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# QUESTIONS & SOLUTIONS OF AIEEE 2011

## PART-II PAPER-1

**Date : 11-05-2011**

**Duration : 3 Hours**

**Max. Marks : 360**

### IMPORTANT INSTRUCTIONS

1. Immediately fill the particulars on this page of the Test Booklet with Blue / Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360**.
5. There are three parts in the question paper A, B, C consisting of **Chemistry, Physics and Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
6. Candidates will be awarded marks as stated above in Instructions No. 5 for correct response of each question.  $\frac{1}{4}$  (**one fourth**) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 6 above.
8. Use **Blue/Black Ball Point Pen only** for writing particulars/markings responses on **Side-1** and **Side-2** of the Answer Sheet. **Use of pencil is strictly prohibited.**
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, paper, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 3 pages at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is A. Make sure that the CODE printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. Do not fold or make any stray marks on the Answer Sheet.

Name of the Candidate (in Capital letters) : \_\_\_\_\_

Roll Number : in figures : 

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 in words : \_\_\_\_\_

Examination Centre Number : 

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Name of Examination Centre (in Capital letters) : \_\_\_\_\_

Candidate's Signature : \_\_\_\_\_ Invigilator's Signature : \_\_\_\_\_

## PART-A (CHEMISTRY)

1. Identify the incorrect statement from the following:

- (1) Ozone absorbs the intense ultraviolet radiation of the sun.
- (2) Depletion of ozone layer is because of its chemical reactions with chlorofluoro alkanes.
- (3) Ozone absorbs infrared radiation.
- (4) Oxides of nitrogen in the atmosphere can cause the depletion of ozone layer.

Ans. (3)

Sol. Layer of  $O_3$  in upper atmosphere absorbs UV radiations from sun, thus protects the human being living on the earth.

Nitrogen oxides (particularly nitric oxide) combine very rapidly with ozone and there is, thus, the possibility that nitrogen oxides emitted from the exhaust systems of supersonic jet aeroplanes might be slowly depleting the concentration of the ozone layer in the upper atmosphere.

The use of chlorofluoro carbon in aerosol and refrigerator and there, subsequent escape into the atmosphere is responsible for making holes in the ozone layer. Ozone at all does not absorb infrared radiation.

2. When  $r$ ,  $P$  and  $M$  represent rate of diffusion, pressure and molecular mass, respectively, then the ratio of the rates of diffusion ( $r_A/r_B$ ) of two gases A and B, is given as:

- (1)  $(P_A/P_B) (M_B/M_A)^{1/2}$
- (2)  $(P_A/P_B)^{1/2} (M_B/M_A)$
- (3)  $(P_A/P_B) (M_A/M_B)^{1/2}$
- (4)  $(P_A/P_B)^{1/2} (M_A/M_B)$

Ans. (1)

Sol.  $r \propto \frac{P}{\sqrt{M}}$

$$\frac{r_A}{r_B} = \frac{P_A}{P_B} \sqrt{\frac{M_B}{M_A}}$$

3. Consider thiol anion ( $RS^\ominus$ ) and alkoxy anion ( $RO^\ominus$ ). Which of the following statements is correct ?

- (1)  $RS^\ominus$  is less basic but more nucleophilic than  $RO^\ominus$ .
- (2)  $RS^\ominus$  is more basic and more nucleophilic than  $RO^\ominus$ .
- (3)  $RS^\ominus$  is more basic but less nucleophilic than  $RO^\ominus$ .
- (4)  $RS^\ominus$  is less basic and less nucleophilic than  $RO^\ominus$ .

Ans. (3)

Sol.  $RS^\ominus$  is more nucleophilic than  $R-O^\ominus$  due to larger size of orbitals and polarization but  $RS^\ominus$  is less basic than  $R-O^\ominus$  as the negative charge get stabilized due to larger size of sulphur atom.

4. The change in the optical rotation of freshly prepared solution of glucose is known as:

- (1) racemisation
- (2) specific rotation
- (3) mutarotation
- (4) tautomerism

Ans. (3)

Sol. Freshly prepared glucose solution get equilibrated to an unequal mixture of (+) and (–) glucose. This phenomenon is called mutarotation.

5. The molality of a urea solution in which 0.0100 g of urea,  $[(\text{NH}_2)_2\text{CO}]$  is added to  $0.3000 \text{ dm}^3$  of water at STP is:
- (1)  $5.55 \times 10^{-4}$       (2) 33.3 m      (3)  $3.33 \times 10^{-2} \text{ m}$       (4) 0.555 m

Ans. (1)

Sol. Molality =  $\frac{0.01/60}{0.3} = \frac{0.01}{60 \times 0.3}$  ;       $d = 1 \text{ g/ml}$

$= 5.55 \times 10^{-4} \text{ m}$

6. The molecular velocity of any gas is:
- (1) inversely proportional to absolute temperature.  
 (2) directly proportional to square of temperature.  
 (3) directly proportional to square root of temperature.  
 (4) inversely proportional to the square root of temperature.

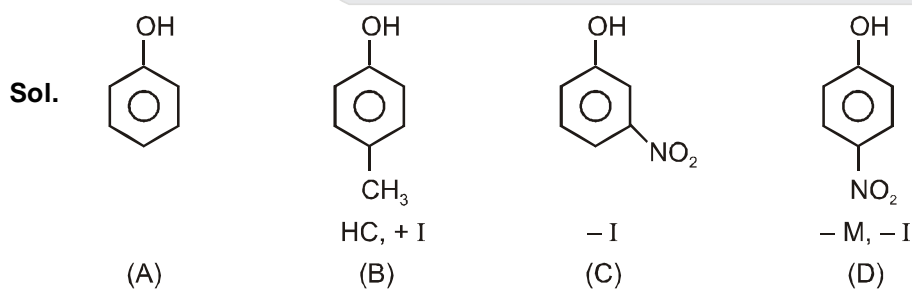
Ans. (3)

Sol.  $v = \sqrt{\frac{8RT}{\pi M}}$

$v \propto \sqrt{T}$

7. The correct order of acid strength of the following compounds:
- (A) Phenol      (B) p-Cresol      (C) m-Nitrophenol      (D) p-Nitrophenol
- is :
- (1)  $D > C > A > B$       (2)  $B > D > A > C$       (3)  $A > B > D > C$       (4)  $C > B > A > D$

Ans. (1)



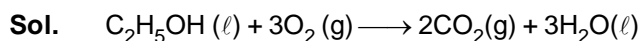
Therefore acidity order is :       $D > C > A > B$

8. The value of enthalpy change ( $\Delta H$ ) for the reaction

$\text{C}_2\text{H}_5\text{OH}_{(l)} + 3\text{O}_{2(g)} \rightarrow 2\text{CO}_{2(g)} + 3\text{H}_2\text{O}_{(l)}$   
 at  $27^\circ\text{C}$  is  $-1366.5 \text{ kJ mol}^{-1}$ . The value of internal energy change for the above reaction at this temperature will be :

- (1)  $-1369.0 \text{ kJ}$  (2)  $-1364.0 \text{ kJ}$  (3)  $-1361.5 \text{ kJ}$  (4)  $-1371.5 \text{ kJ}$

Ans. (2)



$$\Delta n_g = 2 - 3 = -1$$

$$\Delta U = \Delta H - \Delta n_g RT$$

$$= -1366.5 - (-1) \times \frac{8.314}{10^3} \times 300$$

$$= -1366.5 + 0.8314 \times 3 = -1364 \text{ KJ}$$

9. Thermosetting polymer, Bakelite is formed by the reaction of phenol with :

- (1)  $\text{CH}_3\text{CHO}$  (2)  $\text{HCHO}$  (3)  $\text{HCOOH}$  (4)  $\text{CH}_3\text{CH}_2\text{CHO}$

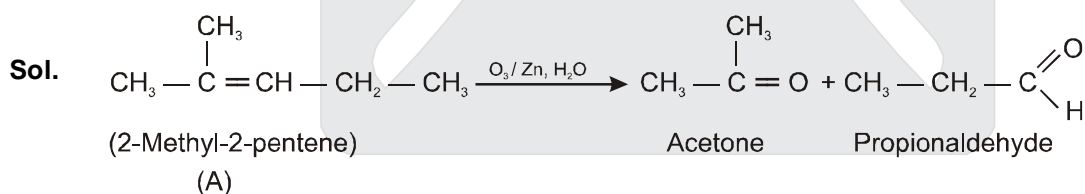
Ans. (2)

Sol. Bakelite is a polymer of phenol and  $\text{HCHO}$  (formaldehyde).

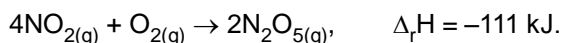
10. Ozonolysis of an organic compound 'A' produces acetone and propionaldehyde in equimolar mixture. Identify 'A' from the following compounds :

- (1) 1-Pentene (2) 2-Pentene  
 (3) 2-Methyl-2-pentene (4) 2-Methyl-1-pentene

Ans. (3)



11. Consider the reaction :

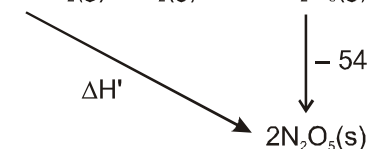
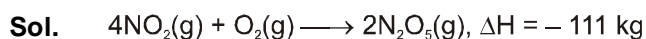


If  $\text{N}_2\text{O}_{5(s)}$  is formed instead of  $\text{N}_2\text{O}_{5(g)}$  in the above reaction, the  $\Delta_r H$  value will be:

(given,  $\Delta H$  of sublimation for  $\text{N}_2\text{O}_5$  is  $54 \text{ kJ mol}^{-1}$ )

- (1)  $+54 \text{ kJ}$  (2)  $+219 \text{ kJ}$  (3)  $-219 \text{ kJ}$  (4)  $-165 \text{ kJ}$

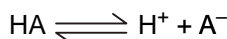
Ans. (4)



$$-111 - 54 = \Delta H'$$

$$\Delta H' = -165 \text{ KJ}$$

12. An acid HA ionises as



The pH of 1.0 M solution is 5. Its dissociation constant would be:

- (1) 5                      (2)  $5 \times 10^{-8}$                       (3)  $1 \times 10^{-5}$                       (4)  $1 \times 10^{-10}$

Ans. (4)

Sol.  $K_a = \frac{[\text{H}^+]^2}{C - [\text{H}^+]}$ ,  $[\text{H}^+] \ll C$

$$= \frac{(10^{-5})^2}{1} = 10^{-10}$$

13. The correct order of electron gain enthalpy with negative sign of F, Cl, Br and I, having atomic number 9, 17, 35 and 53 respectively, is:

- (1)  $\text{F} > \text{Cl} > \text{Br} > \text{I}$                       (2)  $\text{Cl} > \text{F} > \text{Br} > \text{I}$                       (3)  $\text{Br} > \text{Cl} > \text{I} > \text{F}$                       (4)  $\text{I} > \text{Br} > \text{Cl} > \text{F}$

Ans. (2)

Sol. As we move in a group from top to bottom, electron gain enthalpy becomes less negative because the size of the atom increases and the added electron would be at larger distance from the nucleus. Negative electron gain enthalpy of F is less than Cl. This is due to the fact that when an electron is added to F, the added electron goes to the smaller  $n = 2$  energy level and experiences significant repulsion from the other electrons present in this level. In Cl, the electron goes to the larger  $n = 3$  energy level and consequently occupies a larger region of space leading to much less electron-electron repulsion. So the correct order is  $\text{Cl} > \text{F} > \text{Br} > \text{I}$ .

