q. 98. $\int_0^2 \left|x^2+2x-3\right| dx$

Q. 99. Evaluate: $\int_0^{\pi/2} \sin 2x.\log\tan x \, dx$

Q. 100. Prove that: $\int_0^{\frac{\pi}{2}} \log(\sin x)dx = \int_0^{\frac{\pi}{2}} \log(\cos x)dx = \frac{-\pi}{2}\log 2$

Q. 101. Prove that $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta)d\theta = \frac{\pi}{8}\log 2$.

Q. 102. Evaluate: $\int_0^1 e^{2-3x}dx$ as the limit of a sum.

Q. 103. Evaluate: $\int_1^4 (x^2-x)dx$ as the limit of sums.

Application of Derivatives

Q. 1. The volume of a cube is increasing aa constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

Q. 2. Use differentials to find the approximate value of $\sqrt{0.037}$

Q. 3. It is given that for the function f(x) = $x^3 - 6x^2 + ax + b$ on [1, 3], Rolle's theorem holds with c = $2+\frac{1}{\sqrt{3}}$. Find the values of a and b if f(1)= f(3) = 0

Q. 4. Find a point on the curve y = $(x-3)^2$, where the tangent is parallel to the line joining (4, 1) and (3, 0).

Q. 5. Find the intervals in which the function f(x) = $x^4 - 8x^3 + 22x^2 - 24x + 21$ is decreasing or increasing.

Q. 6. Find the local maximum or local minimum of the function. $f(x) = \sin^4 x + \cos^4 x, \quad 0 < x < \frac{\pi}{2}$

Q. 7. Find the point on the curve y2 = 4x which is nearest to the point (2, 1).

Q. 8. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum.

Q. 9. A balloon which always remain spherical has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

Q. 10. Find the intervals in which $f(x) = (x+1)^3 (x-3)^3$ is strictly increasing or decreasing.

Q. 11. Prove that the curves $x = y^2$ and xy = k cut at right angles if 8k2 = 1

Q. 12. Using differentials, find the approximate value of $(26.57)^{1/3}$

Q. 13. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Q. 14. Find the equation of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x0,y0)

Q. 15. Find the intervals of the function $f(x) \sin x + \cos x, 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

Q. 16. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

Q. 17. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Q. 18. Show that the right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of the base.

Q. 19. The sum of the perimeter of a circle, and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

Q. 20. A window is in the form of a rectangle surmounded by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Q. 21. Sand is pouring from a pipe at the rate of 12 cm3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

Q. 22. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

Q. 23. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

Q. 24. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

Q. 25. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

Q. 26. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per sq metres for the base and Rs 45 per square metre for sides. What is the cost of least expensive tank

Continuity & Differentiation

Q. 1. Find the values of a and b such that the function defined by f(x) = (5, if x £ 2 ax + b if 2<x<10 21, if x □ 10) is a continuous function

Q. 2. Find $\frac{dy}{dx}$ of $\sin^2 y + \cos(xy) = p$

Q. 3. Differentiate w.r.t. x x cosx)^x + (x sinx)$^{1/x}$

Q. 4. If x = $\sqrt{a^{\sin^{-1} t}}$, = $\sqrt{a^{\cos^{-1} t}}$ show that $\frac{dy}{dx} = \frac{-y}{x}$

Q. 5. If y = $(\tan^{-1}x)^2$, show that $(x^2 + 1)^2 y^2 + 2x (x^2 + 1) y1 = 2$.

Q. 6. Differentiate sin-1 $\left(\frac{2^{x+1}}{1+4^x}\right)$ w.r.t. x

Q. 7. If x $\sqrt{1+y} + y\sqrt{1+x} = 0$ for -1<x<1, show that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

Q. 8. Find $\frac{dy}{dx}$, if y = a $^{t + 1A}$, x = $(t + 1/t)^a$

Q. 9. Discuss the continuity of the function given by :-

$f(x) = |x-1| + |x-2|$ at $x = 1$, and $x = 2$.

Q. 10. If the function f(x) is given by f(x) =

$$\begin{Bmatrix} (3ax+b) \text{ if } x > 1 \\ 11 \text{ if } x = 1 \\ 5ax - 2b \text{ if } x < 1 \end{Bmatrix}$$

is continuous at x = 1, find the values of a and b.

