

AIEEE – 2010 TEST PAPER WITH ANSWER & SOLUTIONS (HELD ON SUNDAY 25TH APRIL, 2010)

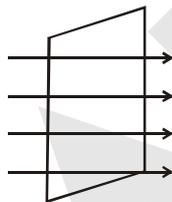
Direction : Questions number 1 – 3 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is :-
- (1) planar
 - (2) convex
 - (3) concave
 - (4) convex near the axis and concave near the periphery

Ans. (1)

Sol. Parallel cylindrical beam gives planar wavefront



2. The speed of the light in the medium is :-
- (1) maximum on the axis of the beam
 - (2) minimum on the axis of the beam
 - (3) the same everywhere in the beam
 - (4) directly proportional to the intensity I

Ans. (2)

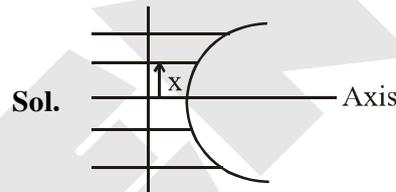
Sol. $\mu = \frac{c}{v} \Rightarrow v = \frac{c}{\mu}$

Since I is decreasing so μ also decreases and hence v increases

So v is minimum on the axis of the beam.

3. As the beam enters the medium, it will :-
- (1) travel as a cylindrical beam
 - (2) diverge
 - (3) converge
 - (4) diverge near the axis and converge near the periphery

Ans. (3)



converge when it enters in the medium.

When light is moving and as it enters the medium then along the axis velocity is decreasing so as we move away from the centre (that is x in figure) the wave covers less distance and hence shape is convex.

Directions : Questions number 4 – 5 are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays

into two daughter nuclei of equal mass $\frac{M}{2}$

each. Speed of light is c .

4. The speed of daughter nuclei is :-

(1) $c \sqrt{\frac{\Delta m}{M + \Delta m}}$ (2) $c \frac{\Delta m}{M + \Delta m}$

(3) $c \sqrt{\frac{2\Delta m}{M}}$ (4) $c \sqrt{\frac{\Delta m}{M}}$

Ans. (3)

IIT-JEE

Announcement

**Classes for
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(XII Appeared / Result Awaited)
starting from
29th April 2010**

Objective : This course is for those students who want to take lead by making an early step towards TARGET IIT-JEE 2011 by utilising precious 60 days prior to start of Phase-II.

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Sol. Total kinetic energy of products = Total energy

$$\text{released } \frac{p^2}{2m} + \frac{p^2}{2m} = (\text{mass defect}) c^2$$

(where $m = \frac{M}{2}$ given)

$$2 \left(\frac{p^2}{2m} \right) = \left[(M + \Delta m) - \left(\frac{M}{2} + \frac{M}{2} \right) \right] \times c^2$$

$$2 \times \left[\frac{p^2}{2 \left(\frac{M}{2} \right)} \right] = (\Delta m) c^2$$

$$\frac{2 \left(\frac{M}{2} v \right)^2}{M} = (\Delta m) c^2$$

$$\Rightarrow v = c \sqrt{\frac{2\Delta m}{M}}$$

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2 . Then :-

- (1) $E_1 = 2E_2$ (2) $E_2 = 2E_1$
 (3) $E_1 > E_2$ (4) $E_2 > E_1$

Ans. (4)

Sol. Because energy is releasing
 \Rightarrow Binding energy per nucleon of product $>$ that of parent
 $\Rightarrow E_2 > E_1$.

Directions : Questions number 6 – 7 contain **Statement-1** and **Statement-2**. Of the four choices given after the statements, choose the one that best describes the two statements.

6. Statement-1 : When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{\max} . When the ultraviolet light is replaced by X-rays, both V_0 and K_{\max} increase.

Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

- (1) Statement-1 is true, Statement-2 is false
 (2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1
 (3) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
 (4) Statement-1 is false, Statement-2 is true

Ans. (1)

7. Statement-1 : Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

- (1) Statement-1 is true, Statement-2 is false
 (2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1
 (3) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
 (4) Statement-1 is false, Statement-2 is true

Ans. (2)

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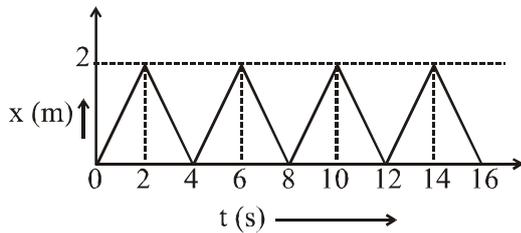
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8. The figure shows the position-time ($x - t$) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is :-

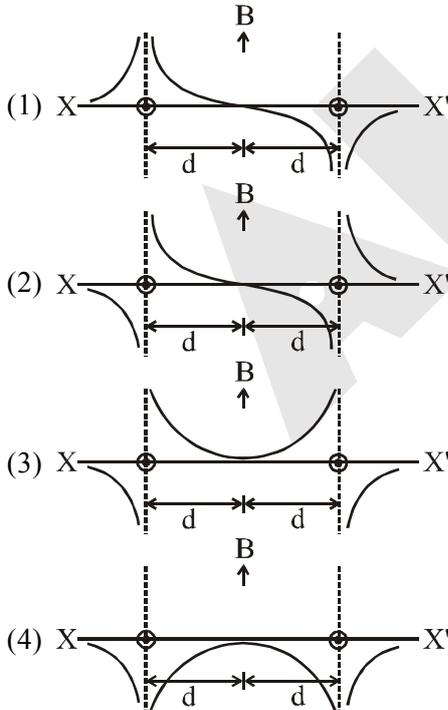


- (1) 0.2 Ns (2) 0.4 Ns (3) 0.8 Ns (4) 1.6 Ns

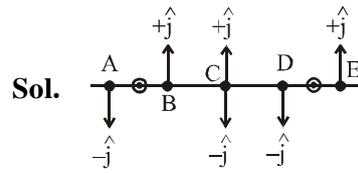
Ans. (3)

Sol. Impulse = $\Delta P = m(V_f - V_i)$
 $= 0.4 [1 - (-1)] = 0.8 \text{ N-s}$

9. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by:-



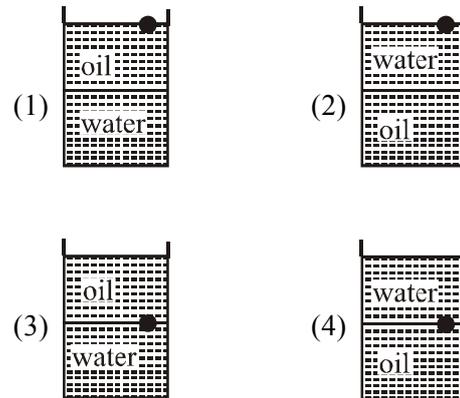
Ans. (2)



considering A,B,C,D,E points in plane where C is the mid point.

At mid point magnetic field is zero and right and left side of mid point magnetic field is in opposite direction and for both conductors magnetic field in right & left side are opposite in direction.

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ?



Ans. (3)

Sol. In equilibrium ball will remain at the interface of water and oil.

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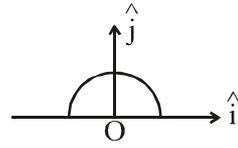


11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field \vec{E} at the centre O is :-

(1) $\frac{q}{2\pi^2 \epsilon_0 r^2} \hat{j}$

(2) $\frac{q}{4\pi^2 \epsilon_0 r^2} \hat{j}$

(3) $-\frac{q}{4\pi^2 \epsilon_0 r^2} \hat{j}$



(4) $-\frac{q}{2\pi^2 \epsilon_0 r^2} \hat{j}$

Ans. (4)

Sol. $\vec{E} = \frac{2k\lambda}{r} (-\hat{j}) = \frac{2}{(4\pi \epsilon_0 r)} \frac{q}{(\pi r)} (-\hat{j})$

$= -\frac{q}{2\pi^2 \epsilon_0 r^2} (+\hat{j}) = -\frac{q}{2\pi^2 \epsilon_0 r^2} (\hat{j})$

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to $32V$, the efficiency of the engine is :-

- (1) 0.25 (2) 0.5 (3) 0.75 (4) 0.99

Ans. (3)

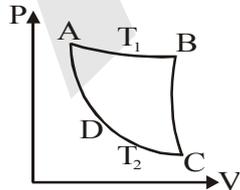
Sol. $T_B = T_1, T_C = T_2, \gamma = 1.4$
 $V_B = V, V_C = 32V$

$T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$

$\frac{T_C}{T_B} = \frac{T_2}{T_1} = \left(\frac{V_B}{V_C}\right)^{\gamma-1}$

$= \left(\frac{1}{32}\right)^{\gamma-1} = \frac{1}{4}$

$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$



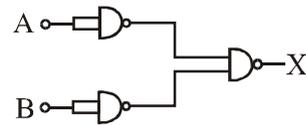
13. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are:-

- (1) 4, 4, 2 (2) 5, 1, 2
 (3) 5, 1, 5 (4) 5, 5, 2

Ans. (2)

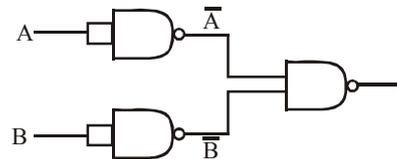
Sol. In 23.023 Number of significant figures will be 5 because all the zero's between non zero digits are significant. In 0.0003 number of significant figures will be one because all the zero's before and after decimal point are insignificant if the number is less than one. In 2.1×10^{-3} number of significant figure are two because power of 10 is not considered as significant figure.