14. The frequency of light emitted for the transition  $n = 4$  to  $n = 2$  of  $\text{He}^+$  is equal to the transition in H atom corresponding to which of the following?

- (1)  $n = 2$  to  $n = 1$                       (2)  $n = 3$  to  $n = 2$                       (3)  $n = 4$  to  $n = 3$                       (4)  $n = 3$  to  $n = 1$

Ans. (1)

Sol.  $h\nu = \Delta E = 13.6 z^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

$$\nu_{\text{He}^+} = \nu_{\text{H}} \times z^2 \left( \frac{1}{\left(\frac{n_1}{2}\right)^2} - \frac{1}{\left(\frac{n_2}{2}\right)^2} \right)$$

$$= \nu_{\text{H}} \left( \frac{1}{\left(\frac{2}{2}\right)^2} - \frac{1}{\left(\frac{4}{2}\right)^2} \right)$$

For H-atom

$$n_1 = 1, n_2 = 2$$

15. A 5% solution of cane sugar (molar mass 342) is isotonic with 1% of a solution of an unknown solute. The molar mass of unknown solute in g/mol is:  
 (1) 171.2 (2) 68.4 (3) 34.2 (4) 136.2

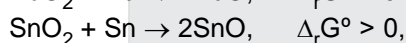
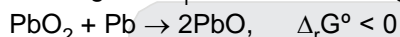
Ans. (2)

Sol.  $\pi_1 = \pi_2$   
 $C_1 = C_2$

$$\frac{5/342}{0.1} = \frac{1/M}{0.1}$$

$$\frac{5}{342} = \frac{1}{M} \Rightarrow M = \frac{342}{5} = 68.4 \text{ gm/mol}$$

16. In view of the signs of  $\Delta_r G^\circ$  for the following reactions :



which oxidation states are more characteristics for lead and tin ?

- (1) For lead +2, for tin +2 (2) For lead +4, for tin +4  
 (3) For lead +2, for tin +4 (4) For lead +4, for tin +2

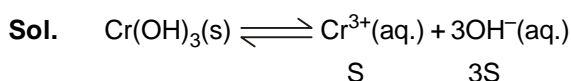
Ans. (3)

Sol. Negative  $\Delta_r G^\circ$  value indicates that + 2 oxidation state is more stable for  $\text{Pb}^{2+}$ . Also it is supported by inert pair effect that +2 oxidation state is more stable for Pb and + 4 oxidation state is more stable for Sn.

17. The  $K_{sp}$  for  $\text{Cr}(\text{OH})_3$  is  $1.6 \times 10^{-30}$ . The molar solubility of this compound in water is :

- (1)  $4\sqrt[4]{1.6 \times 10^{-30}}$  (2)  $4\sqrt[4]{1.6 \times 10^{-30} / 27}$   
 (3)  $1.6 \times 10^{-30} / 27$  (4)  $2\sqrt[4]{1.6 \times 10^{-30}}$

Ans. (2)



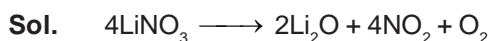
$$27 \text{ S}^4 = K_{sp}$$

$$\text{S} = \left( \frac{K_{sp}}{27} \right)^{1/4} = \left( \frac{1.6 \times 10^{-30}}{27} \right)^{1/4}$$

18. The products obtained on heating  $\text{LiNO}_3$  will be :

- (1)  $\text{Li}_2\text{O} + \text{NO}_2 + \text{O}_2$  (2)  $\text{Li}_3\text{N} + \text{O}_2$  (3)  $\text{Li}_2\text{O} + \text{NO} + \text{O}_2$  (4)  $\text{LiNO}_3 + \text{O}_2$

Ans. (1)



19. Resistance of 0.2 M solution of an electrolyte is  $50 \Omega$ . The specific conductance of the solution is  $1.3 \text{ S m}^{-1}$ . If resistance of the 0.4 M solution of the same electrolyte is  $260 \Omega$ , its molar conductivity is :
- (1)  $6.25 \times 10^{-4} \text{ S m}^2 \text{ mol}^{-1}$  (2)  $625 \times 10^{-4} \text{ S m}^2 \text{ mol}^{-1}$   
 (3)  $62.5 \text{ S m}^2 \text{ mol}^{-1}$  (4)  $6250 \text{ S m}^2 \text{ mol}^{-1}$

Ans. (1)

Sol.  $k = \frac{1}{R} \times \frac{\ell}{A}$

$$1.3 = \frac{1}{50} \times \frac{\ell}{A}$$

$$\frac{\ell}{A} = 65 \text{ m}^{-1}$$

$$\Lambda_m = \frac{K}{1000 \times \text{molarity}} \left( \frac{K}{1000 \times \text{मोलरता}} \right)$$

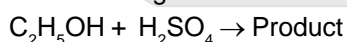
$$= \frac{\left( \frac{1}{2604} \times 65 \right)}{1000 \times 0.40} = 6.25 \times 10^{-4} = \frac{1}{4 \times 0.4 \times 1000} = \frac{1}{1600} = 6.25 \times 10^{-4} \text{ S m}^2 \text{ mol}^{-1}$$

20. Among the ligands  $\text{NH}_3$ , en,  $\text{CN}^-$  and CO the correct order of their increasing field strength, is :
- (1)  $\text{NH}_3 < \text{en} < \text{CN}^- < \text{CO}$  (2)  $\text{CN}^- < \text{NH}_3 < \text{CO} < \text{en}$   
 (3)  $\text{en} < \text{CN}^- < \text{NH}_3 < \text{CO}$  (4)  $\text{CO} < \text{NH}_3 < \text{en} < \text{CN}^-$

Ans. (1)

Sol. Ligands can be arranged in a series in the orders of increasing field strength as given below :  
 $\text{I}^- < \text{Br}^- < \text{S}^{2-} < \text{Cl}^- < \text{NO}_3^- < \text{F}^- < \text{OH}^- < \text{EtOH} < \text{C}_2\text{O}_4^{2-} < \text{H}_2\text{O} < \text{NCS}^- < \text{EDTA} < \text{NH}_3 < \text{en} < \text{NO}_2^- < \text{CN}^- < \text{CO}$   
 Such a series is termed as spectrochemical series. It is an experimentally determined series based on the absorption of light by complexes with different ligands.

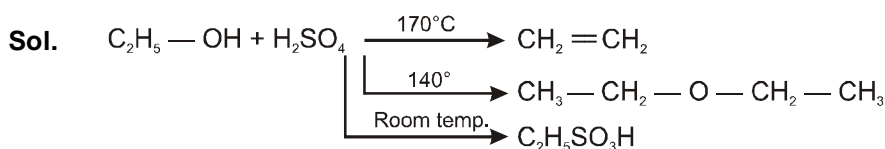
21. Consider the following reaction:



Among the following, which one cannot be formed as a product under any conditions?

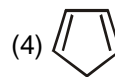
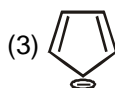
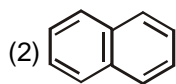
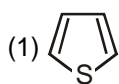
- (1) Ethylene (2) Acetylene  
 (3) Diethyl ether (4) Ethyl-hydrogen sulphate

Ans. (2)

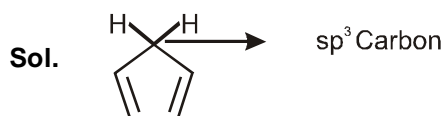


Acetylene is not formed under any conditions.

22. The non aromatic compound among the following is :



Ans. (4)



Cyclopentadiene does not obey Huckel's Rule, as it has  $sp^3$  carbon in the ring.

23. The number of types of bonds between two carbon atoms in calcium carbide is:

(1) One sigma, one pi

(2) Two sigma, one pi

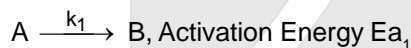
(3) Two sigma, two pi

(4) One sigma, two pi

Ans. (4)

Sol. Calcium carbide exists as  $Ca^{2+}$  and  $C_2^{2-}$ . According to the molecular orbital model,  $C_2^{2-}$  should have a bond order of 3 (configuration  $\pi_u^2 \pi_g^2 \sigma_g^2$ ). MOT configuration suggests that it contains one  $\sigma$  and two  $\pi$ -bonds  $[:C \equiv C:]^{2-}$ .