Q. 11. If y = $[x + \sqrt{x^2 + a^2}]^n$, then prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$

Q. 12. Prove :

$$\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]=\sqrt{a^2-x^2}$$

Q. 13. Find $\frac{dy}{dx}$, when y = sec^{-1}

$$\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$$

Q. 14. If $e^x + e^y = e^{x+y}$, prove that

$$\frac{dy}{dx}=\frac{e^x(e^y-1)}{e^y(e^x-1)}$$

Q. 15. Given that cos

$$\frac{x}{2}.\cos\frac{x}{4}.\cos\frac{x}{8}------=\frac{\sin x}{x}$$

prove that $\frac{1}{2^2}\sec^2\frac{x}{2}+\frac{1}{2^4}\sec^2\frac{x}{4}+------=\operatorname{cosec}^2x-\frac{1}{x^2}$

Q. 16. If x=a(q + sinq), y= a(1+ cosq), prove that

$$\frac{d^2y}{dx^2}=\frac{-a}{y^2}$$

Q. 17.

$$If\ x^y=e^{x-y}\ ,show that\ \frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$$

Q. 18. Find the value of 'k' if

$$f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x} & if\ \ x\neq\dfrac{\pi}{2}\\ 3 & if\ \ x=\dfrac{\pi}{2}\end{cases}$$ is continuous at x = $\frac{\pi}{2}$

Q. 19. If

$$y=\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]\ find\ \frac{dy}{dx}$$

Q. 20. If Cos y = x Cos^2 (a + y) , with Cos a $\neq$ 1, prove that

$$\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$$

Determinants

Q. 1. Prove that :

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

Q. 2. Find the equation of the line joining A(1,3) and B(0,0) using determinants and find if D (K, 0) is a point such that area of a triangle ABD is 3 square units.

Q. 3. If

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}

Q. 4. Prove that :

$$\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & \omega \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

Q. 5. Solve by matrix method:

$2x + y + z = 1$
$x - 2y - z = 3/2$
$3y - 5z = 9$

Q. 6. Prove that :

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

Q. 7. Prove that :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab.$$

Q. 8. Solve :

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

Q. 9. Using determinants, find the area of the triangle whose vertices are (1, 4), (2, 3), (-5, 3). Are the given points collinear.

Q. 10. If the points (a_1, b_1), (a_2, b_2) and $(a_1 + a_2, (b_1 + b_2)$ are collinear, Show that $a_1b_2 = a_2b_1$.

Q. 11. If a, b, c are all positive and are p^{th}, q^{th} and r^{th} terms of G.P., then show that

$$\Delta = \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = o$$

Q. 12. If

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ z-y & x-y & 0 \end{vmatrix}$$

= 0, then Prove that a, b, c are in G.P or x, y, z are in G.P

Q. 13. If x, y, z are different and

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ x & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0, \textit{ then show that } 1 + xyz = 0$$

Q. 14. Show that points A ($a,\ b + c$), B ($b,\ c + a$), C ($c,\ a + b$) are collinear.

Q. 15. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Q. 16. Show that the following system of equations is consistent $2x - y + 3z = 5$, $3x + 2y - z = 7$, $4x + 5y - 5z = 9$. Also, find the solution.

Q. 17. Using matrix method, solve the following system of equations for x, y and z :

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10,\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10,\ \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

Q. 18. Find whether the following system of equations is consistent or not, find the solution of the system also. $3x-y+2z=3$, $x-2y-z = 1$, $2x+y + 3z = 5$.

Q. 19. Determine the product

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

and use it solve the system of equations :

$x-y+z=4$
$x-2y-2z=9$
$2x+y+3z=1$

Q. 20. If

A = $\begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}$

, find A-1, using A solve the following system of linear equations.

2x – y + z + 3 = 0
3x – z + 8 = 0
2x + 6y – 2 = 0

Q. 21.

$$\text{Prove that} \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

Q. 22.

$$\text{Show that} \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz\,(x+y+z)^3$$

Matrices

Q. 1. Construct a 3´4 matrix, whose elements are given by aij= $\frac{1}{2}|-3i+j|$

Q. 2. Construct a 2´3 matrix A = [aij] whose elements are given by aij

= $\frac{(j-2i)^3}{4j}$, i # j = |i + 2j|, i = j

Q. 3. If

A = $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X, such that 2A + 3X = 5B.

Q. 4. If

A = $\begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

, then show that I+A= (I – A)

$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Q. 5. Express the matrix

A= $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of symmetric and skew-symmetric matrix

Q. 6. Obtain the inverse of the matrix

A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

using elementary transformations.