14. The combination of gates shown below yields:-



- (1) NAND gate (2) OR gate
 (3) NOT gate (4) XOR gate

Ans. (2)



Sol.

$x = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}}$

$x = A + B$

\Rightarrow OR gate

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15. If a source of power 4kW produces 10^{20} photons/second, the radiation belongs to apart of the spectrum called :-

- (1) γ -rays (2) X-rays
(3) ultraviolet rays (4) microwaves

Ans. (2)

Sol. No. of photons emitting per second from a source of power P is

$$n = (5 \times 10^{24}) P\lambda$$

$$\Rightarrow \text{wavelength emitting } \lambda = \frac{n}{(5 \times 10^{24}) P}$$

$$\left[\text{or } \lambda = \frac{nhc}{P} \right]$$

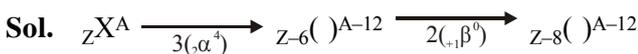
$$\Rightarrow \lambda = \frac{10^{20}}{5 \times 10^{24} \times 4 \times 10^3} = 0.5 \times 10^{-9} \text{ m} = 50 \text{ \AA}$$

And this wavelength comes in X ray region.

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α -particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be:-

- (1) $\frac{A-Z-4}{Z-2}$ (2) $\frac{A-Z-8}{Z-4}$
(3) $\frac{A-Z-4}{Z-8}$ (4) $\frac{A-Z-12}{Z-4}$

Ans. (3)



$$\therefore \frac{\text{Number of neutrons}}{\text{Number of protons}} = \frac{(A-12) - (Z-8)}{Z-8}$$

$$= \frac{A-Z-4}{Z-8}$$

17. Let there be a spherically symmetric charge distribution with charge density varying as

$$\rho(r) = \rho_0 \left(\frac{5}{4} - \frac{r}{R} \right) \text{ upto } r = R, \text{ and } \rho(r) = 0 \text{ for}$$

$r > R$, where r is the distance from the origin. The electric field at a distance r ($r < R$) from the origin is given by :

$$(1) \frac{\rho_0 r}{3\epsilon_0} \left(\frac{5}{4} - \frac{r}{R} \right) \quad (2) \frac{4\pi\rho_0 r}{3\epsilon_0} \left(\frac{5}{3} - \frac{r}{R} \right)$$

$$(3) \frac{\rho_0 r}{4\epsilon_0} \left(\frac{5}{3} - \frac{r}{R} \right) \quad (4) \frac{4\rho_0 r}{3\epsilon_0} \left(\frac{5}{4} - \frac{r}{R} \right)$$

Ans. (3)

$$\text{Sol. Total charge } Q = \int_0^r \rho \, dv = \int_0^r \rho_0 \left(\frac{5}{4} - \frac{r}{R} \right) 4\pi r^2 dr$$

$$= 4\pi\rho_0 \int_0^r \left(\frac{5r^2}{4} - \frac{r^3}{R} \right) dr$$

$$= 4\pi\rho_0 \left[\frac{5r^3}{12} - \frac{r^4}{4R} \right]$$

$$E = \frac{KQ}{r^2} = \frac{1}{4\pi\epsilon_0 r^2} 4\pi\rho_0 \left[\frac{5}{12} r^3 - \frac{r^4}{4R} \right]$$

$$= \frac{\rho_0 r}{4\epsilon_0} \left[\frac{5}{3} - \frac{r}{R} \right]$$

18. In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30° . On taking out the inductor from the circuit the current leads the voltage by 30° . The power dissipated in the LCR circuit is :

- (1) 242 W (2) 305 W (3) 210 W (4) Zero W

Ans. (1)

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Sol. $\tan \phi = \frac{X_L - X_C}{R}$

$\tan 30^\circ = \frac{X_C}{R} \Rightarrow X_C = \frac{R}{\sqrt{3}}$

$\tan 30^\circ = \frac{X_L}{R} \Rightarrow X_L = \frac{R}{\sqrt{3}}$

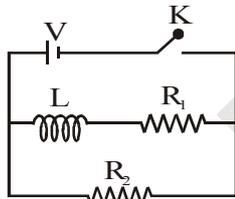
$X_L = X_C \Rightarrow$ Condition for resonance

So $\phi = 0^\circ$

$P = VI \cos 0^\circ$

$P = \frac{V^2}{R} = \frac{(220)^2}{200} = 242W$

19. In the circuit show below, the key K is closed at $t = 0$. The current through the battery is :



(1) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(2) $\frac{V R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

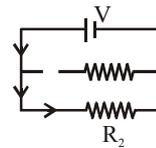
(3) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$

(4) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

Ans. (3)

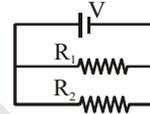
Sol. At $t = 0$ inductor behaves as broken wire then

$i = \frac{V}{R_2}$



at $t = \infty$ Inductor behaves as conducting wire

$i = \frac{V}{R_1 R_2 / (R_1 + R_2)}$



20. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is :
 (1) $y^2 = x^2 + \text{constant}$ (2) $y = x^2 + \text{constant}$
 (3) $y^2 = x + \text{constant}$ (4) $xy = \text{constant}$

Ans. (1)

Sol. $V = K (y\hat{i} + x\hat{j})$

$V_x = Ky$

$\frac{dx}{dt} = Ky$

similarly $\frac{dy}{dt} = Kx$

Hence $\frac{dy}{dx} = \frac{x}{y}$

$\Rightarrow y dy = x dx$, by integrating
 $y^2 = x^2 + c$.

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be :

- (1) 2 (2) 1 (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Ans. (4)

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Sol. $U = \frac{1}{2} CV^2$

$$\frac{U_0}{2} = \frac{1}{2} CV_0^2 e^{-2t_1/RC}$$

$$-\frac{1}{2} = e^{-2t_1/RC} \quad (U_0 = \frac{1}{2} CV_0^2)$$

$$\frac{2t_1}{RC} = \ln 2$$

$$t_1 = \frac{RC \ln 2}{2} \quad \dots(1)$$

and $\frac{q_0}{4} = q_0 e^{-t_2/RC}$

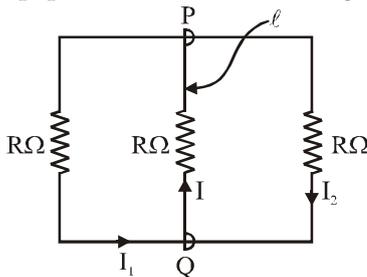
$$-\frac{t_2}{RC} = 2 \ln 2$$

$$t_2 = 2RC \ln 2 \quad \dots(2)$$

from equation (1) and (2)

$$\frac{t_1}{t_2} = \frac{1}{4}$$

22. A rectangular loop has a sliding connector PQ of length ℓ and resistance $R\Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1 , I_2 and I are



$$(1) I_1 = I_2 = \frac{B\ell v}{6R}, I = \frac{B\ell v}{3R}$$

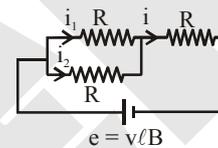
$$(2) I_1 = -I_2 = \frac{B\ell v}{R}, I = \frac{2B\ell v}{R}$$

$$(3) I_1 = I_2 = \frac{B\ell v}{3R}, I = \frac{2B\ell v}{3R}$$

$$(4) I_1 = I_2 = I = \frac{B\ell v}{R}$$

Ans. (3)

Sol. Circuit can be reduced as



$$i = \frac{e}{3R/2} = \frac{2v\ell B}{3R}$$

$$i_1 = i_2 = \frac{i}{2} = \frac{v\ell B}{3R}$$

23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02(\text{m}) \sin \left[2\pi \left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})} \right) \right].$$
 The

tension in the string is :

- (1) 6.25 N (2) 4.0 N
(3) 12.5 N (4) 0.5 N

Ans. (1)