24. A reactant (A) forms two products :



If  $E_{a2} = 2 E_{a1}$ , then  $k_1$  and  $k_2$  are related as :

(1)  $k_2 = k_1 e^{E_{a1}/RT}$

(2)  $k_2 = k_1 e^{E_{a2}/RT}$

(3)  $k_1 = A k_2 e^{E_{a1}/RT}$

(4)  $k_1 = 2 k_2 e^{E_{a2}/RT}$

Ans. (3)

Sol.  $K_1 = A_1 e^{-E_{a1}/RT}$

$$K_2 = A_2 e^{-E_{a2}/RT}$$

$$\frac{K_1}{K_2} = \frac{A_1}{A_2} e^{(E_{a2} - E_{a1})/RT}$$

$$K_1 = K_2 A \times e^{E_{a1}/RT}$$



25. Copper crystallises in fcc lattice with a unit cell edge of 361 pm. The radius of copper atom is:

- (1) 108 pm (2) 128 pm (3) 157 pm (4) 181 pm

Ans. (2)

Sol. FCC lattice

$$a = 361 \text{ pm}$$

$$a\sqrt{2} = 4r$$

$$r = \frac{361 \times \sqrt{2}}{4} = 127.6 \approx 128 \text{ pm.}$$

26. The mass of potassium dichromate crystals required to oxidise  $750 \text{ cm}^3$  of 0.6 M Mohr's salt solution is :  
(Given molar mass potassium dichromate = 294, Mohr's salt = 392)

- (1) 0.45 g (2) 22.05 g (3) 2.2 g (4) 0.49 g

Ans. (2)

$$\text{Sol. } 0.6 \times 0.75 \times 1 = \frac{w}{294} \times 6$$

$$w = 22.05 \text{ g}$$

27. What is the best description of the change that occurs when  $\text{Na}_2\text{O(s)}$  is dissolved in water ?

- (1) Oxide ion accepts sharing in a pair of electrons  
(2) Oxide ion donates a pair of electrons  
(3) Oxidation number of oxygen increases  
(4) Oxidation number of sodium decreases

Ans. (2)

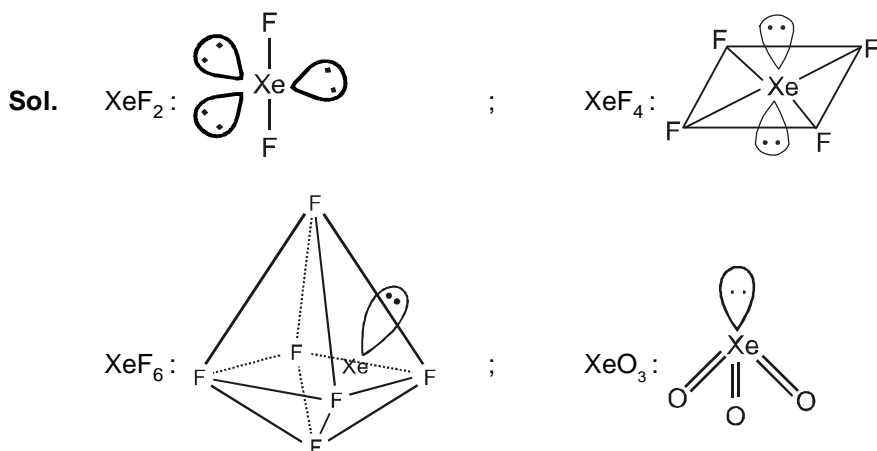


$\text{O}^{2-}$  acts as Lewis base.

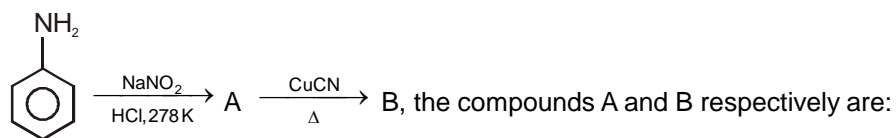
28. Which of the following has maximum number of lone pairs associated with Xe ?

- (1)  $\text{XeF}_4$  (2)  $\text{XeF}_6$  (3)  $\text{XeF}_2$  (4)  $\text{XeO}_3$

Ans. (3)



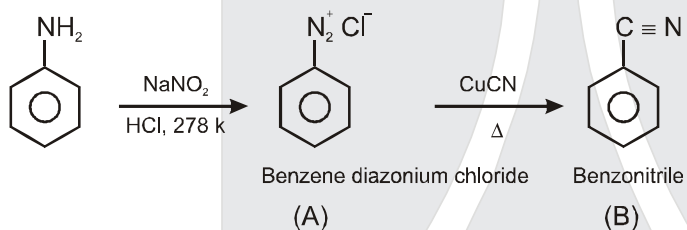
29. In the chemical reactions



- (1) Benzene diazonium chloride and benzonitrile
- (2) Nitrobenzene and chlorobenzene
- (3) Phenol and bromobenzene
- (4) Fluorobenzene and phenol

Ans. (1)

Sol.

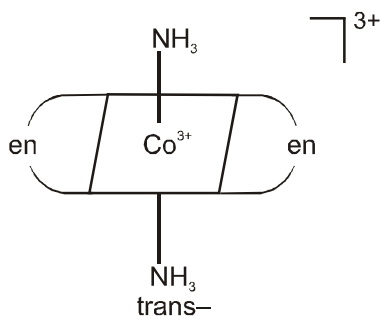
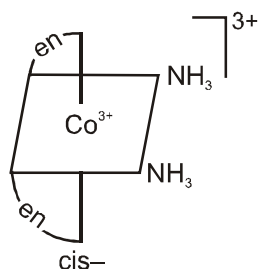


30. Which one of the following complex ions has geometrical isomers ?

- (1)  $[\text{Ni}(\text{NH}_3)_5\text{Br}]^+$
  - (2)  $[\text{Co}(\text{NH}_3)_2(\text{en})_2]^{3+}$
  - (3)  $[\text{Cr}(\text{NH}_3)_4(\text{en})]^{3+}$
  - (4)  $[\text{Co}(\text{en})_3]^{3+}$
- (en - ethylenediamine)

Ans. (2)

Sol.



## PART-B (PHYSICS)

31. At time  $t = 0$  s a particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to :

(1) constant                      (2)  $t$                       (3)  $\frac{1}{\sqrt{t}}$                       (4)  $\sqrt{t}$

**Ans. (3)**

**Sol.** K.E. =  $ct$

$$\frac{1}{2}mv^2 = ct$$

$$\frac{p^2}{2m} = ct$$

$$p = \sqrt{2ctm}$$

$$F = \frac{dp}{dt} = \sqrt{2cm} \times \frac{1}{\sqrt{t}}$$

$$F \propto \frac{1}{\sqrt{t}}$$

32. At two points P and Q on a screen in Young's double slit experiment, waves from slits  $S_1$  and  $S_2$  have a path difference of 0 and  $\frac{\lambda}{4}$  respectively. The ratio of intensities at P and Q will be :

(1) 2 : 1                      (2)  $\sqrt{2} : 1$                       (3) 4 : 1                      (4) 3 : 2

**Ans. (1)**

**Sol.**

$$\Delta x_1 = 0$$

$$\Delta \phi = 0^\circ$$

$$I_1 = I_0 + I_0 + 2I_0 \cos 0^\circ = 4I_0$$

$$\Delta x_2 = \frac{\lambda}{4}$$

$$\Delta \theta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \left(\frac{\pi}{2}\right)$$

$$I_2 = I_0 + I_0 + 2I_0 \cos \frac{\pi}{2} = 2I_0$$

$$\frac{I_1}{I_2} = \frac{4I_0}{2I_0} = \frac{2}{1}$$

33. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is :

(1)  $\sqrt{\frac{Gm}{4R}}$                       (2)  $\sqrt{\frac{Gm}{3R}}$                       (3)  $\sqrt{\frac{Gm}{2R}}$                       (4)  $\sqrt{\frac{Gm}{R}}$

**Ans. (1)**

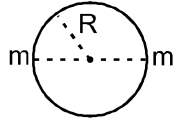
**Sol.**  $\frac{Gm^2}{(2R)^2} = m\omega^2 R$

$$\frac{Gm^2}{4R^3} = \omega^2$$

$$\omega = \sqrt{\frac{Gm}{4R^3}}$$

$$v = \omega R$$

$$v = \sqrt{\frac{Gm}{4R^3}} \times R = \sqrt{\frac{Gm}{4R}}$$



- 34.** The minimum force required to start pushing a body up a rough (frictional coefficient  $\mu$ ) inclined plane is  $F_1$  while the minimum force needed to prevent it from sliding down is  $F_2$ . If the inclined plane makes an angle  $\theta$

from the horizontal such that  $\tan \theta = 2\mu$  then the ratio  $\frac{F_1}{F_2}$  is :

(1) 1

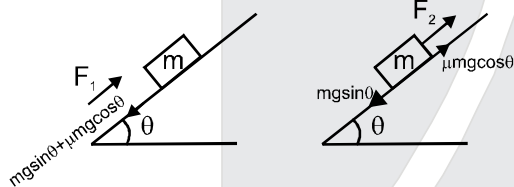
(2) 2

(3) 3

(4) 4

**Ans. (3)**

**Sol.**



$$F_1 = mg \sin \theta + \mu mg \cos \theta$$

$$F_2 = mg \sin \theta - \mu mg \cos \theta$$

$$\frac{F_1}{F_2} = \frac{\sin \theta + \mu \cos \theta}{\sin \theta - \mu \cos \theta}$$

$$\frac{\tan \theta + \mu}{\tan \theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = \frac{3\mu}{\mu} = 3.$$

- 35.** If  $400 \Omega$  of resistance is made by adding four  $100 \Omega$  resistances of tolerance 5%, then the tolerance of the combination is :

(1) 5%

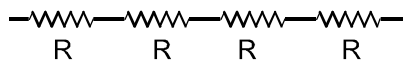
(2) 10%

(3) 15%

(4) 20%

**Ans. (1)**

**Sol.**



$$R = 100 \pm 5$$

$$4R = 400 \pm 20$$

Tolerance of combination is also 5%.

- 36.** An electric charge  $+q$  moves with velocity  $\vec{v} = 3\hat{i} + 4\hat{j} + \hat{k}$ , in an electromagnetic field given by :

$\vec{E} = 3\hat{i} + \hat{j} + 2\hat{k}$  and  $\vec{B} = \hat{i} + \hat{j} + 3\hat{k}$ . The y - component of the force experienced by  $+q$  is :

(1) 11 q

(2) 5 q

(3) 3 q

(4) 2 q

**Ans. (1)**

**Sol.**  $\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$

$$= q \left[ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3\hat{i} + \hat{j} + \hat{k} & 3 & 4 & 1 \\ 1 & 1 & -3 \end{vmatrix} \right]$$

$$= q[3\hat{i} + \hat{j} + 2\hat{k} + \hat{i} - 12 - 1 - \hat{j}(-9 - 1) + \hat{k}(3 - 4)]$$

$$= q[3\hat{i} + \hat{j} + 2\hat{k} - 13\hat{i} + 10\hat{j} - \hat{k}]$$

$$= q[-10\hat{i} + 11\hat{j} + \hat{k}]$$

$$= F_y = 11q\hat{j}.$$

- 37.** The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are  $4 \times 10^{-7}$  ohm metre and  $8 \times 10^{-7}$  m<sup>2</sup> respectively. The potential gradient will be equal to :

- (1) 1 V/m (2) 0.5 V/m (3) 0.1 V/m (4) 0.2 V/m

**Ans. 3**

**Sol.**  $x = \frac{V}{\ell} = \frac{IR}{\ell} = \frac{IR}{\ell} \left( \frac{\rho \ell}{A} \right) = \frac{I\rho}{A}$

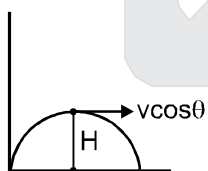
$$x = \frac{0.2 \times 4 \times 10^{-7}}{8 \times 10^{-7}} = \frac{0.8}{8} = 0.1 \text{ V/m.}$$