Q. 7. If

f(x)= $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Prove that f(x). f(y) = f(x + y)

Q. 8. Show that the matrix B¢AB is symmetric or skew-symmetric according as A is symmetric or skew symmetric.

Q. 9. If A and B are invertible matrices of the same order, then prove that $(AB)^{-1} = B^{-1}A^{-1}$

Q. 10. Let $f(x) = x^2 - 5x + 6$. Find f(A) If

A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Q. 11. If

A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Show that $A^2 - 5A + 7I = 0$, Use this to find A^4.

Q. 12. Express the matrix

A = $\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$

as the sum of a symmetric and a skew-symmetric matrix.

Q. 13. Find the values of x, y, z if the matrix

A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

satisfy the equation $\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ A'A = I_3.

Q. 14. Show that :

$$\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Q. 18. Find the inverse of

$\begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$, if $a^2 + b^2 + c^2 + d^2 = 1$.

Q. 19. Using the method of reduction (i.e elementary row transformations),find the inverse of

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & -1 \end{bmatrix}$$

Q. 20. For what value of k the matrix $A = \begin{bmatrix} 2 & k \\ 3 & 5 \end{bmatrix}$ has no inverse.

Q. 21. Prove that the product of matrices

$\begin{bmatrix} \cos^2\theta & \cos\theta.\sin\theta \\ \cos\theta.\sin\theta & \sin^2\theta \end{bmatrix}$ and $\begin{bmatrix} \cos^2\phi & \cos\phi.\sin\phi \\ \cos\phi.\sin\phi & \sin^2\phi \end{bmatrix}$

is the null matrix, when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

Q. 22. A matrix X has a + b rows and a + 2 columns while the matrix Y has b + 1 rows and a + 3 columns. Both matrices XY and YX exist. Find a and b. Can you say XY and YX are of the same type? Are they equal.

Q. 23. Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverse Trigonometric Functions

Q. 1. Find the value of : $\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$.

Q. 2. Prove : $\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $|x| < \frac{1}{\sqrt{3}}$

Q. 3. If $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x.

Q. 4. Find the value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right)$.

Q. 5. Prove : $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$

Q. 6. Solve : $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

Q. 7. Prove : $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Q. 8. Solve : sin-1 (1 –x) – $2\sin^{-1}x = \frac{\pi}{2}$.

Q. 9. Evaluate: tan-1$\sqrt{3}$ - $\sec^{-1}(-2) + \operatorname{cosec}^{-1}\frac{2}{\sqrt{3}}$.

Q. 10. Prove :

$$\tan^{-1}\left\{\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right\} = \frac{\pi}{4} + \frac{x}{2}$$

Q. 11. Simplify :

$$\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), \quad -\frac{\pi}{4} < x < \frac{\pi}{4}.$$

Q. 12. Prove: sec2 $(\tan^{-1}2)$ + cosec2 $(\cot^{-1}3) = 15$.

Q. 13. Simplify :

$$\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$$

Q. 14. Prove :

$$\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

Q. 15. If $\sin(\sin^{-1}\frac{1}{5} + \cos^{-1}x) = 1$, then find the value of x.

Q. 16. Prove that :

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\theta/2\right) = \cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right)$$

Q. 17. Find the principal value of Sec^{-1}

Q. 18. Find value of

$$\text{Sin}\left[\tan^{-1}\frac{1-x^2}{2x} + \cos^{-1}\frac{1-x^2}{1+x^2}\right]$$

Q. 19. If 2 $\tan^{-1}$ (Cos q) = $\tan^{-1}$ (2 cosec q), find q.

Q. 20.

Simplify $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

Relations & Functions

Q. 1. Show that the relation R in the set A = $\{1,2,3,4,5\}$given by R=$\{(a,b):|a-b| \text{ is even}\}$, is an equivalence relation.

Q. 2. Show that the relation R in R defined by R=$\{(a,b):a\le b\}$, is reflexive and transitive but not symmetric.

Q. 3. Let A= R $-$\{3\} and B = R$-$\{1\} , Consider the function *f*: A ® B defined by *f* (x) =$\frac{x-2}{x-3}$. Show that *f* is one–one and onto

Q. 4. If *f*: R ® R be given by *f* (x) = $(3-x^3)^{\frac{1}{3}}$, find the value of *fof* (x)

Q. 5. Consider *f* : R ® R given by f(x) = 4x+3, show that *f* is invertible . Find the inverse of *f*.

Q. 6. Let f: N $\rightarrow R$ be a function defined as f (x) = $4x^2 + 12x + 15$ .Show that f: N $\rightarrow S$ where S is the range of f, is invertible .Find the inverse of f .