Sol. $y = 0.2 \sin \left[2\pi \left(\frac{t}{0.04} - \frac{x}{0.50} \right) \right]$

$$v = \sqrt{\frac{T}{m}} = \frac{\omega}{k}$$

$$\sqrt{\frac{T}{0.04}} = \frac{1}{0.50}$$

$$T = \left(\frac{0.50}{0.04} \right)^2 \times 0.04$$

$$= (12.5)^2 \times 0.04 = 6.25 \text{ Newton.}$$

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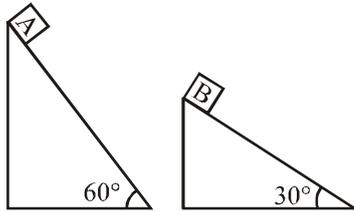
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24. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



- (1) 4.9 ms^{-2} in vertical direction.
- (2) 4.9 ms^{-2} in horizontal direction
- (3) 9.8 ms^{-2} in vertical direction
- (4) Zero

Ans. (1)

Sol. relative vertical acceleration of A with respect to B

$$= g (\sin^2 60^\circ - \sin^2 30^\circ)$$

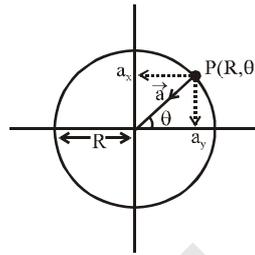
$$= 9.8 \left(\frac{3}{4} - \frac{1}{4} \right) = 4.9 \text{ m/s}^2$$

25. For a particle in uniform circular motion, the acceleration \vec{a} at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x-axis).

- (1) $\frac{v^2}{R} \hat{i} + \frac{v^2}{R} \hat{j}$
- (2) $-\frac{v^2}{R} \cos \theta \hat{i} + \frac{v^2}{R} \sin \theta \hat{j}$
- (3) $-\frac{v^2}{R} \sin \theta \hat{i} + \frac{v^2}{R} \cos \theta \hat{j}$
- (4) $-\frac{v^2}{R} \cos \theta \hat{i} - \frac{v^2}{R} \sin \theta \hat{j}$

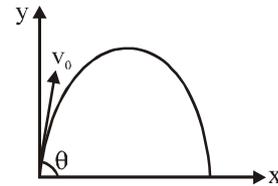
Ans. (4)

Sol.



$$\vec{a} = -\frac{V^2}{R} \cos \theta \hat{i} - \frac{V^2}{R} \sin \theta \hat{j}$$

26. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin \theta}{g}$, the angular momentum of the particle is :



- (1) $\frac{1}{2} mg v_0 t^2 \cos \theta \hat{i}$
- (2) $- mg v_0 t^2 \cos \theta \hat{j}$
- (3) $mg v_0 t \cos \theta \hat{k}$
- (4) $-\frac{1}{2} mg v_0 t^2 \cos \theta \hat{k}$

Where \hat{i} , \hat{j} and \hat{k} are unit vectors along x, y and z-axis respectively.

Ans. (4)

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Sol. \therefore Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

where

$$\vec{r} = u \cos \theta t \hat{i} + \left(u \sin \theta t - \frac{1}{2} g t^2 \right) \hat{j}$$

$$\vec{p} = m \left[u \cos \theta \hat{i} + (u \sin \theta - g t) \hat{j} \right]$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$= -\frac{1}{2} m g V_0 t^2 \cos \theta \hat{k}$$

27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3} , the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3} , the dielectric constant of the liquid is :

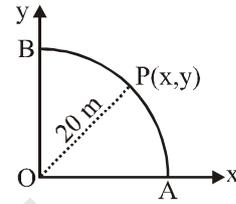
- (1) 1 (2) 4 (3) 3 (4) 2

Ans. (4)

Sol. $K = \frac{\rho}{\rho - \sigma} = \frac{1.6}{1.6 - 0.8} = 2$

$\rho \rightarrow$ density of sphere, $\sigma \rightarrow$ density of liquid

28. A point P moves in counter clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 2\text{s}$ is nearly :



- (1) 14 m/s^2 (2) 13 m/s^2
(3) 12 m/s^2 (4) 7.2 m/s^2

Ans. (1)

Sol. As $S = t^3 + 5$

$$\frac{ds}{dt} = 3t^2 = V$$

$$\therefore a_t = \frac{dv}{dt} = 6t$$

at $t = 2\text{sec}$

$$|\vec{a}| = \sqrt{a_c^2 + a_t^2}$$

$$= \sqrt{\left(\frac{V^2}{R}\right)^2 + a_t^2}$$

$$= \sqrt{\frac{(3t^2)^2}{R} + \left(\frac{dv}{dt}\right)^2}$$

$$= \sqrt{\frac{9t^4}{R} + 36t^2}$$

$$= \sqrt{144 + (7.2)^2}$$

$$= 14 \text{ m/s}^2.$$

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29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constant and x is the distance between the atoms. if the dissociation energy of the molecule is

$D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is :

- (1) $\frac{b^2}{6a}$ (2) $\frac{b^2}{2a}$ (3) $\frac{b^2}{12a}$ (4) $\frac{b^2}{4a}$

Ans. (4)

Sol. $U = \frac{a}{x^{12}} - \frac{b}{x^6}$

$$F = -\frac{dU}{dx} = +12\frac{a}{x^{13}} - \frac{6b}{x^7} = 0$$

$$\Rightarrow x = \left(\frac{2a}{b}\right)^{1/6}$$

$$U(x = \infty) = 0$$

$$U_{\text{equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = -\frac{b^2}{4a}$$

$$\therefore U(x = \infty) - U_{\text{equilibrium}} = 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$$

30. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2 . The respective temperature coefficients of their series and parallel combinations are nearly :

(1) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

(2) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

(3) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$

(4) $\alpha_1 + \alpha_2, \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$

Ans. (1)

Sol. $\alpha_S = \frac{\alpha_1 R_{01} + \alpha_2 R_{02}}{R_{01} + R_{02}}$

$R_{01} = R_{02} = R_0$ (given)

$$\alpha_S = \frac{\alpha_1 + \alpha_2}{2}$$

For parallel combination

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{R_0(1 + \alpha_1 t)} + \frac{1}{R_0(1 + \alpha_2 t)}$$

$$\frac{1}{\frac{R_0}{2}(1 + \alpha_p t)} = \frac{1}{R_0(1 + \alpha_1 t)} + \frac{1}{R_0(1 + \alpha_2 t)}$$

$2(1 + \alpha_p t)^{-1} = (1 + \alpha_1 t)^{-1} + (1 + \alpha_2 t)^{-1}$
using binomial expansion

$$2 - 2\alpha_p t = 1 - \alpha_1 t + 1 - \alpha_2 t$$

$$\Rightarrow \alpha_p = \frac{\alpha_1 + \alpha_2}{2}$$

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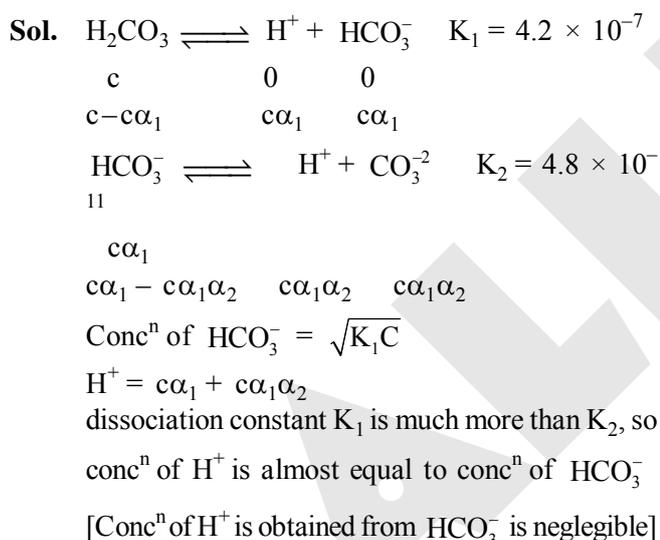
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31. In aqueous solution the ionization constants for carbonic acid are
 $K_1 = 4.2 \times 10^{-7}$ and $K_2 = 4.8 \times 10^{-11}$
 Select the correct statement for a saturated 0.034 M solution of the carbonic acid :-
 (1) The concentration of H^+ is double that of CO_3^{2-}
 (2) The concentration of CO_3^{2-} is 0.034 M
 (3) The concentration of CO_3^{2-} is greater than that of HCO_3^-
 (4) The concentrations of H^+ and HCO_3^- are approximately equal

Ans. (4)