- 38.** A particle of mass 'm' is projected with a velocity  $v$  making an angle of  $30^\circ$  with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is :

- (1) zero (2)  $\frac{mv^3}{\sqrt{2}g}$  (3)  $\frac{\sqrt{3}}{16} \frac{mv^3}{g}$  (4)  $\frac{\sqrt{3}}{2} \frac{mv^2}{g}$

**Ans. (3)**

**Sol.**



$$L_0 = Pr_{\perp}$$

$$L_0 = mv \cos \theta H$$

$$= mg \frac{\sqrt{3}}{2} \cdot \frac{v^2 \sin^2 30^\circ}{2g} = \frac{\sqrt{3}mv^3}{16g}.$$

- 39.** The specific heat capacity of a metal at low temperature (T) is given as :

$$C_p (\text{kJK}^{-1} \text{kg}^{-1}) = 32 \left( \frac{T}{400} \right)^3$$

A 100 gram vessel of this metal is to be cooled from 20°K to 4°K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is :

- (1) greater than 0.148 kJ (2) between 0.148 kJ and 0.028 kJ  
(3) less than 0.028 kJ (4) equal to 0.002 kJ

**Ans. (4)**

**Sol.**  $Q = \int mcdT$

$$= \int_{20}^4 0.1 \times 32 \times \left( \frac{T^3}{400^3} \right) dT$$

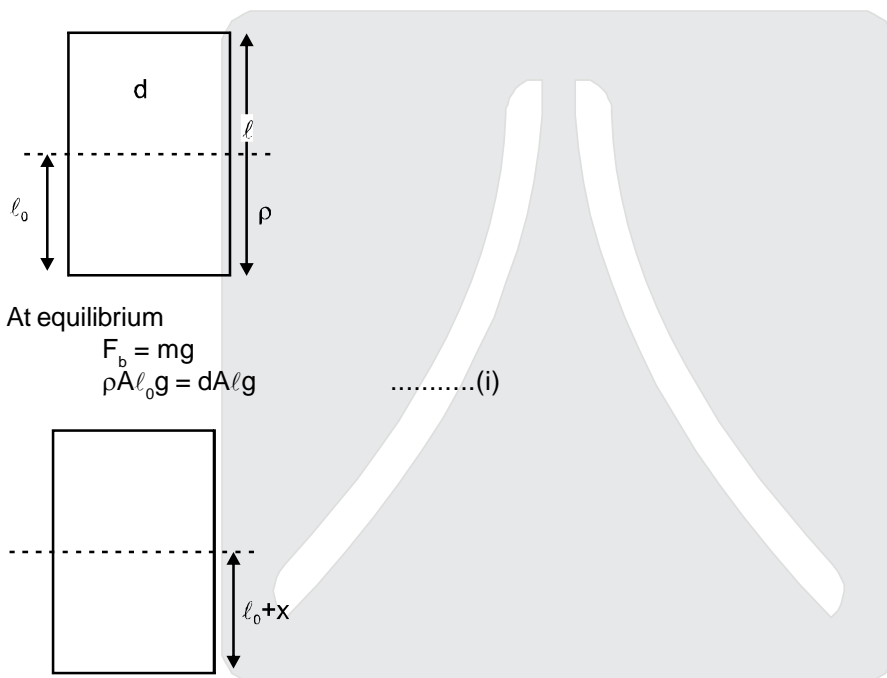
$$\approx 0.002 \text{ k J.}$$

- 40.** A wooden cube (density of wood 'd') of side ' $\ell$ ' floats in a liquid of density ' $\rho$ ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period ' $T$ '. Then, ' $T$ ' is equal to :

(1)  $2\pi \sqrt{\frac{\ell d}{\rho g}}$       (2)  $2\pi \sqrt{\frac{\ell \rho}{dg}}$       (3)  $2\pi \sqrt{\frac{\ell d}{(\rho - d)g}}$       (4)  $2\pi \sqrt{\frac{\ell \rho}{(\rho - d)g}}$

**Ans. (1)**

**Sol.**



Restoring force,

$$F = mg - F_b'$$

$$F = mg - \rho A (\ell_0 + x)g$$

$$dA \ell a = dA \ell g - \rho A \ell_0 g - \rho g A x$$

$$a = -\frac{\rho g}{d \ell} x$$

$$\omega = \sqrt{\frac{\rho g}{d \ell}}$$

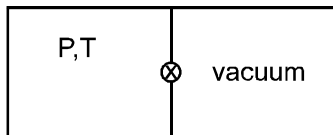
$$T = 2\pi \sqrt{\frac{\ell d}{\rho g}} \quad \dots\dots\dots(i)$$

- 41.** A container with insulating walls is divided into equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure P and temperature T, whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be :

(1)  $\frac{P}{2}, \frac{T}{2}$       (2) P, T      (3) P,  $\frac{T}{2}$       (4)  $\frac{P}{2}, T$

**Ans. (4)**

Sol



It is the free expansion

So, T remain constant

$$P_1 V_1 = P_2 V_2$$

$$P \frac{V}{2} = P_2 (V)$$

$$P_2 = \left( \frac{P}{2} \right)$$

42. In a Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength  $\lambda$ . In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the

first case is  $I_1$  and in the second case is  $I_2$ , then the ratio  $\frac{I_1}{I_2}$  is :

(1) 2

(2) 1

(3) 0.5

(4) 4

Ans. (1)

Sol. For coherent sources :

$$I_1 = 4I_0$$

For incoherent sources

$$I_2 = 2I_0$$

$$\frac{I_1}{I_2} = \frac{4}{2} = 2$$

43. The output of an OR gate is connected to both the inputs of a NAND gate. The combination will serve as a :  
 (1) NOT gate (2) NOR gate (3) AND gate (4) OR gate

Ans. (2)

Sol.  $\overline{(\overline{A} + \overline{B})} = \text{NOR gate}$

When both inputs of NAND gate are connected, it behaves as NOT gate OR + NOT = NOR.

44. Two positive charges of magnitude 'q' are placed at the ends of a side (side 1) of a square of side '2a'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is :

(1) zero

$$(2) \frac{1}{4\pi\epsilon_0} \frac{2qQ}{a} \left( 1 + \frac{1}{\sqrt{5}} \right)$$

$$(3) \frac{1}{4\pi\epsilon_0} \frac{2qQ}{a} \left( 1 - \frac{2}{\sqrt{5}} \right)$$

$$(4) \frac{1}{4\pi\epsilon_0} \frac{2qQ}{a} \left( 1 - \frac{1}{\sqrt{5}} \right)$$

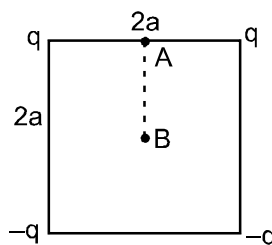
Ans. (4)

Sol. Potential at point A,

$$V_A = \frac{2Kq}{a} - \frac{2Kq}{a\sqrt{5}}$$

Potential at point B,

$$V_B = 0$$



$$\therefore \text{Using work energy theorem,}$$

$$W_{AB}(\text{electric}) = Q(V_A - V_B)$$

$$= \frac{2KqQ}{a} \left[ 1 - \frac{1}{\sqrt{5}} \right] = \left( \frac{1}{4\pi\epsilon_0} \right) \frac{2Qq}{a} \left[ 1 - \frac{1}{\sqrt{5}} \right]$$

45. Combination of two identical capacitors, a resistor R and a dc voltage source of voltage 6V is used in an experiment on a (C – R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time needed for reducing the voltage of the fully charged series combination by half is :  
 (1) 10 second (2) 5 second (3) 2.5 second (4) 20 second

**Ans. (3)**

**Sol.** Time constant for parallel combination = 2RC

$$\text{Time constant for series combination} = \frac{RC}{2}$$

In first case :

$$V = V_0 e^{-\frac{t_1}{2RC}} = \frac{V_0}{2} \quad \dots\dots\dots \text{(i)}$$

In second case :

$$V = V_0 e^{-\frac{t_2}{(RC/2)}} = \frac{V_0}{2} \quad \dots\dots\dots \text{(ii)}$$

From (i) & (ii),

$$\frac{t_1}{2RC} = \frac{t_2}{(RC/2)} \Rightarrow t_2 = \frac{t_1}{4} = \frac{10}{4} = 2.5 \text{ sec.}$$

46. A beaker contains water up to a height  $h_1$  and kerosene of height  $h_2$  above water so that the total height of (water + kerosene) is  $(h_1 + h_2)$ . Refractive index of water is  $\mu_1$  and that of kerosene is  $\mu_2$ . The apparent shift in the position of the bottom of the beaker when viewed from above is :

- (1)  $\left(1 + \frac{1}{\mu_1}\right) h_1 - \left(1 + \frac{1}{\mu_2}\right) h_2$  (2)  $\left(1 - \frac{1}{\mu_1}\right) h_1 + \left(1 - \frac{1}{\mu_2}\right) h_2$   
 (3)  $\left(1 + \frac{1}{\mu_1}\right) h_2 - \left(1 + \frac{1}{\mu_2}\right) h_1$  (4)  $\left(1 - \frac{1}{\mu_1}\right) h_2 + \left(1 - \frac{1}{\mu_2}\right) h_1$

**Ans. (2)**

**Sol.**

$\mu_2$	Kerosene	$h_2$
$\mu_1$	Water	$h_1$

Apparent shift :

$$= h_1 \left( 1 - \frac{1}{\mu_1} \right) + h_2 \left( 1 - \frac{1}{\mu_2} \right)$$

47. A metal rod of Young's modulus Y and coefficient of thermal expansion  $\alpha$  is held at its two ends such that its length remains invariant. If its temperature is raised by  $t^\circ\text{C}$ , the linear stress developed in it is :

- (1)  $\frac{Y}{\alpha t}$  (2)  $Y\alpha t$  (3)  $\frac{1}{(Y\alpha t)}$  (4)  $\frac{\alpha t}{Y}$

**Ans. (2)**



**Sol.**  $\frac{\Delta \ell}{\ell} = \alpha \cdot \Delta T$

and  $Y = \frac{F/A}{\Delta \ell / \ell}$

So,  $F = AY\alpha t$

Thermal stress  $\left(\frac{F}{A}\right) = Y\alpha t$ .