Q. 7. Show that binary operation a * b = $\frac{a+b}{2}$ " a,b ò N is commutative but not associative.

Q. 8. Show that *f*: N $\rightarrow$N defined by *f* (x) = $\begin{cases}\frac{n+1}{2}\end{cases}$, if n is odd

$\begin{cases}\frac{n}{2}\end{cases}$, if n is even is many-one onto function.

Q. 9. Show that *f* : R –\{0\} $\rightarrow$R–\{0\} given by *f* (x) = 3/x is invertible and it is inverse of itself.

Q. 10. On the set M = A (x) = $\left\{\begin{bmatrix} x & x \\ x & x \end{bmatrix} : x \in R\right\}$of 2 ´ 2 matrices, find the identity element for the multiplication of matrices as a binary operation. Also, find the inverse of an element of M.

Q. 11. Let *f* (x) = [x] and g(x) = êx ê find go*f* (-5/3) – *f*og (-5/3)

Q. 12. Show that the function *f*: R $\rightarrow$ R defined by *f* (x) = $3x^3 + 5$ for x $\in$ R is a bijection.

Q. 13. Show that the relation R on the set R of all real numbers, defined as R = \{ (a, b): a £ b^2 \} is neither reflexive nor symmetric nor transitive.

Q. 14. Show that the function *f*:N ® Ngiven by *f* (1) = *f* (2) =1 and *f* (x) = x-1,for every x > 2 is onto but not one-one.

Q. 15. If $\oplus$ is a binary operation on R defined by a $\oplus$ b = a/4 + b/7 for a, b $\in$ R, find the value of (2 $\oplus$ 5) $\oplus$ 7

Integration

Q. 1.

$$Evalute \int \frac{Sin^8 x - \cos^8 x}{1 - 2\sin^2 x . \cos^2 x} dx.$$

Q. 2.

$$Evalute \int \frac{1}{\sqrt{\sin^3 x . \sin (x + a)}} dx.$$

Q. 3.

$$Evalute \int \tan^3 x \, dx.$$

Q. 4.

$$Evalute \int \frac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)} dx.$$

Q. 5.

$$Evalute \int \frac{e^x (x^2 + 1)}{(x + 1)^2} dx.$$

Q. 6.

$$Evalute \int e^{\tan^{-1} x} \left(\frac{1 + x + x^2}{1 + x^2} \right) dx.$$

Q. 7

$$Evalute \int \frac{Sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{Sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx \quad x \in [0, 1]$$

Q. 8.

$$Evalute \int \frac{\sqrt{x^2 + 1} \left[\log \left(x^2 + 1\right) - 2\log x\right]}{x^4} dx.$$

Q. 9.

$$Evalute \int \frac{x}{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}} dx.$$

Q. 10.

$$If\ \frac{d}{dx} f(x) = \sin 5x + \cos 2x - \frac{3}{x^2},\ such\ that\ f(\pi) = \frac{1}{5}$$

then find $f(x)$

Algebra of Matrices

Q. 1. If

$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

, Prove that

$(A+B)^2 \neq A^2 + B^2 + 2AB.$

Q. 2. If

$A = \begin{bmatrix} 0 & 1 \\ -1 & \end{bmatrix}$, find x and y such that $(xI + yA)^2 = A.$

Q. 3. Let f(x) = x2 – 5x + 6. Find f(A) if

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}.$$

Q. 4. Let A and two matricessuch that they commute. Show that for any positive integer n (AB)n = An Bn.

Q. 5. Find the value of x, if

$$\begin{bmatrix} 2 & 1 & 7 \\ 0 & 0 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = I.$$

Q. 6. Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric. Hence represent

$$A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

as above.

Q. 7. Using elementary row transformation find the inverse of the matrix

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

Algebra of Matrices Cont.

Q. 1.

$\begin{bmatrix} x & X^{+}y \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ -4 & 3 \end{bmatrix}$, find X and Y.

Q. 2.

$\begin{bmatrix} x-y & X^{+}2z \\ X^{+} & z^{+}3w \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ 4 & 15 \end{bmatrix}$, find X, Y, Z and W.