32. Solubility product of silver bromide is 5.0×10^{-13} . The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is :-
 (1) $5.0 \times 10^{-8} \text{ g}$ (2) $1.2 \times 10^{-10} \text{ g}$
 (3) $1.2 \times 10^{-9} \text{ g}$ (4) $6.2 \times 10^{-5} \text{ g}$

Ans. (3)

Sol. $Br^- = \frac{K_{sp}(\text{AgBr})}{C_{Ag^+}}$

$$Br^- = \frac{5 \times 10^{-13}}{0.05} = 10^{-11}$$

$$Conc^n = [KBr] = 10^{-11}$$

$$\text{Mole of KBr} = 10^{-11}$$

$$\text{weight of KBr} = 10^{-11} \times 120 = 1.2 \times 10^{-9} \text{ g}$$

33. The correct sequence which shows decreasing order of the ionic radii of the elements is :-

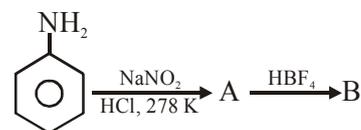
- (1) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$
 (2) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$
 (3) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$
 (4) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$

Ans. (1)

Sol. For an isoelectronic series size $\propto \frac{-ve \text{ charge}}{+ve \text{ charge}}$

$$So = O^{2-} > F^- > Na^+ > Mg^{+2} > Al^{+3}$$

34. In the chemical reactions,



the compounds 'A' and 'B' respectively are :-

- (1) Nitrobenzene and chlorobenzene
 (2) Nitrobenzene and fluorobenzene
 (3) Phenol and benzene
 (4) Benzene diazonium chloride and fluorobenzene

Ans. (4)

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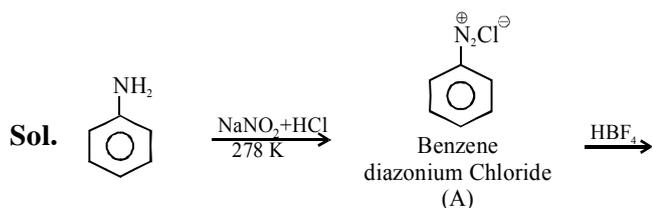
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35. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K , how many moles of water are in the vapour phase when equilibrium is established ?

(Given : Vapour pressure of H_2O at 300 K is 3170 Pa ; $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$) :-

- (1) $1.27 \times 10^{-3} \text{ mol}$ (2) $5.56 \times 10^{-3} \text{ mol}$
(3) $1.53 \times 10^{-2} \text{ mol}$ (4) $4.46 \times 10^{-2} \text{ mol}$

Ans. (1)

Sol. $PV = nRT$

$$3170 \times 10^{-3} = n \times 8.314 \times 300$$

$$n = \frac{3170 \times 10^{-3}}{8.314 \times 300}; \quad V = 1 \text{ Lt} = 1 \text{ dm}^3 = 10^{-3} \text{ m}^3$$

$$n = 1.27 \times 10^{-3} \text{ mol}$$

36. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous ZnCl_2 , is :-

- (1) 1-Butanol
(2) 2-Butanol
(3) 2-Methylpropan-2-ol
(4) 2-Methylpropanol

Ans. (3)

Sol. 3° -Alcohol reacts faster with Luca's reagent [$\text{ZnCl}_2 + \text{Conc. HCl}$]

37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is ($K_f = 1.86 \text{ K kg mol}^{-1}$) :-

- (1) 0.0186 K (2) 0.0372 K
(3) 0.0558 K (4) 0.0744 K

Ans. (3)

Sol. Given

$$\text{Na}_2\text{SO}_4 (\alpha = 100\%)$$

$$i = 3$$

$$k_f = 1.86$$

$$w/m = 0.01$$

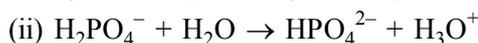
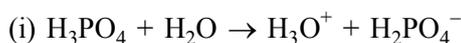
$$W = 1 \text{ kg} = 1000 \text{ g}$$

$$\Delta T_f = \frac{1000 \times k_f \times i \times w}{m \times W}$$

$$\Delta T_f = \frac{1000 \times 1.86 \times 3 \times 0.01}{1000}$$

$$\Delta T_f = 5.58 \times 0.01 = 0.0558 \text{ K}$$

38. Three reactions involving H_2PO_4^- are given below :-



In which of the above does H_2PO_4^- act as an acid ?

- (1) (i) only (2) (ii) only
(3) (i) and (ii) (4) (iii) only

Ans. (2)

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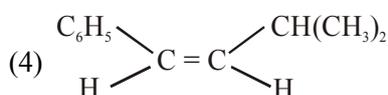
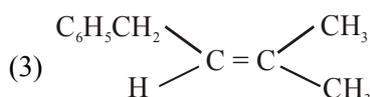
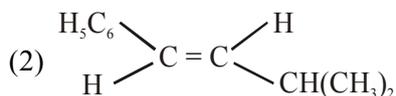
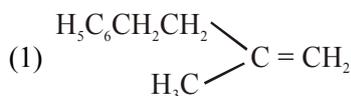
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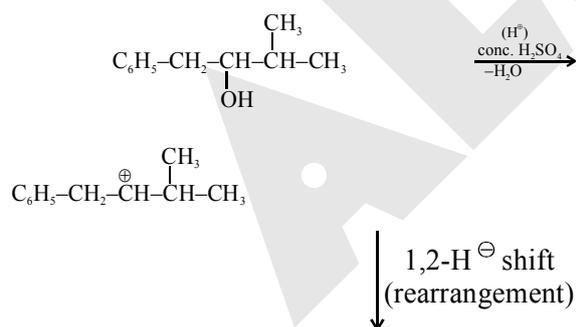


39. The main product of the following reaction is

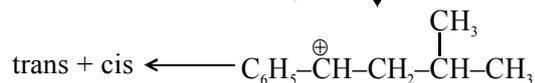


Ans. (2)

Sol.



1,2-H⁺ shift
(rearrangement)



trans is major product due to thermodynamically more stable as well as carbocation show anti elimination.

40. The energy required to break one mole of Cl-Cl bonds in Cl₂ is 242 kJ mol⁻¹. The longest wavelength of light capable of breaking a single Cl-Cl bond is

$$(C = 3 \times 10^8 \text{ ms}^{-1} \text{ and } N_A = 6.02 \times 10^{23} \text{ mol}^{-1})$$

- (1) 494 nm (2) 594 nm
(3) 640 nm (4) 700 nm

Ans. (1)

Sol. ∴ For 1 mole of Cl₂ energy required = 242 × 10³ Joule
∴ For 1 molecule of Cl₂

$$\text{energy required} = \frac{242 \times 10^3 \times 3 \times 10^8}{6.023 \times 10^{23}} \text{ Joule}$$

$$\therefore E = \frac{hc}{\lambda}$$

$$\frac{242 \times 10^3}{6.02 \times 10^{23}} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$\lambda = 0.494 \times 10^{-6} \text{ m} = 494 \text{ nm}$$

41. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is :-

- (1) 29.5 (2) 59.0 (3) 47.4 (4) 23.7

Ans. (4)

$$\text{Sol. } N_x V_x = 20 \times 0.1 - 15 \times 0.1 = 0.5$$

$$\%N = \frac{1.4 N_x V_x}{\text{mass of organic compound taken}}$$

$$\%N = \frac{1.4 \times .5}{29.5 \text{ mg}}$$

$$\%N = \frac{1.4 \times .5}{29.5 \text{ g}} \times 1000 = 23.72$$

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42. Ionisation energy of He^+ is $19.6 \times 10^{-18} \text{ J atom}^{-1}$. The energy of the first stationary state ($n = 1$) of Li^{2+} is :-

- (1) $8.82 \times 10^{-17} \text{ J atom}^{-1}$
- (2) $4.41 \times 10^{-16} \text{ J atom}^{-1}$
- (3) $-4.41 \times 10^{-17} \text{ J atom}^{-1}$
- (4) $-2.2 \times 10^{-15} \text{ J atom}^{-1}$

Ans. (3)

Sol. $E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}$

$E \propto Z^2$

IP of $\text{He}^+ = -E_1 = 19.6 \times 10^{-18} \text{ J atom}^{-1}$ (given)

E_1 for $\text{Li}^{2+} = ?$

$$\frac{E_{(\text{He}^+)}}{E_{(\text{Li}^{2+})}} = \frac{(2)^2}{(3)^2} = \frac{4}{9}$$

$$E_{(\text{Li}^{2+})} = \frac{9}{4} (-19.6 \times 10^{-18}) = -44.1 \times 10^{-18} \\ = -4.41 \times 10^{-17} \text{ J atom}^{-1}$$

43. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1}) :-

- (1) 144.5 kPa
- (2) 72.0 kPa
- (3) 36.1 kPa
- (4) 96.2 kPa

Ans. (2)

Sol. If A = Heptane; B = octane

$$P_S = P_A^o X_A + P_B^o X_B$$

$$X_A = \frac{25/100}{\frac{25}{100} + \frac{35}{114}} = \frac{.25}{.25 + 0.30} = \frac{.25}{.55} = \frac{5}{11}$$

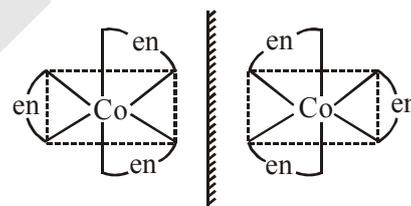
then $X_B = \frac{6}{11}$

$$P_S = 105 \times \frac{5}{11} + 45 \times \frac{6}{11} \\ = 47.727 + 24.54 \approx 72 \text{ kPa}$$

44. Which one of the following has an optical isomer ?

- (1) $[\text{Zn}(\text{en})_2]^{2+}$
 - (2) $[\text{Zn}(\text{en})(\text{NH}_3)_2]^{2+}$
 - (3) $[\text{Co}(\text{en})_3]^{3+}$
 - (4) $[\text{Co}(\text{H}_2\text{O})_4(\text{en})]^{3+}$
- (en = ethylenediamine)

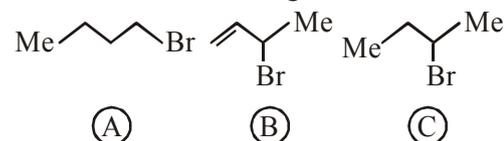
Ans. (3)



Sol.

Octahedral complex having 3 bidentate ligand shows optical isomerism

45. Consider the following bromides :-



The correct order of S_N1 reactivity is

- (1) $A > B > C$
- (2) $B > C > A$
- (3) $B > A > C$
- (4) $C > B > A$

Ans. (2)

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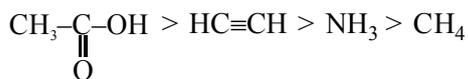
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Ans. (1)

Sol. Acidic strength order :



Basic order of conjugate bases is



50. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is :-

- (1) 144 pm (2) 288 pm (3) 398 pm (4) 618 pm

Ans. (1)

Sol. FCC type structure (For NaCl crystal)

$$r_+ + r_- = \frac{a}{2} \quad a = 508 \text{ pm}$$

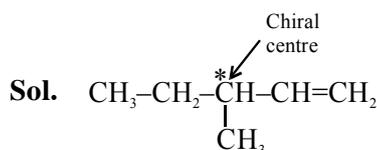
$$110 + r_- = \frac{a}{2} \quad r_+ = 110 \text{ pm}$$

$$r_- = \frac{508}{2} - 110 = 254 - 110 = 144$$

51. Out of the following, the alkene that exhibits optical isomerism is :-

- (1) 2-methyl-2-pentene (2) 3-methyl-2-pentene
(3) 4-methyl-1-pentane (4) 3-methyl-1-pentene

Ans. (4)



3-Methyl-1-pentene

52. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, then reaction would be spontaneous when :-

- (1) T = T_e (2) T_e > T
(3) T > T_e (4) T_e is 5 times T

Ans. (3)

Sol. ΔG = ΔH - TΔS [ΔH = +ve ; ΔS = +ve]

$$\Delta G = +ve - T_e(+ve)$$

if T > T_e then ΔG = -ve (spontaneous)

53. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively :-

- (1) 48% and 26% (2) 30% and 26%
(3) 26% and 32% (4) 32% and 48%

Ans. (3)

Sol. For FCC or CCP

$$4r = \sqrt{2} a$$

$$\text{P.F.} = \frac{n \times \frac{4}{3}\pi r^3}{a^3}$$

$$n = 4 ; a = 2\sqrt{2} r$$

$$\text{P.F.} = \frac{4 \times \frac{4}{3}\pi r^3}{(2\sqrt{2} r)^3}$$

$$\text{P.F.} = 74\%$$

$$\text{free space} = 100 - 74 = 26\%$$

For BCC

$$4r = \sqrt{3} a$$

$$\text{P.F.} = \frac{2 \times \frac{4}{3}\pi r^3}{\left(\frac{4}{\sqrt{3}} r\right)^3}$$

$$\text{P.F.} = 68\%$$

$$\text{Free space} = 32\%$$

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54. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is :-

- (1) natural rubber (2) teflon
(3) nylon 6, 6 (4) polystyrene

Ans. (3)

Sol. Nylon-66 (poly amides) has strong intermolecular forces like hydrogen bonding

55. At 25° C, the solubility product of Mg(OH)₂ is 1.0 × 10⁻¹¹. At which pH, will Mg²⁺ ions start precipitating in the form of Mg(OH)₂ from a solution of 0.001 M Mg²⁺ ions ?

- (1) 8 (2) 9 (3) 10 (4) 11

Ans. (3)

Sol. $K_{sp} = [Mg^{+2}] [OH^-]^2$

$$1 \times 10^{-11} = 10^{-3} \times [OH^-]^2$$

$$[OH^-]^2 = 10^{-8}$$

$$OH^- = 10^{-4}$$

$$pOH = 4 \qquad [pH + pOH = 14]$$

$$pH = 10$$

56. The correct order of $E_{M^{2+}/M}^\circ$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is :-

- (1) Cr > Mn > Fe > Co
(2) Mn > Cr > Fe > Co
(3) Cr > Fe > Mn > Co
(4) Fe > Mn > Cr > Co

Ans. (2)

Sol. Order of standard reduction potential

$$Mn > Cr > Fe > Co$$

(according to electrochemical series)

57. Biuret test is not given by

- (1) proteins (2) carbohydrates
(3) polypeptides (4) urea

Ans. (2)

Sol. Carbohydrates does not give biuret test because of absence of peptide ($-\overset{\text{O}}{\parallel}{C}-NH-$) bond.

58. The time for half life period of a certain reaction $A \longrightarrow \text{Products}$ is 1 hour. When the initial concentration fo the reactant 'A' is 2.0 mol L⁻¹, How much time does it take for its concentration to come from 0.50 to 0.25 mol L⁻¹ if it is a zero order reaction ?

- (1) 1 h (2) 4 h (3) 0.5 h (4) 0.25 h

Ans. (4)

Sol. $t_{1/2} = \frac{a}{2k_0}$ (for zero order reaction)

$$k_0 = \frac{a}{2 \times t_{1/2}} = \frac{2}{2 \times 1} = 1$$

$$k_0 = \frac{A_0 - [A]_t}{t} \Rightarrow k_0 = \frac{0.50 - 0.25}{t} = 1$$

$$t = 0.25 \text{ hr.}$$

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59. A solution containing 2.675 g of $\text{CoCl}_3 \cdot 6\text{NH}_3$ (molar mass = 267.5 g mol⁻¹) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of AgNO_3 to give 4.78 g of AgCl (molar mass = 143.5 g mol⁻¹). The formula of the complex is :-

(At. mass of Ag = 108 u)

- (1) $[\text{CoCl}(\text{NH}_3)_5]\text{Cl}_2$
- (2) $[\text{Co}(\text{NH}_3)_6]\text{Cl}_3$
- (3) $[\text{CoCl}_2(\text{NH}_3)_4]\text{Cl}$
- (4) $[\text{CoCl}_3(\text{NH}_3)_3]$

Ans. (2)

Sol. Moles of $\text{CoCl}_3 \cdot 6\text{NH}_3 = \frac{2.675}{267.5} = \frac{1}{100}$ mole
 = 0.01 mole

Moles of $\text{AgCl} = \frac{4.78}{143.5} = \frac{3}{100}$ mole = 0.03 mole

0.01 mole of compound gives moles of AgCl = 0.03 mole

So 1mole of compound gives moles of

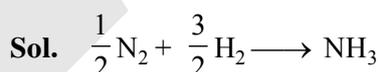
$$\text{AgCl} = \frac{0.03}{0.01} = 3 \text{mole}$$

so structural formula of compound having 3 Cl^- ions out side of coordination sphere, so formula is $[\text{Co}(\text{NH}_3)_6]\text{Cl}_3$

60. The standard enthalpy of formation of NH_3 is $-46.0 \text{ kJ mol}^{-1}$. If the enthalpy of formation of H_2 from its atoms its atoms is -436 kJ mol^{-1} and that of N_2 is -712 kJ mol^{-1} , the average bond enthalpy of N-H bond in NH_3 is :-

- (1) $-1102 \text{ kJ mol}^{-1}$ (2) -964 kJ mol^{-1}
- (3) $+352 \text{ kJ mol}^{-1}$ (4) $+1056 \text{ kJ mol}^{-1}$

Ans. (3)



$$\Delta H = H_f(\text{NH}_3) - \frac{1}{2}H_f(\text{N}_2) - \frac{3}{2}H_f(\text{H}_2)$$

$$-46 = H_f(\text{NH}_3) - \frac{1}{2}(-712) - \frac{3}{2} \times (-436)$$

$$H_f(\text{NH}_3) = -46 - 356 - 654 = -1056 \text{ KJ}$$

Enthalpy of formation of $\text{NH}_3 = -1056 \text{ KJ}$

Enthalpy of dissociation of $\text{NH}_3 = 1056 \text{ KJ}$

Average bond enthalpy of N-H bond

$$= \frac{1056}{3} = 352$$

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61. Consider the following relations :-
 $R = \{(x, y) \mid x, y \text{ are real numbers and } x = wy \text{ for some rational number } w\}$;

$S = \{(\frac{m}{n}, \frac{p}{q}) \mid m, n, p \text{ and } q \text{ are integers such that } n, q \neq 0 \text{ and } qm = pn\}$.

