(MATHEMATICS)

Code : 65/1

General Instructions :

(i) All questions are compulsory.

(ii) The question paper consists of **29** questions divided into three sections A, B and C. Section A comprises of **10** questions of **one mark** each, Section B comprises of **12** questions of **four marks** each and Section C comprises of **7** questions of **six marks** each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the questions.

(iv) There is no overall choice, However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.

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1. Write the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$. **1**

Sol. $\tan^{-1}(\sqrt{3}) = \pi/3$

$\cot^{-1}(-\sqrt{3}) = \pi - \pi/6$

Hence

$\pi/3 - (\pi - \pi/6) = -\pi/2$

2. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$. **1**

Sol. $\because \quad \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

$= \tan^{-1}(2\sin(2\cdot\pi/6))$

$= \tan^{-1}\left(2\cdot\sin\frac{\pi}{3}\right)$

$= \tan^{-1}\left(2\cdot\frac{\sqrt{3}}{2}\right) = \tan^{-1}\sqrt{3} = \pi/3$

3. For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix ? **1**

Sol. The value of determinant of skew symmetric matrix of odd order is always equal to zero.

$$\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{vmatrix} = 0$$

$-1(0 - 3x) - 2(3 - 0) = 0$

$\Rightarrow 3x - 6 = 0 \quad \Rightarrow \boxed{x = 2}$

4. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k. **1**

Sol. Given $A^2 = kA$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \Rightarrow \boxed{k = 2}$$

5. Write the differential equation representing the family of curves y = mx, where m is an arbitrary constant. **1**

Sol. $y = mx$ .....(1)

differentiating with respect to x, we get

$dy/dx = m$

$\therefore$ differential equation of curve

$$y = \frac{xdy}{dx}$$

6. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32} \cdot A_{32}$. **1**

Sol. $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

$A_{32} = (-1)^{3+2} M_{32}$ where M_{32} is the minor of a_{32}.

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$$

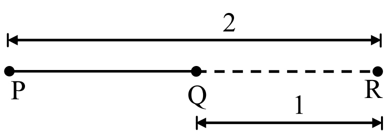
$$A_{32} = -\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} \Rightarrow A_{32} = -(8 - 30)$$

$$\boxed{A_{32} = 22}$$

$\therefore$ $a_{32}A_{32} = 5(22) = 110$

7. P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$ respectively. Write the position vector of point R which divides the line segment PQ in the ratio 2 : 1externally. **1**

Sol. P.V. of P is $3\vec{a} - 2\vec{b}$



P.V. of Q is $\vec{a} + \vec{b}$

Point R divides segment PQ in ratio 2 : 1 externally.

$$\text{P.V. of R} = \frac{(\text{P.V. of p})1 - (\text{P.V.of Q})(2)}{1-2}$$

$$\text{P.V. of R} = \frac{(3\vec{a} - 2\vec{b})(1) - (\vec{a} + \vec{b})(2)}{1-2} = \frac{\vec{a} - 4\vec{b}}{-1}$$

$$\text{P.V. of R} = 4\vec{b} - \vec{a}$$

8. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a})\cdot(\vec{x} + \vec{a}) = 15$. **1**

Sol. Given $|\vec{a}| = 1$

$(\vec{x} - \vec{a})\cdot(\vec{x} + \vec{a}) = 15$

$|\vec{x}|^2 - |\vec{a}|^2 = 15$

$|\vec{x}|^2 - 1 = 15$

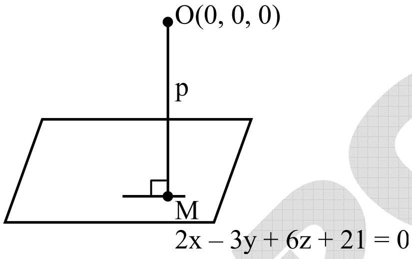
$|\vec{x}|^2 = 15 + 1$

$|\vec{x}|^2 = 16$

$|\vec{x}| = 4$

9. Find the length of the perpendicular drawn from the origin to the plane $2x - 3y + 6z + 21 = 0$. **1**

Sol. $p = \left|\dfrac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$



$p = \left|\dfrac{0+0+0+21}{\sqrt{2^2+3^2+6^2}}\right| \Rightarrow p = \dfrac{21}{\sqrt{49}} \Rightarrow p = \dfrac{21}{7} \Rightarrow p = 3$

10. The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, find the marginal revenue, when x = 5, and write which value does the equations indicate. **1**

Sol. $R(x) = 3x^2 + 36x + 5$

$MR = \dfrac{dR}{dx} = 6x^2 + 36$

when $x = 5$

$MR = 30 + 36 = 66$

11. Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers. **4**

Sol. $f : R^+ \to [4, \infty)$

$f(x) = x^2 + 4$

$f'(x) = 2x > 0$ $\quad\therefore$ (one - one)

As $f(x) = x^2 + 4 \ge 4$

$\Rightarrow$ Rage $= [4, \infty)$ = co-domain

$\therefore$ onto

So f is invertible.