**48.** A travelling wave represented by  $y = A \sin((\omega t - kx))$  is superimposed on another wave represented by  $y = A \sin(\omega t + kx)$ . The resultant is :

(1) A wave travelling along + x direction

(2) A wave travelling along - x direction

(3) A standing wave having nodes at  $x = \frac{n\lambda}{2}$ ,  $n = 0, 1, 2, \dots$

(4) A standing wave having nodes at  $x = \left(n + \frac{1}{2}\right) \frac{\lambda}{2}$ ;  $n = 0, 1, 2, \dots$

**Ans. (4)**

**Sol.**  $Y = A \sin(\omega t - kx) + A \sin(\omega t + kx)$   
 $Y = 2A \sin \omega t \cos kx$  standing wave  
 For nodes  $\cos kx = 0$

$$\frac{2\pi}{\lambda} \cdot x = (2n + 1) \frac{\pi}{2}$$

$$\therefore x = \frac{(2n + 1)\lambda}{4}, n = 0, 1, 2, 3, \dots$$

**49.** A thin circular disk of radius R is uniformly charged with density  $\sigma > 0$  per unit area. The disk rotates about its axis with a uniform angular speed  $\omega$ . The magnetic moment of the disk is :

- (1)  $\pi R^4 \sigma \omega$  (2)  $\frac{\pi R^4}{2} \sigma \omega$  (3)  $\frac{\pi R^4}{4} \sigma \omega$  (4)  $2\pi R^4 \sigma \omega$

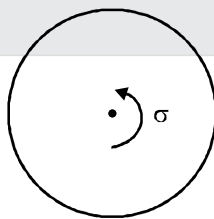
**Ans. (3)**

**Sol.**  $\frac{q}{2M} = \frac{\text{Magnetic dipole moment}}{\text{Angular momentum}}$

$\therefore$  Magnetic dipole moment(M)

$$M = \frac{q}{2M} \cdot \left(\frac{MR^2}{2}\right) \cdot \omega$$

$$= \frac{1}{4} \sigma \cdot \pi R^4 \omega.$$



**50.** An aluminium sphere of 20 cm diameter is heated from  $0^\circ\text{C}$  to  $100^\circ\text{C}$ . Its volume changes by (given that coefficient of linear expansion for aluminium  $\alpha_{Al} = 23 \times 10^{-6}/^\circ\text{C}$ )

- (1) 2.89 cc (2) 9.28 cc (3) 49.8 cc (4) 28.9 cc

**Ans. (4)**

**Sol.**  $\Delta V = V_0(3\alpha) \Delta T$

$$= \frac{4}{3} \pi (10)^3 \times 3 \times 23 \times 10^{-6} \times 100$$

$$\Delta V = 28.9 \text{ cc.}$$

51. Two mercury drops (each of radius 'r') merge to form a bigger drop. The surface energy of the bigger drop, if T is the surface tension, is :

(1)  $4\pi r^2 T$  (2)  $2\pi r^2 T$  (3)  $2^{8/3}\pi r^2 T$  (4)  $2^{5/3}\pi r^2 T$

Ans. (3)

Sol.  $2 \cdot \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = 2^{1/3} r$

S.E. =  $T \cdot 4\pi R^2$

$T \cdot 4\pi \cdot 2^{2/3} r^2$

$T \cdot 2^{8/3} \pi r^2$

52. If a ball of steel (density  $\rho = 7.8 \text{ g cm}^{-3}$ ) attains a terminal velocity of  $10 \text{ cm s}^{-1}$  when falling in a water (Coefficient of Viscosity  $\eta_{\text{water}} = 8.5 \times 10^{-4} \text{ Pa.s}$ ) then its terminal velocity in glycerine ( $\rho = 1.2 \text{ g cm}^{-3}$ ,  $\eta = 13.2 \text{ Pa.s}$ ) would be, nearly:

(1)  $6.25 \times 10^{-4} \text{ cm s}^{-1}$  (2)  $6.45 \times 10^{-4} \text{ cm s}^{-1}$  (3)  $1.5 \times 10^{-5} \text{ cm s}^{-1}$  (4)  $1.6 \times 10^{-5} \text{ cm s}^{-1}$

Ans. (1)

Sol.  $V\rho g = 6\pi\eta rv + v\rho_\ell g$

$Vg(\rho - \rho_\ell) = 6\pi\eta rv$

$Vg(\rho - \rho_\ell) = 6\pi\eta'rv'$

$V'\eta' = \frac{(\rho - \rho_\ell')}{(\rho - \rho_\ell)} \times v\eta$

$V' = \frac{(\rho - \rho_\ell')}{(\rho - \rho_\ell)} \times \frac{v\eta}{\eta'}$

$= \frac{(7.8 - 1.2)}{(7.8 - 1)} \times \frac{10 \times 8.5 \times 10^{-4}}{13.2}$

$v' = 6.25 \times 10^{-4} \text{ cm/s.}$

53. A horizontal straight wire 20 m long extending from east to west falling with a speed of  $5.0 \text{ M/s}$ , at right angles to the horizontal component of the earth's magnetic field  $0.30 \times 10^{-4} \text{ Wb m}^{-2}$ . The instantaneous Value of the e.m. f. induced in the wire will be :

(1) 3 mV (2) 4.5 mV (3) 1.5 mV (4) 6.0 mV

Ans. (1)

Sol.  $\vec{W} \times \vec{B} \rightarrow \vec{E}$

$\varepsilon_{\text{ind}} = Bv\ell$

$= 0.3 \times 10^{-4} \times 5 \times 20$

$= 3 \times 10^{-3} \text{ V}$

$= 3 \text{ mV.}$

54. After absorbing a slowly moving neutron of Mass  $m_n$  (momentum  $\approx 0$ ) a nucleus of mass M breaks into two nuclei of masses  $m_1$  and  $5m_1$  ( $6m_1 = M + m_n$ ) respectively. If the de Broglie wavelength of the nucleus with mass  $m_1$  is  $\lambda$ , the de Broglie wavelength of the nucleus will be:

(1)  $5\lambda$  (2)  $\lambda/5$  (3)  $\lambda$  (4)  $25\lambda$

Ans. (3)

**Sol.**

$$P_i = 0$$

$$P_f = P_1 + P_2$$

$$P_i = P_f$$

$$0 = P_1 + P_2$$

$$(P_1 = -P_2)$$

$$\lambda_1 = \frac{h}{P_1}$$

$$\lambda_2 = \frac{h}{P_2}$$

$$|\lambda_1| = |\lambda_2|$$

$$\lambda_1 = \lambda_2 = \lambda.$$

**55.** Which of the following four alternatives is not correct?

We need modulation:

- (1) to reduce the time lag between transmission and reception of the information signal
- (2) to reduce the size of antenna
- (3) to reduce the fractional band width, that is the ratio of the signal band width to the centre frequency
- (4) to increase the selectivity.

**Ans. (1)**

**Sol.** Low frequencies cannot be transmitted to long distances. Therefore, they are super imposed on a high frequency carrier signal by a process known as modulation.  
Speed of electro-magnetic waves will not change due to modulation. So time lag between transmission and reception of the information signal.

**56.** If a spring of stiffness 'k' is cut into two parts 'A' and 'B' of length  $\ell_A : \ell_B = 2 : 3$ , then the stiffness of spring 'A' is given by :

- (1)  $\frac{3k}{5}$
- (2)  $\frac{2k}{5}$
- (3) k
- (4)  $\frac{5}{2k}$

**Ans. (4)**

**Sol.**  $\ell_A = \frac{2\ell}{5}, \ell_B = \left(\frac{3\ell}{5}\right)$

$$K\ell = K_A\ell_A = K_B\ell_B$$

$$K\ell = K_A\left(\frac{2\ell}{5}\right)$$

$$K_A = \frac{5K}{2} \Rightarrow K_B = \frac{5K}{3}.$$

**57. Statement - 1 :**

A nucleus having energy  $E_1$  decays by  $\beta^-$  emission to daughter nucleus having energy  $E_2$ , but the  $\beta^-$  rays are emitted with a continuous energy spectrum having end point energy  $E_1 - E_2$ .

**Statement - 2:**

To conserve energy and momentum in  $\beta$ -decay at least three particles must take part in the transformation.

- (1) Statement-1 is correct but statement-2 is not correct.
- (2) Statement-1 and statement-2 both are correct and statement-2 is the correct explanation of statement-1.
- (3) Statement-1 is correct, statement-2 is correct and statement-2 is not the correct explanation of statement-1
- (4) Statement-1 is incorrect, statement-2 is correct.

**Ans. (2)**

**Sol. Statement-1:** Energy of  $\beta^-$  particle from 0 to maximum so  $E_1 - E_2$  is the continuous energy spectrum.

**Statement-2 :** For energy conservation and momentum at least three particles daughter nucleus  $+\beta^-$  and antineutrino.

58. When monochromatic red light is used instead of blue light in a convex lens, its focal length will :  
 (1) increase (2) decrease  
 (3) remain same (4) does not depend on colour of light

**Ans. (1)**

**Sol.**

$$\mu_R < \mu_B$$

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f_B} > \frac{1}{f_R}$$

$$f_R > f_B$$

59. **Statement - 1 :**

On viewing the clear blue portion of the sky through a Calcite Crystal, the intensity of transmitted light varies as the crystal is rotated.

**Statement - 2:**

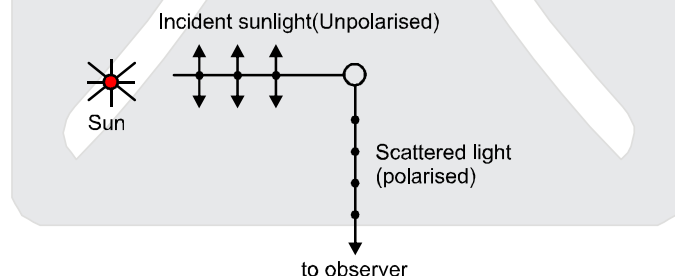
The light coming from the sky is polarized due to scattering of sun light by particles in the atmosphere. The scattering is largest for blue light

- (1) Statement-1 is true, statement-2 is false.  
 (2) Statement-1 is true, statement-2 is true, statement-2 is the correct explanation of statement-1  
 (3) Statement-1 is true, statement-2 is true, statement-2 is not the correct explanation of statement-1  
 (4) Statement-1 is false, statement-2 is true.

**Ans. (2)**

**Sol.**

The light from a clear blue portion of the sky shows a rise and fall of intensity when viewed through a polaroid which is rotated.