Then :

- (1) R is an equivalence relation but S is not an equivalence relation
- (2) Neither R nor S is an equivalence relation
- (3) S is an equivalence relation but R is not an equivalence relation
- (4) R and S both are equivalence relations

Ans. (3)

Sol. For R, $xRy \Rightarrow x = wy$

For reflexive

$xRx \Rightarrow x = wx$

Which is true then $w = 1$

For symmetric

consider $x = 0, y \neq 0$

$xRy \Rightarrow 0Ry \Rightarrow 0 = wy$

which is true when $w = 0$

Now

$yRx \Rightarrow yR0 \Rightarrow y = w \times 0$

There is no rational value of w

for which $y = w \times 0$

Hence relation is not symmetric and hence not an equivalence relation

Now for S

For reflexive

$\frac{m}{n} S \frac{m}{n} \Rightarrow mn = nm$

which is true

For symmetric

Let $\frac{m}{n} S \frac{p}{q} \Rightarrow qm = np$

$\frac{p}{q} S \frac{m}{n} \Rightarrow pn = mq$

which is true

Relation is symmetric

For transitive

Let $\frac{m}{n} S \frac{p}{q} \Rightarrow qm = np$... (1)

$\frac{p}{q} S \frac{r}{s} \Rightarrow ps = rq$... (2)

From Equation (1) and equation (2)

$\Rightarrow ms = nr$

$\therefore \frac{m}{n} S \frac{r}{s}$

S is transitive

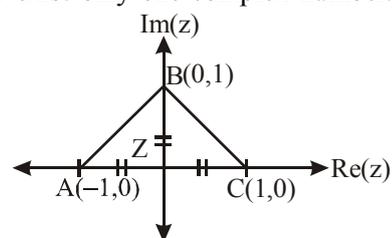
\therefore S is equivalence.

62. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals :-

- (1) 0
- (2) 1
- (3) 2
- (4) ∞

Ans. (2)

Sol. z is the circumcentre (0, 0) of triangle ABC so their exist only one complex number.



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- 63.** If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$
 (1) -2 (2) -1
 (3) 1 (4) 2

Ans. (3)

Sol. Roots of equation $x^2 - x + 1 = 0$ are
 $\alpha = -\omega, \beta = -\omega^2,$
 $\alpha^{2009} + \beta^{2009} = (-\omega)^{2009} + (-\omega^2)^{2009}$
 $= -(\omega^2 + \omega) = 1$

- 64.** Consider the system of linear equations :

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ 2x_1 + 3x_2 + x_3 &= 3 \\ 3x_1 + 5x_2 + 2x_3 &= 1 \end{aligned}$$

The system has

- (1) Infinite number of solutions
 (2) Exactly 3 solutions
 (3) A unique solution
 (4) No solution

Ans. (4)

Sol. Δ or $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0$

$$\Delta_x = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$$

So system will have no solutions.

- 65.** There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is :-
 (1) 3 (2) 36
 (3) 66 (4) 108

Ans. (4)

Sol. Urn A \rightarrow 3 Red balls

Urn B \rightarrow 9 Blue balls

So the number of ways = selection of 2 balls from urn A & B each.

$$= {}^3C_2 \cdot {}^9C_2 = 108$$

- 66.** Let $f : (-1, 1) \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

- (1) 4 (2) -4
 (3) 0 (4) -2

Ans. (2)

Sol. $g(x) = [f(2f(x) + 2)]^2$
 $g'(x) = 2f(2f(x) + 2) \cdot f'(2f(x) + 2) \cdot 2f'(x)$
 Put $x = 0$
 $g'(0) = 2f(2f(0) + 2) \cdot f'(2f(0) + 2) \cdot 2f'(0)$
 $= 2f(2(-1) + 2) \cdot f'(2(-1) + 2) \cdot 2f'(0)$
 $= 2f(0) \cdot f'(0) \cdot 2f'(0)$
 $= 4(-1)(1)(1) = -4$

- 67.** Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a positive increasing function

with $\lim_{x \rightarrow \infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} =$

- (1) 1 (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 3

Ans. (1)

Sol. For +ve numbers
 $x < 2x < 3x$
 $f(x) < f(2x) < f(3x)$ ($\because f(x)$ is increasing)
 $\frac{f(x)}{f(x)} < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$
 $\lim_{x \rightarrow \infty} \frac{f(x)}{f(x)} < \lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} < \lim_{x \rightarrow \infty} \frac{f(3x)}{f(x)}$

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$$1 < \lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} < 1$$

By sandwich theorem, $\lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} = 1$

68. Let $p(x)$ be a function defined on \mathbb{R} such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and

$p(1) = 41$. Then $\int_0^1 p(x) dx$ equals :-

- (1) $\sqrt{41}$ (2) 21 (3) 41 (4) 42

Ans. (2)

Sol. $p'(x) = p'(1-x)$

$$\int p'(x) dx = \int p'(1-x) dx$$

$$\Rightarrow p(x) = -p(1-x) + c$$

Put $x = 0$

$$\Rightarrow p(0) = -p(1) + c \Rightarrow 1 = -41 + c$$

$$\Rightarrow c = 42$$

$$\therefore p(x) = -p(1-x) + 42$$

$$\Rightarrow p(x) + p(1-x) = 42$$

$$\Rightarrow \int_0^1 \{p(x) + p(1-x)\} dx = \int_0^1 42 dx$$

$$\text{(By using property } \int_0^a f(x) dx = \int_0^a f(a-x) dx \text{)}$$

$$\Rightarrow 2 \int_0^1 p(x) dx = 42 \Rightarrow \int_0^1 p(x) dx = 21$$

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \dots = a_{10} = 150$ and a_{10}, a_{11}, \dots are in an AP with common difference -2 , then the time taken by him to count all notes is :-

- (1) 24 minutes (2) 34 minutes
(3) 125 minutes (4) 135 minutes

Ans. (2)

Sol. $4500 = 150 \times 10 + \{148 + 146 + \dots \text{ upto } n \text{ terms}\}$
 $= 1500 + \frac{n}{2} \{296 + (n-1)(-2)\}$

$$\Rightarrow n^2 - 149n + 3000 = 0$$

$$\Rightarrow (n-24)(n-125) = 0$$

$$\Rightarrow n = 24 \quad \because n \neq 125$$

$$\text{So total time taken} = 10 + 24 = 34 \text{ min.}$$

70. The equation of the tangent to the curve

$$y = x + \frac{4}{x^2}, \text{ that is parallel to the x-axis, is :-}$$

- (1) $y = 0$ (2) $y = 1$ (3) $y = 2$ (4) $y = 3$

Ans. (4)

Sol. $y = x + \frac{4}{x^2}$

$$\frac{dy}{dx} = 1 - \frac{8}{x^3}$$

Equation of tangent is parallel to x-axis

$$\therefore \frac{dy}{dx} = 0$$

$$\Rightarrow 1 - \frac{8}{x^3} = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2$$

$$\text{At, } x = 2, y = 2 + \frac{4}{4} = 3 \Rightarrow y_1 = 3$$

\therefore point is (2, 3)

equation of tangent is :

$$y - y_1 = 0(x - x_1)$$

$$y = 3$$

71. The area bounded by the curves $y = \cos x$ and

$$y = \sin x \text{ between the ordinates } x = 0 \text{ and } x = \frac{3\pi}{2}$$

is :-

(1) $4\sqrt{2} - 2$ (2) $4\sqrt{2} + 2$

(3) $4\sqrt{2} - 1$ (4) $4\sqrt{2} + 1$

Ans. (1)