Q. 3.

$If\ A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} and\ B = \begin{bmatrix} 0 & 8 \\ 4 & 15 \end{bmatrix}$, prove that $(A+B)^2 \neq A^2 + 2AB + B^2$.

Q. 4. Find x such that:

$$\begin{bmatrix} 1 & 1 & X \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0.$$

Q. 5. Find x such that:

$$\begin{bmatrix} X & 4 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0.$$

Q. 6.

$If\ A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find X and Y such that $A^2 = xA + yI$

Q. 7.

$$2\begin{bmatrix} x & p \\ y & q \end{bmatrix} + 5\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 0 & -5 \\ 6 & 4 \end{bmatrix}$$

Q. 8.

If X + Y = $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \& X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$, find matrices X and Y.

Q. 9. Find a matrix X such that 2A + 2B + X = 0, where

$A = \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$

Q. 10.

Let A = $\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$. Find X and Y such that, X + Y = A and X is a symmetric matrix and Y is a skew symmetric matrix.

Q.11.

Let A = $\begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ and I be the identity matrix of order 2.

Show that $[I + A] = [I - A] \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Q. 12.

Let f(x) = x2 + 5x + 6I, find f(A) if $A = \begin{bmatrix} 2 & 0 & 4 \\ 1 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$

Q. 13.

If f(x) = $\begin{bmatrix} \cos X & \sin X \\ -\sin X & \cos X \end{bmatrix}$ then prove that f(x).f(y) = f(x+y).

Q. 14.

If f (x) = $\begin{bmatrix} \cos X & -\sin X & 0 \\ \sin X & \cos X & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then prove that f(x). f(y) = f(x + y).

Q. 15. If A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ prove by mathematical induction that $(aI + bA)^n = a^n I + na^{n-1} bA$ where I is the identity matrix of the order 2 and n is a positive integer.

Q. 16.

i.

If A = $\begin{bmatrix} 0 & 0 \\ 7 & 0 \end{bmatrix}$, show that A^2 is a zero matrix.

ii.

If A = $\begin{bmatrix} ab & 0^2 \\ -a^2 & -ab \end{bmatrix}$, show that A^2 is a zero matrix

iii.

If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, show that A^2 = $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

Q. 17.

If A = $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -a \end{bmatrix}$ and (A + B) 2 = A2 + B2, find a and b.

Q. 18.

If A = $\begin{bmatrix} 0 & 5 & 7 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix}$, then prove that A4 = 0, where 0 stands for a null matrix.

Q. 19.

If A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then show that A3 = 4A.

Q. 20.

If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 3 & 3 \end{bmatrix}$, then prove that A^3 A^2 + 7A + 2I = 0.

Q. 21.

$$If\ A = \begin{bmatrix} 4 & 2 & 9 \\ -2 & 3 & 5 \\ 0 & 2 & -7 \end{bmatrix},\ B = \begin{bmatrix} -3 & 2 & 4 \\ -1 & -4 & -2 \\ 2 & 4 & 8 \end{bmatrix} and\ C = \begin{bmatrix} 1 & 2 & 7 \\ 8 & 5 & 6 \\ 3 & 4 & 9 \end{bmatrix}$$

Find each of the following:

i. (A + C)’
ii. 2A’ – B’
iii. (A + B + C)’

Q. 22. Prove that any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

Q. 23. A store has in stock 240 shirts, 192 jerseys and 384 trousers. The selling price of one shirt, one jersey and one trouser are Rs 350, Rs 500 and Rs 400 respectively. Using matrices, find the money the store owner will receive by selling all the items of his stock.
[Ans. Rs 333600]

Q. 24. Three shopkeepers A, B and C go to a store to buy stationary items. A purchases 12 dozen notebooks, 6 dozen pens and 10 dozen pencils; B purchases 20 dozen notebooks, 10 dozen pens and 15 dozen pencils and C purchases 10 dozen notebooks, 10 dozen pens and 25 dozen pencils. If per dozen rates of notebooks, pen and pencil are Rs 72, Rs 48 and Rs 18
respectively, use matrix multiplication to calculate bill of each.
[Ans. Rs 1332, Rs 2190 and Rs 1650]

Q. 25. Express the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ **as the sum of symmetric and skew symmetric matrix.**