60. **Statement - 1 :**

Two longitudinal waves given by equations :  $y_1(x, t) = 2a \sin(\omega t - kx)$  and  $y_2(x, t) = a \sin(2\omega t - 2kx)$  will have equal intensity.

**Statement - 2:**

Intensity of waves of given frequency in same medium is proportional to square of amplitude only.

- (1) Statement-1 is true, statement-2 is false.  
 (2) Statement-1 is true, statement-2 is true, statement-2 is the correct explanation of statement-1  
 (3) Statement-1 is true, statement-2 is true, statement-2 is not the correct explanation of statement-1  
 (4) Statement-1 is false, statement-2 is true.

**Ans. (1)**

**Sol.**

$$\text{Since, } I \propto A^2 \omega^2$$

$$I_1 \propto (2a)^2 \omega^2$$

$$I_2 \propto a^2 (2\omega)^2$$

$$I_1 = I_2$$

Intensity depends on frequency also.

## PART-C (MATHEMATICS)

61. Let  $f$  be a function defined by  $f(x) = (x-1)^2 + 1, (x \geq 1)$ .

**Statement - 1 :**

The set  $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$ .

**Statement - 2 :**

$f$  is a bijection and  $f^{-1}(x) = 1 + \sqrt{x-1}, x \geq 1$ .

- (1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.  
 (2) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1  
 (3) Statement-1 is true, Statement-2 is false  
 (4) Statement-1 is false, Statement-2 is true .

**Sol.**

(1)

$$f(x) = (x-1)^2 + 1, x \geq 1$$

$f : [1, \infty) \rightarrow [1, \infty)$  is a bijective function

$$\Rightarrow y = (x-1)^2 + 1 \Rightarrow (x-1)^2 = y-1$$

$$\Rightarrow x = 1 \pm \sqrt{y-1} \Rightarrow f^{-1}(y) = 1 \pm \sqrt{y-1}$$

$$\Rightarrow f^{-1}(x) = 1 + \sqrt{x-1} \quad \{\because x \geq 1\}$$

so statement-2 is correct

$$\text{Now } f(x) = f^{-1}(x) \Rightarrow f(x) = x \Rightarrow (x-1)^2 + 1 = x$$

$$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$$

so statement-1 is correct

62. If  $\omega \neq 1$  is the complex cube root of unity and matrix  $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$ , then  $H^{70}$  is equal to -

(1) 0

(2)  $-H$

(3)  $H^2$

(4)  $H$

**Sol.**

(4)

$$H^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$$

$$\text{If } H^k = \begin{bmatrix} \omega^k & 0 \\ 0 & \omega^k \end{bmatrix}, \text{ then } H^{k+1} = \begin{bmatrix} \omega^{k+1} & 0 \\ 0 & \omega^{k+1} \end{bmatrix}$$

So by mathematical induction,

$$H^{70} = \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H$$

63. Let  $[.]$  denote the greatest integer function then the value of  $\int_0^{1.5} x[x^2] dx$  is : .

(1) 0

(2)  $\frac{3}{2}$

(3)  $\frac{3}{4}$

(4)  $\frac{5}{4}$

**Sol.**

(3)

$$\int_0^1 x[x^2] dx + \int_1^{\sqrt{2}} x[x^2] dx + \int_{\sqrt{2}}^{1.5} x[x^2] dx$$

$$\int_0^1 x \cdot 0 dx + \int_1^{\sqrt{2}} x dx + \int_{\sqrt{2}}^{1.5} 2x dx$$

$$0 + \left[ \frac{x^2}{2} \right]_1^{\sqrt{2}} + \left[ x^2 \right]_{\sqrt{2}}^{1.5}$$

$$\frac{1}{2}(2-1) + (2.25-2)$$

$$\frac{1}{2} + .25$$

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

64. The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact is given by :

(1)  $2y - 3x = 0$

(2)  $y = \frac{6}{x}$

(3)  $x^2 + y^2 = 13$

(4)  $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$

Sol. (2)

$$Y - y = \frac{dy}{dx}(X - x)$$

$$\text{X-intercept is } \left(x - \frac{y}{dy/dx}, 0\right)$$

$$\text{Y-intercept is } \left(0, y - \frac{xdy}{dx}\right)$$

According to statment

$$x - \frac{y}{dy/dx} = 2x \text{ and } y - \frac{xdy}{dx} = 2y$$

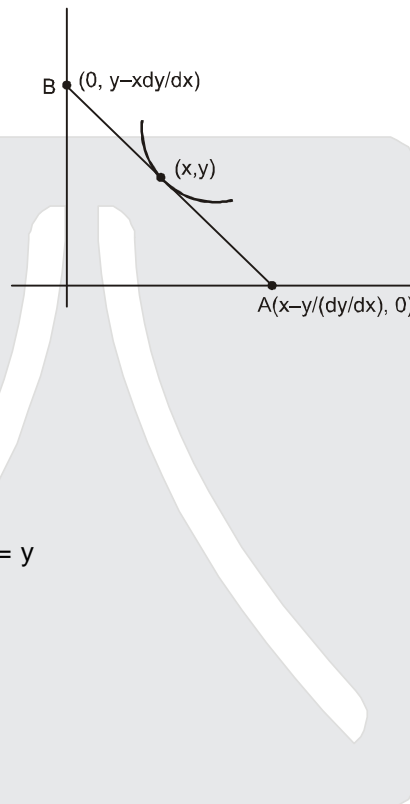
$$\frac{-y}{dy/dx} = x$$

$$\frac{dx}{x} + \frac{dy}{y} = 0$$

$$\ell ny = -\ell nx + \ell nc$$

$$y = \frac{c}{x} \Rightarrow c = 6$$

$$\text{Hence } y = \frac{6}{x}$$



65. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively :

(1) 32, 2

(2) 32, 4

(3) 28, 2

(4) 28, 4

Sol. (1)

$$\text{Correct mean} = \text{observed mean} + 2$$

$$30 + 2 = 32$$

$$\text{Correct S.D.} = \text{observed S.D.} = 2$$

66. The lines  $x + y = |a|$  and  $ax - y = 1$  intersect each other in the first quadrant. Then the set of all possible values of  $a$  is the interval :

(1)  $(0, \infty)$

(2)  $[1, \infty)$

(3)  $(-1, \infty)$

(4)  $(-1, 1]$

Sol. (2)

$$x + y = |a|$$

$$ax - y = 1$$

$$\text{if } a > 0$$

$$x + y = a$$

$$ax - y = 1$$

$$x(1 + a) = 1 + a \text{ as } x = 1$$

$$y = a - 1$$

It is first quadrant

$$\text{so } a - 1 \geq 0$$

$$a \geq 1$$

$$a \in [1, \infty)$$

If  $a < 0$

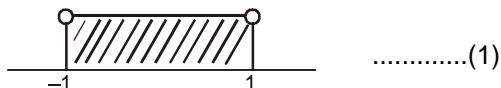
$$x + y = -a$$

$$ax - y = 1$$

+

$$x(1 + a) = 1 - a$$

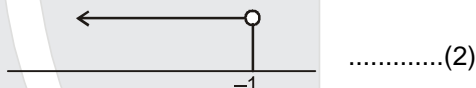
$$x = \frac{1-a}{1+a} > 0 \Rightarrow \frac{a-1}{a+1} < 0$$



$$y = -a - \frac{1-a}{1+a}$$

$$= \frac{-a - a^2 - 1 + a}{1+a} > 0$$

$$-\left(\frac{a^2+1}{a+1}\right) > 0 \Rightarrow \frac{a^2+1}{a+1} < 0$$



from (1) and (2)  $a \in \{\emptyset\}$

So correct answer is  $a \in [1, \infty)$

67. If the vector  $p\hat{i} + \hat{j} + \hat{k}$ ,  $\hat{i} + q\hat{j} + \hat{k}$  and  $\hat{i} + \hat{j} + r\hat{k}$  ( $p \neq q \neq r \neq 1$ ) are coplanar, then the value of  $pqr - (p+q+r)$  is-
- (1) 2 (2) 0 (3) -1 (4) -2

Sol.

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

$$\Rightarrow p(qr - 1) + 1(1 - r) + 1(1 - q) = 0$$

$$\Rightarrow pqr - p + 1 - r + 1 - q = 0$$

$$\Rightarrow pqr - (p + q + r) = -2$$

68. The distance of the point  $(1, -5, 9)$  from the plane  $x - y + z = 5$  measured along a straight line  $x = y = z$  is :

(1)  $10\sqrt{3}$

(2)  $5\sqrt{3}$

(3)  $3\sqrt{10}$

(4)  $3\sqrt{5}$

Sol. (1)

Line through  $P(1, -5, 9)$  parallel to  $x = y = z$  is

$$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda \text{ (say)}$$

$$Q(x = 1 + \lambda, y = -5 + \lambda, z = 9 + \lambda)$$

Given plane  $x - y + z = 5$

$$\therefore 1 + \lambda + 5 - \lambda + 9 + \lambda = 5$$

$$\Rightarrow \lambda = -10$$

$$\therefore Q(-9, -15, -1)$$

$$\therefore PQ = \sqrt{(1+9)^2 + (15-5)^2 + (9+1)^2}$$

$$= \sqrt{300} = 10\sqrt{3}$$

69. Let  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  be three non-zero vectors which are pairwise non-collinear. If  $\vec{a} + 3\vec{b}$  is collinear with  $\vec{c}$  and  $\vec{b} + 2\vec{c}$  is collinear with  $\vec{a} + 3\vec{b} + 6\vec{c}$  is :

- (1)  $\vec{a}$  (2)  $\vec{c}$  (3)  $\vec{0}$  (4)  $\vec{a} + \vec{c}$

Sol. (3)

$$\vec{a} + 3\vec{b} = \lambda \vec{c} \quad \dots\dots(1)$$

$$\vec{b} + 2\vec{c} = \mu \vec{a} \quad \dots\dots(2)$$

$$(1) - 3(2) \text{ gives } (1 + 3\mu)\vec{a} - (\lambda + 6)\vec{c} = \vec{0}$$

As  $\vec{a}$  and  $\vec{c}$  are non collinear

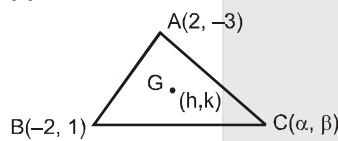
$$\therefore 1 + 3\mu = 0 \text{ and } \lambda + 6 = 0$$

$$\text{From (1) } \vec{a} + 3\vec{b} + 6\vec{c} = \vec{0}$$

70. If A(2, -3) and B(-2, 1) are two vertices of a triangle and third vertex moves on the line  $2x + 3y = 9$ , then the locus of the centroid of the triangle is :