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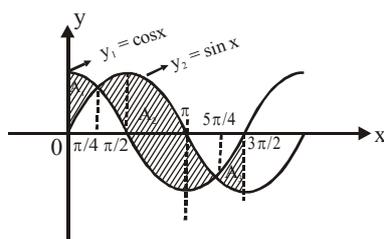
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Sol.

$$A = A_1 + A_2 + A_3$$

$$A_1 = \left| \int_0^{\pi/4} (\cos x - \sin x) dx \right|$$

$$= \left| \sin x + \cos x \right|_0^{\pi/4}$$

$$= \left| \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (0 + 1) \right| = |\sqrt{2} - 1| = \sqrt{2} - 1$$

$$A_2 = \left| \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx \right|$$

$$= \left| (\sin x - \cos x) \Big|_{\pi/4}^{5\pi/4} \right| = \left| -(\cos x + \sin x) \Big|_{\pi/4}^{5\pi/4} \right|$$

$$= \left| \left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \right] \right|$$

$$= \left| \left[-\sqrt{2} - \sqrt{2} \right] \right| = 2\sqrt{2}$$

$$A_3 = \left| \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x) dx \right| = \sqrt{2} - 1$$

so $A_1 + A_2 + A_3 = 4\sqrt{2} - 2$

72. Solution of the differential equation

$$\cos x dy = y(\sin x - y)dx, 0 < x < \frac{\pi}{2} \text{ is :}$$

- (1) $\sec x = (\tan x + c) y$
- (2) $y \sec x = \tan x + c$
- (3) $y \tan x = \sec x + c$
- (4) $\tan x = (\sec x + c) y$

Ans. (1)

Sol. $\cos x dy = y(\sin x - y)dx$

$$\Rightarrow \frac{dy}{dx} = \frac{y \sin x - y^2}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} = y \tan x - y^2 \sec x$$

$$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} = \frac{1}{y} \tan x - \sec x$$

$$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} - \frac{1}{y} \tan x = -\sec x$$

$$\Rightarrow -\frac{1}{y^2} \frac{dy}{dx} + \frac{1}{y} \tan x = \sec x \quad \dots (1)$$

Put $\frac{1}{y} = t$ in equation (1)

$$\Rightarrow -\frac{1}{y^2} \frac{dy}{dx} = \frac{dt}{dx} \quad \dots (2)$$

From equation (1) & (2), we get,

$$\Rightarrow \frac{dt}{dx} + t \cdot \tan x = \sec x$$

$$\therefore \text{I.F.} = e^{\int \tan x dx}$$

$$= e^{\log |\sec x|} = \sec x$$

\therefore solution of differential equation is :

$$t \cdot \sec x = \int \sec x \cdot \sec x \cdot dx + c$$

$$\frac{1}{y} \sec x = \tan x + c$$

$$\sec x = y (\tan x + c)$$

73. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector \vec{b} satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is :

- (1) $-\hat{i} + \hat{j} - 2\hat{k}$
- (2) $2\hat{i} - \hat{j} + 2\hat{k}$
- (3) $\hat{i} - \hat{j} - 2\hat{k}$
- (4) $\hat{i} + \hat{j} - 2\hat{k}$

Ans. (1)

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Sol. $(\vec{a} \times \vec{b}) + \vec{c} = 0$

$(\vec{a} \times \vec{b}) = -\vec{c}$

$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = -\vec{a} \times \vec{c}$

$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2 \vec{b} = -\vec{a} \times \vec{c}$

$\Rightarrow 3(\vec{j} - \vec{k}) - 2\vec{b} = -(-2\vec{i} - \vec{j} - \vec{k})$

$(\vec{a} \times \vec{c} = -2\vec{i} - \vec{j} - \vec{k})$

$\Rightarrow 2\vec{b} = -2\vec{i} + 2\vec{j} - 4\vec{k}$

$\Rightarrow \vec{b} = -\vec{i} + \vec{j} - 2\vec{k}$

74. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and

$\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

- (1) $(-3, 2)$ (2) $(2, -3)$ (3) $(-2, 3)$ (4) $(3, -2)$

Ans. (1)

Sol. Give $\vec{a} \perp \vec{b}$, $\vec{a} \perp \vec{c}$ & $\vec{b} \perp \vec{c}$

so $\vec{a} \cdot \vec{c} = 0$ & $\vec{b} \cdot \vec{c} = 0$

$\Rightarrow \lambda - 1 + 2\mu = 0$ & $2\lambda + 4 + \mu = 0$

$\Rightarrow \lambda = -3$ & $\mu = 2$

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is :-

- (1) $x = 1$ (2) $2x + 1 = 0$
(3) $x = -1$ (4) $2x - 1 = 0$

Ans. (3)

Sol. Locus of point of intersection of perpendicular tangent is directrix of the parabola.

so $x = -1$

76. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point $(13, 32)$. The line K is parallel to L and has

the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is :

- (1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$ (3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

Ans. (4)

Sol. Line L passes through $(13, 22)$

$\frac{13}{5} + \frac{32}{b} = 1$

$\Rightarrow b = -20$

so equation of L is $\frac{x}{5} - \frac{y}{20} = 1$

$\Rightarrow 4x - y = 20$

slope of L is $m_1 = 4$

slope of $\frac{x}{c} + \frac{y}{3} = 1$ is $m_2 = -\frac{3}{c}$

$\Rightarrow -\frac{3}{c} = 4 \Rightarrow c = -\frac{3}{4}$

equation of line K is

$-\frac{4x}{3} + \frac{4}{3} = 1$

$4x - y = -3$

Distance between L & k is $\left| \frac{20+3}{\sqrt{16+1}} \right| = \frac{23}{\sqrt{17}}$

77. A line AB in three-dimensional space makes angle 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals :-

- (1) 30° (2) 45° (3) 60° (4) 75°

Ans. (3)

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Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$... (i)

$\alpha = 45^\circ, \beta = 120^\circ$

Put in equation (i)

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = \frac{1}{4}$

$\Rightarrow \gamma = 60^\circ$

78. Let S be a non-empty subset of R. Consider the following statement :

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statement is the negation of the statement P ?

(1) There is a rational number $x \in S$ such that $x \leq 0$

(2) There is no rational number $x \in S$ such that $x \leq 0$

(3) Every rational number $x \in S$ satisfies $x \leq 0$

(4) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational number.

Ans. (3)

Sol. For Every rational number $x \in S$ satisfies $x \leq 0$.

79. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$,

where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) $\frac{25}{16}$ (2) $\frac{56}{33}$ (3) $\frac{19}{12}$ (4) $\frac{20}{7}$

Ans. (2)

Sol. $\cos(\alpha + \beta) = \frac{4}{5}$

$\Rightarrow \tan(\alpha + \beta) = \frac{3}{4}$

$\sin(\alpha - \beta) = \frac{5}{13}$

$\Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$

$\tan 2\alpha = \tan\{(\alpha + \beta) + (\alpha - \beta)\}$

$= \frac{\tan(\alpha + \beta)\tan(\alpha - \beta)}{1 - \tan(\alpha - \beta)\tan(\alpha + \beta)}$

$= \frac{\frac{3}{4} + \frac{6}{12}}{1 - \frac{3}{4} \times \frac{5}{12}} = \frac{56}{33}$

80. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if :-

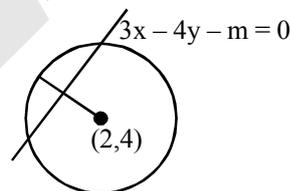
(1) $-85 < m < -35$ (2) $-35 < m < 15$

(3) $15 < m < 65$ (4) $35 < m < 85$

Ans. (2)

Sol. Equation of circle $x^2 + y^2 - 4x - 8y - 5 = 0$

Centre (2, 4) ; radius = 5



If line intersect the circle at two distinct points then length of perpendicular from centre to the line is less than radius of circle.

$\left| \frac{6 - 16 - m}{5} \right| < 5$

$\Rightarrow -35 < m < 15$

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance for the combined data set is :-

(1) $\frac{5}{2}$ (2) $\frac{11}{2}$ (3) 6 (4) $\frac{13}{2}$

Ans. (2)

Sol. Given $n_1 = 5, n_2 = 5$

$\sigma_1^2 = 4, \sigma_2^2 = 5$

$\bar{x}_1 = 2, \bar{x}_2 = 4$

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(XII Appeared / Result Awaited)
starting from
29th April 2010**

Objective : This course is for those students who want to take lead by making an early step towards TARGET IIT-JEE 2011 by utilising precious 60 days prior to start of Phase-II.