Q. 26. If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that A2 – 4A – 5I = 0

Q. 27. A contractor gets his supply of building materials from three firms A, B and C. He receives 35 truck load of stones and 14 truck load of sand from A, 30 truck load of stones and 8 truck load of sand from B and 29 truck load of stones and 9 truck load of sand from C. The stones cost Rs 1000 per truck and sand Rs 300 per truck. Using matrix multiplication find the amount received by each firm from the contractor.
[Ans. Rs 39200, Rs 32400 and Rs 31700]

Q. 28. Tarun wants to invest Rs 30000 in two different types of bonds paying 5% p.a. and7%p.a. Using matrix multiplication find how much amount should he invest in each type ofbond to earn an annual interest of Rs 2000.
[Ans. Rs 5000 in 5%, Rs 25000 in 7%]

Q. 29.

If A = $\begin{bmatrix} 1 & 0 & 5 \\ 2 & -1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 2 & 5 \end{bmatrix}$
Find A', B' and (A + B)'. Is (A + B)' = A' + B'?

Q. 30.

If A = $\begin{bmatrix} 3 & 0 & 5 \\ 2 & 4 & 3 \\ 4 & 6 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 & 1 \\ 0 & 2 & 4 \\ 1 & 2 & 5 \end{bmatrix}$ verify (AB)' = B'.A'.

Q. 31.

If A = $\begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$ prove by mathematical induction that $A^n = \begin{bmatrix} a^n & na^{n-1} \\ 0 & a^n \end{bmatrix}$ *for* $n \in N$.

Q. 32.

If A = $\begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$ prove by mathematical induction that $A^n = \begin{bmatrix} \cos nA & \sin nA \\ -\sin nA & \cos nA \end{bmatrix}$ *for* $n \in N$.

Q. 33.

If X = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ prove by mathematical induction that $X^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ *for* $n \in N$.

Differential Calculus I

Q. 1. Discuss the continuity of

$$f(x) = \begin{cases} |x| + 3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x \le 3. \\ 6x + 2, & \text{if } x \ge 3 \end{cases}$$

Q. 2. If

$$y = \frac{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}{\sqrt{a^2 + x^2} - \sqrt{a^2 - x^2}}, \text{ show that } \frac{dy}{dx} = \frac{-2a^2}{x^3}\left\{1 + \frac{a^2}{\sqrt{a^4 - x^4}}\right\}.$$

Q. 3. Differentiate the following function w.r.t. x….

$$\tan^{-1}\left\{\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right\}.$$

Q. 4. If $\sqrt{1 - x^6} + \sqrt{1 - y^6} = a(x^3 - y^3)$ then show that

$$\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1 - y^6}{1 - x^6}}.$$

Q. 5.

$$y = xx^2 - 3 + (x - 3)\,x2 \text{ for } x > 3, \text{ find } \frac{dy}{dx}.$$

Q. 6. Differentiate $Sin - 1(2\,ax\sqrt{1 - a^2x^2})$ with respect to $\sqrt{1 - a^2x^2}$.

Q. 7. If $y = e^{a\cos - 1x}$, $-1 \le x \le 1$, show that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0.$

Q. 8. Find the value of a and b such that the function defined by

$$f(x)=\begin{cases}5, & if\ x\le 2\\ ax+b, & if\ 2<x<10\\ 21, & if\ x\ge 10\end{cases}\quad ia\ a\ continuous\ function.$$

Differential Calculus II

Q. 1. Discuss the continuity of

$$f(x)=\begin{cases}-2, & if\ x\le -1\\ 2x, & if\ -1<x\le 1.\\ 2, & if\ x>1\end{cases}$$

Q. 2. Find the relationship between a and b so that the function f defined by

$$f(x)=\begin{cases}ax+1, & if\ x\le 3\\ bx+3, & if\ x>3\end{cases}\ is\ continuous\ at\ x=3.$$

Q. 3. If

$y=\left[x+\sqrt{x^2+a^2}\right]^n$ then show that $\dfrac{dy}{dx}=\dfrac{ny}{\sqrt{x^2+a^2}}.$

Q. 4. $y=\tan^{-1}\left(\dfrac{3a^2x-x^3}{a^3-3ax^2}\right)-\dfrac{a}{\sqrt{3}}<x<\dfrac{a}{\sqrt{3}},\ find\ \dfrac{dy}{dx}.$

Q. 5. If cos y = x Cos (a + y), with

$Cos\ a\ne\pm 1,$ prove that $\dfrac{dy}{dx}=\dfrac{Cos^2\ (a+y)}{\sin\ a}$

Q. 6. Differentiate $(\log x)^x+x\log^x$ w.r.to x.

Q. 7.

$x=a\left[Cost+\dfrac{1}{2}\log\ \tan^2\dfrac{t}{2}\right]and\ y=a\ Sin\ t,\ show\ that\ \dfrac{dy}{dx}=\tan t.$

Q. 8. If $(x-a)^2+(y-b)^2=c^2$, for some c > 0, prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

is a constant independent of a and b.