- (1)  $x - y = 1$  (2)  $2x + 3y = 1$  (3)  $2x + 3y = 3$  (4)  $2x - 3y = 1$

Sol. (2)



$$\alpha = 3h$$

$$\beta - 2 = 3k$$

$$\beta = 3k + 2$$

third vertex on the line

$$2x + 3y = 9$$

$$2\alpha + 3\beta = 9$$

$$2(3h) + 3(3k + 2) = 9$$

$$2h + 3k = 1$$

$$2x + 3y - 1 = 0$$

71. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points. then :

- (1)  $N \leq 100$  (2)  $100 < N \leq 140$  (3)  $140 < N \leq 190$  (4)  $N > 190$

Sol. (1)

$${}^{10}C_3 - {}^6C_3$$

$$= \frac{10 \times 9 \times 8}{6} - \frac{6 \times 5 \times 4}{6} = 120 - 20 = 100$$

72. Define F(x) as the product of two real functions  $f_1(x) = x$ ,  $x \in \mathbb{R}$ , and  $f_2(x) = \begin{cases} \sin \frac{1}{x}, & \text{If } x \neq 0 \\ 0, & \text{If } x = 0 \end{cases}$  as follows :

$$F(x) = \begin{cases} f_1(x) \cdot f_2(x), & \text{If } x \neq 0 \\ 0, & \text{If } x = 0 \end{cases}$$

**Statement - 1** : F(x) is continuous on  $\mathbb{R}$ .

**Statement - 2** :  $f_1(x)$  and  $f_2(x)$  are continuous on  $\mathbb{R}$ .

- (1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.  
 (2) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1  
 (3) Statement-1 is true, Statement-2 is false  
 (4) Statement-1 is false, Statement-2 is true

Sol. (2)

$$f(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at  $x = 0$



$$\text{LHL} = \lim_{h \rightarrow 0^+} \left\{ -h \sin \left( -\frac{1}{h} \right) \right\}$$

= 0 × a finite quantity between -1 and 1

$$\text{RHL} = \lim_{h \rightarrow 0^+} h \sin \frac{1}{h}$$

= 0

f(0) = 0

∴ f(x) is continuous on R.

f<sub>2</sub>(x) is not continuous at x = 0

73. **Statement - 1** : For each natural number n, (n + 1)<sup>7</sup> - n<sup>7</sup> - 1 is divisible by 7.

**Statement - 2** : For each natural number n, n<sup>7</sup> - n is divisible by 7.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Sol.

(1)

**Statement 2**

$$P(n) = n^7 - n$$

Put n = 1      1 - 1 = 0 is divisible by 7

Let n = k      P(k) = k<sup>7</sup> - k is divisible by 7

Put n = k + 1

$$\therefore P(k + 1) = (k + 1)^7 - (k + 1)$$

$$= k^7 + {}^7C_1 k^6 + {}^7C_2 k^5 + \dots + {}^7C_6 k + 1 - k - 1$$

$$P(k + 1) = (k^7 - k) + \text{multiple of 7}$$

As 7 is coprime with 1, 2, 3, 4, 5, 6 so <sup>7</sup>C<sub>1</sub>, <sup>7</sup>C<sub>2</sub>, ..., <sup>7</sup>C<sub>6</sub> are all divisible by 7

P(k + 1) is divisible by 7

Hence P(n) = n<sup>7</sup> - n is divisible by 7

**Statement 1**

n<sup>7</sup> - n is divisible by 7

$$\Rightarrow (n + 1)^7 - (n + 1) \text{ is divisible by 7}$$

$$\Rightarrow (n + 1)^7 - n^7 - 1 + (n^7 - n) \text{ is divisible by 7}$$

$$\Rightarrow (n + 1)^7 - n^7 - 1 \text{ is divisible by 7}$$

74. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is -

$$(1) x^2 + y^2 - 2x - 2y + 1 = 0$$

$$(2) x^2 + y^2 - x - y = 0$$

$$(3) x^2 + y^2 + 2x + 2y - 7 = 0$$

$$(4) x^2 + y^2 + x + y - 2 = 0$$

Sol.

(2)

Circle whose diametric end points are (1, 0) and (0, 1) will be of smallest radius.

$$(x - 1)(x - 0) + (y - 0)(y - 1) = 0$$

$$x^2 + y^2 - x - y = 0$$

75. The equation of the hyperbola whose foci are (-2, 0) and (2, 0) and eccentricity is 2 is given by :

$$(1) x^2 - 3y^2 = 3$$

$$(2) 3x^2 - y^2 = 3$$

$$(3) -x^2 + 3y^2 = 3$$

$$(4) -3x^2 + y^2 = 3$$

Sol.

(2)

$$ae = 2$$

$$e = 2$$

$$\therefore a = 1$$

$$b^2 = a^2 (e^2 - 1)$$

$$b^2 = 1 (4 - 1)$$

$$b^2 = 3$$

$$\frac{x^2}{1} - \frac{y^2}{3} = 1$$

$$3x^2 - y^2 = 3$$

76. If the trivial solution is the only solution of the system of equations

$$\begin{aligned}x - ky + z &= 0 \\ kx + 3y - kz &= 0 \\ 3x + y - z &= 0\end{aligned}$$

then the set of all values of  $k$  is :

- (1)  $\mathbb{R} - \{2, -3\}$  (2)  $\mathbb{R} - \{2\}$  (3)  $\mathbb{R} - \{-3\}$  (4)  $\{2, -3\}$

Sol.

(1)

$$\begin{aligned}x - ky + z &= 0 \\ kx + 3y - kz &= 0 \\ 3x + y - z &= 0\end{aligned}$$

this equation will have non trivial solution if

$$\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} = 0$$

$$1(-3 + k) + k(-k + 3k) + 1(k - 9) = 0$$

$$k - 3 + 2k^2 + k - 9 = 0$$

$$2k^2 + 2k - 12 = 0$$

$$k^2 + k - 6 = 0$$

$$k = -3, k = 2$$

so the equation will have only trivial solution  
when  $k \in \mathbb{R} - \{2, -3\}$

77. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of  $x$  to get roots (3, 2). The correct roots of equation are :

- (1) 6, 1 (2) 4, 3 (3) -6, -1 (4) -4, -3

Sol.

(1)

Let the correct equation be  $ax^2 + bx + c = 0$

now sachin's equation  $\Rightarrow$

$$ax^2 + bx + c' = 0 \begin{cases} \nearrow 4 \\ \searrow 3 \end{cases}$$

Rahul's equation  $\Rightarrow$

$$ax^2 + b'x + c = 0 \begin{cases} \nearrow 3 \\ \searrow 2 \end{cases}$$

$$-\frac{b}{a} = 7$$

..... (i)

$$\frac{c}{a} = 6$$

..... (ii)

from (i) and (ii)

correct equation is

$$x^2 - 7x + 6 = 0$$

roots are 6 and 1

78. Let  $a_n$  be the  $n^{\text{th}}$  term of an A.P. If  $\sum_{r=1}^{100} a_{2r} = \alpha$  and  $\sum_{r=1}^{100} a_{2r-1} = \beta$ , then the common difference of the A.P. is

- (1)  $\alpha - \beta$  (2)  $\frac{\alpha - \beta}{100}$  (3)  $\beta - \alpha$  (4)  $\frac{\alpha - \beta}{200}$

Sol.

(2)

Let A.P. be  $a, a + d, a + 2d, \dots$

$$a_2 + a_4 + \dots + a_{200} = \alpha$$

$$\Rightarrow \frac{100}{2} [2(a + d) + (100 - 1)d] = \alpha \quad \dots (i)$$

$$\text{and } a_1 + a_3 + a_5 + \dots + a_{199} = \beta$$

$$\Rightarrow \frac{100}{2} [2a + (100 - 1)d] = \beta \quad \dots(ii)$$

on solving (i) and (ii)

$$d = \frac{\alpha - \beta}{100}$$

79. Consider the differential equation  $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$ . If  $y(1) = 1$ , then  $x$  is given by :

(1)  $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$

(2)  $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

(3)  $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$

(4)  $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

Sol. (3)

$$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$$

$$\text{I.F.} = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$$

$$\text{so } x \cdot e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$$

$$\text{Let } \frac{-1}{y} = t$$

$$\Rightarrow \frac{1}{y^2} dy = dt$$

$$\Rightarrow I = - \int t e^t dt = e^t - t e^t$$

$$= e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$$

$$\Rightarrow x e^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$$

$$\Rightarrow x = 1 + \frac{1}{y} + c \cdot e^{1/y}$$

$$\text{since } y(1) = 1$$

$$\therefore c = -\frac{1}{e}$$

$$\Rightarrow x = 1 + \frac{1}{y} - \frac{1}{e} \cdot e^{1/y}$$

80. Let  $f : \mathbb{R} \rightarrow [0, \infty)$  be such that  $\lim_{x \rightarrow 5} f(x)$  exists and  $\lim_{x \rightarrow 5} \frac{(f(x))^2 - 9}{\sqrt{|x - 5|}} = 0$

Then  $\lim_{x \rightarrow 5} f(x)$  equals :

(1) 0

(2) 1

(3) 2

(4) 3

Sol. (4)

$$\lim_{x \rightarrow 5} \frac{(f(x))^2 - 9}{\sqrt{|x - 5|}} = 0$$

$$\lim_{x \rightarrow 5} [(f(x))^2 - 9] = 0$$

$$\lim_{x \rightarrow 5} f(x) = 3$$

81. **Statement - 1 :**

Determinant of a skew-symmetric matrix of order 3 is zero.

**Statement - 2 :**

For any matrix A,  $\det(A)^T = \det(A)$  and  $\det(-A) = -\det(A)$ .

Where  $\det(2)$  denotes the determinant of matrix B. Then :

(1) Both statements are true

(2) Both statements are false

(3) Statement-1 is false and statement-2

(4) Statement-1 is true and statement-2 is false

Sol.

(4)

**Statement-1 :** Determinant of a skew symmetric matrix of odd order is zero

**Statement-2 :**  $\det(A^T) = \det(A)$

$\det(-A) = (-1)^n \det(A)$  where A is a  $n \times n$  order matrix

82.

The possible values of  $\theta \in (0, \pi)$  such that  $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$  are :

(1)  $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(2)  $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(3)  $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(4)  $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

Sol.