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The variance of combined data

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1n_2}{(n_1 + n_2)^2} (\bar{x}_1 - \bar{x}_2)^2$$

$$= \frac{5 \times 4 + 5 \times 5}{10} + \frac{5 \times 5}{100} \times (2 - 4)^2 = \frac{11}{2}$$

82. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is :-

- (1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{21}$ (4) $\frac{2}{23}$

Ans. (2)

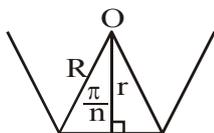
Sol. Required probability = $\frac{{}^3C_1 \cdot {}^4C_1 \cdot {}^2C_1}{{}^9C_3} \cdot 3! = \frac{2}{7}$

83. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is :-

- (1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$
 (2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$
 (3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$
 (4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

Ans. (3)

Sol. $\frac{r}{R} = \cos \frac{\pi}{n}$



When $n = 3$, $\frac{r}{R} = \cos \frac{\pi}{3} = \frac{1}{2}$

When $n = 4$, $\frac{r}{R} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

When $n = 6$, $\frac{r}{R} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

There is no value of n for which $\cos \frac{\pi}{n} = \frac{2}{3}$

84. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is :-

- (1) Less than 4 (2) 5
 (3) 6 (4) At least 7

Ans. (4)

Sol. For Non singular matrix $|A| \neq 0$

$$A = \begin{bmatrix} 1 & \square & \square \\ \square & 1 & \square \\ \square & \square & 1 \end{bmatrix}$$

There are 6 vacant places where 5 zero's and one 1 are to be placed, for $|A| \neq 0$

So, no. of ways = 6

$$|A| = \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} \neq 0$$

So, atleast 7 matrices are possible.

85. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} k - 2x, & \text{if } x \leq -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$$

If f has a local minimum at $x = -1$, then a possible value of k is :

- (1) 1 (2) 0 (3) $-\frac{1}{2}$ (4) -1

Ans. (4)

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Sol. f has a local minimum at $x = -1$

$$\therefore \lim_{x \rightarrow -1} f(x) \geq f(-1)$$

$$k + 2 \leq 1$$

$$k \leq -1$$

$$\therefore k = -1$$

Directions : Questions number 86 to 90 are Assertion - Reason type questions. Each of these questions contains two statements :

Statement-1 (Assertion) and

Statement-2 (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

86. Four numbers are chosen at random (without replacement) from the set $(1, 2, 3, \dots, 20)$.

Statement-1 : The probability that the chosen numbers when arranged in some order will form

an AP is $\frac{1}{85}$

Statement-2 : In the four chosen numbers form an AP, then the set of all possible values of common difference is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$.

- (1) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1.
- (2) Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for statement-1.
- (3) Statement-1 is true, Statement-2 is false.
- (4) Statement-1 is false, Statement-2 is true.

Ans. (3)

Sol. Let terms of an AP

$$a, a + d, a + 2d, a + 3d$$

$$\therefore a \geq 1, \quad a + 3d \leq 20$$

$$3d \leq 19 \quad \Rightarrow d \leq \frac{19}{3}$$

so $d = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$ and ± 6
statement 2 is wrong

if $d = 1$

then $a + 3d \leq 20$

$a \leq 17$

so 17 cases will be there

Total case for $d = \pm 1$ is 34

similarly $d = -1$

so in this case also

17 cases will be there

Similarly

d	± 1	± 2	± 3	± 4	± 5	± 6
cases	34	28	22	16	10	4

So total favourable case = 114

Total exhaustive cases = ${}^{20}C_4 \times 2$

$$\text{required probability} = \frac{114}{{}^{20}C_4 \times 2} = \frac{1}{85}$$

Statement 1 is true.

87. Let $S_1 = \sum_{j=1}^{10} j(j-1) {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j {}^{10}C_j$ and

$$S_3 = \sum_{j=1}^{10} j^2 {}^{10}C_j.$$

Statement-1 : $S_3 = 55 \times 2^9$.

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

- (1) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1.
- (2) Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for statement-1.
- (3) Statement-1 is true, Statement-2 is false.
- (4) Statement-1 is false, Statement-2 is true.

Ans. (3)

$$\text{Sol. } S_3 = \sum_{j=1}^{10} j^2 {}^{10}C_j = \sum_{j=1}^{10} j^2 \times \frac{10}{j} \times {}^9C_{j-1}$$

$$S_3 = 10 \left\{ \sum_{j=1}^{10} (j-1+1) \frac{9}{j-1} {}^8C_{j-2} \right\}$$

$$= 10 \left\{ \sum_{j=2}^{10} 9 \cdot {}^8C_{j-2} + \sum_{j=1}^{10} {}^9C_{j-1} \right\}$$

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$$S_3 = 10 (9 \cdot 2^8 + 2^9)$$

$$10 \{2^8 (11)\} = 110 \times 2^8 = 55 \times 2^9$$

so statement-1 is true

statement-2

$$S_2 = \sum_{j=1}^{10} j \cdot {}^{10}C_j = 10 \sum_{j=1}^{10} {}^9C_{j-1} = 10 \cdot 2^9$$

so, statement-2 is wrong

- 88. Statement-1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$.

Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

- (1) Statement-1 is true, Statement-2 is true ;
Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true ;
Statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Ans. (2)

Sol. Mirror image of B(1, 3, 4) in plane $x - y + z = 5$

$$\frac{x-1}{1} = \frac{y-3}{-1} = \frac{z-4}{1} = -2 \frac{(1-3+4-5)}{1+1+1} = 2$$

$$\Rightarrow x = 3, y = 1, z = 6$$

\therefore mirror image of B (1, 3, 4)

is A (3, 1, 6)

statement-1 is correct

statement-2 is true but it is not the correct explanation.

- 89.** Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function defined

$$\text{by } f(x) = \frac{1}{e^x + 2e^{-x}}$$

Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in \mathbb{R}$.

Statement-2 : $0 < f(x) \leq \frac{1}{2\sqrt{2}}$, for all $x \in \mathbb{R}$.

- (1) Statement-1 is true, Statement-2 is true ;
Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true ;
Statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Ans. (1)

Sol. $f(x) = \frac{1}{e^x + 2e^{-x}}$

$$y = \frac{1}{e^x + 2e^{-x}} \quad \text{Let } e^x = t \in (0, \infty)$$

$$y = \frac{1}{t + \frac{2}{t}} \Rightarrow y = \frac{t}{t^2 + 2} \Rightarrow t^2 y - t + 2y = 0$$

$$D \geq 0$$

$$1 - 8y^2 \geq 0$$

$$\Rightarrow 8y^2 - 1 \leq 0 \Rightarrow y \in \left[\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}} \right]$$

but $y > 0$

$$\therefore y \in \left(0, \frac{1}{2\sqrt{2}} \right]$$

$$\therefore f(0) = \frac{1}{3}$$

$$\therefore f(c) = \frac{1}{3} \quad (c \in \mathbb{R})$$

So Statement-1 is true, Statement-2 is true ;
Statement-2 is a correct explanation for Statement-1.

- 90.** Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define $\text{Tr}(A)$ = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

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Statement-1 : $\text{Tr}(A) = 0$.

Statement-2 : $|A| = 1$.

- (1) Statement-1 is true, Statement-2 is true ;
Statement-2 is a correct explanation for Statement-1.
- (2) Statement-1 is true, Statement-2 is true ;
Statement-2 is not a correct explanation for statement-1.
- (3) Statement-1 is true, Statement-2 is false.
- (4) Statement-1 is false, Statement-2 is true.

Ans. (3)

Sol. Statement-1 :

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^2 = I$$

$$\begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$b(a + d) = 0 \text{ and } c(a + d) = 0$$

$$\Rightarrow a + d = 0 \quad (\because b \neq 0, c \neq 0)$$

So trace matrix A, $\text{tr}(A) = 0$.

So statement-1 is true.

Statement-2 :

$$A^2 = I$$

$$A = A^{-1}$$

but for $A = A^{-1}$

A should be I, but I is not possible because all entries are not zero.

So statement-2 is wrong.

ANSWER KEY (CODE - A) AIEEE 2010 TEST PAPER HELD ON 25-04-2010

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A.	1	2	3	3	4	1	2	3	2	3	4	3	2	2	2	3	3	1	3	1	4	3	1	1	4
Q.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
A.	4	4	1	4	1	4	3	1	4	1	3	3	2	2	1	4	3	2	3	2	4	1	4	1	1
Q.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A.	4	3	3	3	3	2	2	4	2	3	3	2	3	4	4	2	1	2	2	4	1	1	1	1	3
Q.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90										
A.	4	3	3	2	2	2	2	3	4	4	3	3	2	1	3										

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