Differential Calculus III

Q. 1. Discuss the continuity of

$$f(x)=\begin{cases}2x, & if\ x<0\\ 0, & if\ 0\le x\le 1.\\ 4x, & if\ x>1\end{cases}$$

Q. 2. For what value of λ is the function defined by

$$f(x)=\begin{cases}\lambda\ (x^2-2x), & if\ x\le 0\\ 4x+1, & if\ x>0.\end{cases}$$

continuous at x = 0? What about continuity at x = 1?

Q. 3.

$$If\ y=\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}},\ show\ that\ \frac{dy}{dx}=2x+\frac{2x^3}{\sqrt{x^4-4}}.$$

Q. 4.

$$y=Cos^{-1}\left(2x\sqrt{1-x^2}\right)-\frac{1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}},\ Find\ \frac{dy}{dx}.$$

Q. 5.

$$If\ Sin\ y=x\ Sin\ (a+y),\ prove\ that\ \frac{dy}{dx}=\frac{Sin^2\ (a+y)}{Sin\ a}.$$

Q. 6.

$$If\ Sin\ y=\ (Sin\ x)^{\tan x}+(Cos\ x)^{\sec x},\ Find\ \frac{dy}{dx}.$$

Q. 7.

$$If\ u = Sin\ (m\ Cos^{-1}x),\ v = Cos\ (m\ Sin^{-1}x),\ prove\ that\ \frac{du}{dv} = \sqrt{\frac{1-u^2}{1-v^2}}$$

Q. 8.

$$If\ x = a\ (Cost + t\ Sin\ t)\ and\ y = a\ (Sin - t\ Cost),\ Find\ \frac{d^2y}{dx^2}$$

More Questions

Q. 1. find the equation of the line joining)3,1(*A* and)0,0(*B* using determinants and find *k* if)0,(*kD* is a point such that area of triangle *ABD* is 3 sq. units.

Q. 2. If a, b and c are real numbers, and

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

, show that either a + b + c = 0 or a = b = c.

Q. 3. Solve the following system of linear equation

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

Q. 4. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Q. 5. Without expanding show that

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & a & b^2-bc \\ 1 & a & c^2-bc \end{vmatrix} = 0.$$

Q. 6. Show that :

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)\ (b-c)\ (ab+bc+ca).$$

Q. 7. Show that the matrix A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation x2 – 4x – 5 = 0. Hence find A-1

Q. 8. If A =

$\begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$; show that A2 = A-1 . (Without using elementary transformations)

Q.9. Construct a 3 × 3 matrix whose elements aij are given by

$$aij = \begin{cases} i - j, if\ i > j \\ i + j, if\ i < j \end{cases}$$

Q.10. If $\begin{bmatrix} 7 & x^2 \\ y^3 & 18 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ -64 & 18 \end{bmatrix}$ then find the values of x and y .

Q.11. How many number of matrices are possible of order 3 × 3 with each entry 0 or 1.

Q.12. If A is square matrix such that A2 = A , then find the value of (I + A)3 – 7A.

Q.13. Using elementary transformation , find the inverse of the matrix. $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Q.14. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.

Q.15.

$$If\ A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} and\ A + A^T = I\ then\ find\ the\ value\ of\ \alpha$$

Q. 16.

$$If\ f(x) = x^2 - 5x + 7\ find\ f\ (A)\ if\ A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

Q.17.

$$Let\,A = \begin{bmatrix} 0 & -\tan\dfrac{\theta}{2} \\ \tan\dfrac{\theta}{2} & 2 \end{bmatrix} show\ that\ (I + A) = (I - A)\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Q.18.

$$If\ A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, then\ find\ k\ so\ that\ A^2 = 8A + KI.$$

Q.19.

$Let\ A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. Using the principle of mathematical induction , show that

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$