(4)

$$\sin 4\theta + 2\sin 4\theta \cos 3\theta = 0$$

$$\therefore \theta \in (0, \pi)$$

$$\sin 4\theta (1 + 2\cos 3\theta) = 0$$

$$\sin 4\theta = 0$$

or

$$\cos 3\theta = -\frac{1}{2}$$

$$4\theta = n\pi; n \in \mathbb{I}$$

or

$$3\theta = 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{I}$$

$$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$$

or

$$\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$$

83.

The area bounded by the curves  $y^2 = 4x$  and  $x^2 = 4y$  is :

(1)  $\frac{32}{3}$

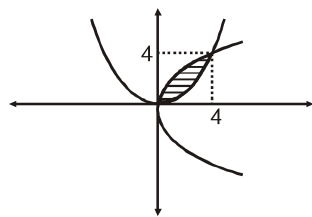
(2)  $\frac{16}{3}$

(3)  $\frac{8}{3}$

(4) 0

Sol.

(2)



$$\text{Area} = \int_0^4 \left( 2\sqrt{x} - \frac{x^2}{4} \right) dx$$

$$= \left( 2 \left( \frac{x^{3/2}}{3/2} \right) - \frac{x^3}{12} \right)_0^4$$

$$= \frac{4}{3} \times 8 - \frac{64}{12}$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

84. Let  $f$  be a function defined by -

$$f(x) = \begin{cases} \frac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \quad x \neq 0$$

**Statement - 1 :**  $x = 0$  is point of minima of  $f$

**Statement - 2 :**  $f'(0) = 0$ .

- (1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.  
 (2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1 (3)  
 Statement-1 is true, statement-2 is false.  
 (4) Statement-1 is false, statement-2 is true.

**Sol. (2)**

$$f(x) = \begin{cases} \frac{\tan x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

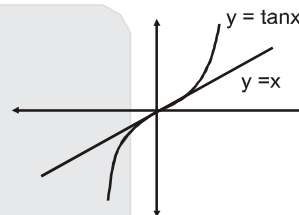
In right neighbourhood of '0'  
 $\tan x > x$

$$\frac{\tan x}{x} > 1$$

In left neighbourhood of '0'  
 $\tan x < x$

$$\frac{\tan x}{x} > 1 \quad \text{as } (x < 0)$$

at  $x = 0$ ,  $f(x) = 1$   
 $\Rightarrow x = 0$  is point of minima  
 so statement 1 is true.  
 statement 2 obvious



85. The only statement among the following that is a tautology is -

- (1)  $A \wedge (A \vee B)$  (2)  $A \vee (A \wedge B)$  (3)  $[A \wedge (A \rightarrow B)] \rightarrow B$  (4)  $B \rightarrow [A \wedge (A \rightarrow B)]$

**Sol. (3)**

A	B	$A \vee B$	$A \wedge B$	$A \wedge (A \vee B)$	$A \vee (A \wedge B)$	$A \rightarrow B$	$A \wedge (A \rightarrow B)$	$[A \wedge (A \rightarrow B)] \rightarrow B$	$B \rightarrow [A \wedge (A \rightarrow B)]$
T	F	T	F	T	T	F	F	T	T
F	T	T	F	F	F	T	F	T	F
T	T	T	T	T	T	T	T	T	T
F	F	F	F	F	F	T	F	T	T

86. Let  $A, B, C$  be pairwise independent events with  $P(C) > 0$  and  $P(A \cap B \cap C) = 0$ .

Then  $P(A^c \cap B^c / C)$ .

- (1)  $P(1) - P(B^c)$  (2)  $P(A^c) + P(B^c)$  (3)  $P(A^c) - P(B^c)$  (4)  $P(A^c) - P(B)$

**Sol. (4)**

$$\begin{aligned}
 P(A^c \cap B^c / C) &= \frac{P((A^c \cap B^c) \cap C)}{P(C)} \\
 &= \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)} \\
 &= \frac{P(C) - P(A) \cdot P(C) - P(B)P(C) + 0}{P(C)} \\
 &= 1 - P(A) - P(B) \\
 &= P(A^c) - P(B)
 \end{aligned}$$

87. Let for  $a \neq a_1 \neq 0$ ,  
 $f(x) = ax^2 + bx + c$ ,  $g(x) = a_1x^2 + b_1x + c_1$  and  $p(x) = f(x) - g(x)$ .  
 If  $p(x) = 0$  only for  $x = -1$  and  $p(-2) = 2$ , then the value of  $p(2)$  is :  
 (1) 3 (2) 9 (3) 6 (4) 18

Sol.

(4)

$$P(x) = 0$$

$$\Rightarrow f(x) = g(x)$$

$$\Rightarrow ax^2 + bx + c = a_1x^2 + b_1x + c_1$$

$$\Rightarrow (a - a_1)x^2 + (b - b_1)x + (c - c_1) = 0.$$

It has only one solution  $x = -1$

$$\Rightarrow b - b_1 = a - a_1 + c - c_1 \quad \dots (1)$$

$$\text{vertex } (-1, 0) \Rightarrow \frac{b - b_1}{2(a - a_1)} = -1 \Rightarrow b - b_1 = 2(a - a_1) \quad \dots (2)$$

$$\Rightarrow f(-2) - g(-2) = 2$$

$$\Rightarrow 4a - 2b + c - 4a_1 + 2b_1 - c_1 = 2$$

$$\Rightarrow 4(a - a_1) - 2(b - b_1) + (c - c_1) = 2 \quad \dots (3)$$

$$\text{by (1), (2) and (3) } (a - a_1) = (c - c_1) = \frac{1}{2}(b - b_1) = 2$$

$$\begin{aligned} \text{Now } P(2) &= f(2) - g(2) \\ &= 4(a - a_1) + 2(b - b_1) + (c - c_1) \\ &= 8 + 8 + 2 = 18 \end{aligned}$$

88. The length of the perpendicular drawn from the point  $(3, -1, 11)$  to the line  $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  is :

- (1)  $\sqrt{29}$  (2)  $\sqrt{33}$  (3)  $\sqrt{53}$  (4)  $\sqrt{66}$

Sol.

(3)

Let feet of perpendicular is

$$(2\alpha, 3\alpha + 2, 4\alpha + 3)$$

$$\Rightarrow D' \text{ ratio of the } \perp \text{ line } < 2\alpha - 3, 3\alpha + 3, 4\alpha - 8 >$$

$$\text{and } D' \text{ ratio of the line } < 2, 3, 4 >$$

$$\Rightarrow 2(2\alpha - 3) + 3(3\alpha + 3) + 4(4\alpha - 8) = 0$$

$$\Rightarrow 29\alpha - 29 = 0$$

$$\Rightarrow \alpha = 1$$

$$\Rightarrow \text{feet of } \perp \text{ is } (2, 5, 7)$$

$$\Rightarrow \text{length } \perp \text{ is } \sqrt{1^2 + 6^2 + 4^2}$$

$$= \sqrt{53}$$

89. Consider the following relation  $R$  on the set of real square matrices of order 3.  
 $R = \{(A, B) | A = P^{-1}BP \text{ for some invertible matrix } P\}$ .

**Statement -1 :**

$R$  is equivalence relation.

**Statement - 2 :**

For any two invertible  $3 \times 3$  matrices  $M$  and  $N$ ,  $(MN)^{-1} = N^{-1}M^{-1}$ .

(1) Statement-1 is true, statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

Sol.

(2)

**for reflexive**

$$(A, A) \in R$$

$$\Rightarrow A = P^{-1}AP$$

which ..... for  $P = I$

$\therefore$  reflexive

**for symmetry**

As  $(A, B) \in R$  for matrix  $P$

$$A = P^{-1}BP$$

$$\begin{aligned}
&\Rightarrow PAP^{-1} = B \\
&\Rightarrow B = PAP^{-1} \\
&\Rightarrow B = (P^{-1}) A (P^{-1}) \\
&\therefore (B, A) \in R \text{ for matrix } P^{-1} \\
&\therefore R \text{ is symmetric}
\end{aligned}$$

**for transitivity**

$$\begin{aligned}
A &= P^{-1} B P \\
\text{and } B &= P^{-1} C P \\
&\Rightarrow A = P^{-1} (P^{-1} C P) P \\
&\Rightarrow A = (P^{-1})^2 C P^2 \\
&\Rightarrow A = (P^2)^{-1} C (P^2) \\
&\therefore (A, C) \in R \text{ for matrix } P^2 \\
&\therefore R \text{ is transitive} \\
&\text{so } R \text{ is equivalence}
\end{aligned}$$

90. If function  $f(x)$  is differentiable at  $x = a$ , then  $\lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$  is :

- (1)  $-a^2 f'(a)$  (2)  $a f(a) - a^2 f'(a)$  (3)  $2a f(a) - a^2 f'(a)$  (4)  $2a f(a) + a^2 f'(a)$

**Sol.**

$$\begin{aligned}
&\lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(x)}{x - a} \\
&= \lim_{x \rightarrow a} \frac{2x f(a) - a^2 f'(x)}{1} \\
&= 2a f(a) - a^2 f'(a)
\end{aligned}$$

**Alter**

$$\begin{aligned}
&\lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(x)}{x - a} \\
&= \lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(a) + a^2 f(a) - a^2 f(x)}{x - a} \\
&= \lim_{x \rightarrow a} \frac{(x^2 - a^2) f(a) - a^2 (f(x) - f(a))}{x - a} \\
&= \lim_{x \rightarrow a} (x + a) f(a) - a^2 \left\{ \frac{f(x) - f(a)}{(x - a)} \right\} \\
&= 2a f(a) - a^2 f'(a)
\end{aligned}$$

**Read the following instructions carefully:**

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet (**Side—1**) with **Blue/Black Ball Point Pen**.
2. For writing/marketing particulars on **Side-2** of the Answer Sheet, use **Blue/Black Ball Point Pen** only.
3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each *incorrect response*, one-fourth ( $\frac{1}{4}$ ) of the total marks allotted to the question would be deducted from the total score. **No deduction** from the total score, however, will be made **if no response** is indicated for an item in the Answer Sheet.
6. Handle the Test Booklet and Answer Sheet with care, *as under no circumstance (except for discrepancy in Test Booklet Code and Answer Sheet Code), will another set be provided.*
7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/ writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 3 pages at the end of the booklet.
8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**
9. Each candidate must show on demand his/her Admit Card to the Invigilator.
10. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.
11. The candidates should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed **the Attendance Sheet** a second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. The candidates are **also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.**
12. Use of Electronic/Manual Calculator and any Electronic Item like mobile phone, pager etc. is prohibited.
13. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
15. **Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination hall/room.**