Further : $y = x^2 + 4$

$\Rightarrow y - 4 = x^2 \quad \Rightarrow x = \pm\sqrt{y-4}$

As $x > 0$ so $x = \sqrt{y-4}$

$\therefore \; y = \sqrt{x-4} = f^{-1}(x)$

Or $f^{-1}(y) = \sqrt{y-4}$

12. Show that : **4**

$$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$$

OR

Solve the following equation :

$$\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

Sol. Let $\frac{1}{2}\sin^{-1}\frac{3}{4} = \theta$ then $\frac{3}{4} = \sin 2\theta$

Now $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \tan\theta$

If $\sin 2\theta = \frac{3}{4}$ then $\frac{2\tan\theta}{1+\tan^2\theta} = \frac{3}{4}$

$8\tan\theta = 3 + 3\tan^2\theta$

$3\tan^2\theta - 8\tan\theta + 3 = 0$

$\tan\theta = \frac{8 \pm \sqrt{64 - 4\times3\times3}}{6}$

$\tan\theta = \frac{8\pm\sqrt{28}}{6} = \frac{4\pm\sqrt{7}}{3}$

$\tan\theta = \frac{4+\sqrt{7}}{3}$ or $\frac{4-\sqrt{7}}{3}$

$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ Hence proved.

OR

$\cos(\tan^{-1}x)$

LHS. let $\tan^{-1}x = \theta \Rightarrow x = \tan\theta$

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+x^2}}$$

Hence $\cos(\tan^{-1}x) = \frac{1}{\sqrt{1+x^2}}$

R.H.S Let $\cot^{-1}\frac{3}{4} = \theta \Rightarrow \frac{3}{4} = \cot\theta$

then $\sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{1}{\sqrt{1+\frac{9}{16}}} = \frac{4}{5}$

Now LHS = RHS

$$\frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$$

$25 = 16 + 16x^2$

$x^2 = \frac{9}{16} \Rightarrow x = \frac{3}{4}$

13. Using properties of determinants, prove the following : **4**

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$$

Sol. $$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$$

LHS $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

Now, apply $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 3x+3y & x+y & x+2y \\ 3x+3y & x & x+y \\ 3x+3y & x+2y & x \end{vmatrix}$$

$$3(x+y)\begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix}$$

(Taking common $3(x + y)$ from C_1)

Now, apply $R_1 \to R_1 - R_3$

$R_2 \to R_2 - R_3$

$$3(x+y)\begin{vmatrix} 0 & -y & 2y \\ 0 & -2y & y \\ 1 & x+2y & x \end{vmatrix}$$

$$3(x+y)\begin{vmatrix} -y & 2y \\ -2y & y \end{vmatrix}$$

$$3y^2(x+y)\begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix}$$

$3y^2(x + y)(-1 + 4) = 9y^2(x + y)$. Hence proved.

14. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$. **4**

Sol. $y^x = e^{y-x}$

$\Rightarrow x \log_e y = y - x$ .....(1)

Differentiating w.r.t. x

$\Rightarrow \log_e y + x \cdot \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx} - 1$

$\Rightarrow \log_e y + 1 = \frac{dy}{dx}\left(1 - \frac{x}{y}\right)$ {from (1) $\frac{x}{y} = \frac{1}{1+\log_e y}$ }

$\Rightarrow \log_e y + 1 = \frac{dy}{dx}\left(1 - \frac{1}{1+\log_e y}\right)$

$\Rightarrow (\log_e y + 1) = \frac{dy}{dx}\left(\frac{\log_e y}{1+\log_e y}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{(1+\log_e y)^2}{\log_e y}$

15. Differentiate the following with respect to x : **4**

$\sin^{-1}\left(\frac{2^{x+1}\cdot 3^x}{1+(36)^x}\right)$

Sol. $y = \sin^{-1}\left[\frac{2^{x+1}\cdot 3^x}{1+(36)^x}\right]$

$y = \sin^{-1}\left[\frac{2^x\cdot 2\cdot 3^x}{1+(36)^x}\right]$

$y = \sin^{-1}\left[\frac{2\cdot(6)^x}{1+(6)^{2x}}\right]$

$y = 2\tan^{-1}(6)^x$

$\frac{dy}{dx} = \frac{2}{1+(6)^{2x}}\cdot 6^x \log 6$

$\frac{dy}{dx} = \frac{2\cdot 6^x \log 6}{1+(36)^x}$

16. Find the value of k, for which $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$ is continuous at x = 0. **4**

OR

If x = a $\cos^3\theta$ and y = a $\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

Sol. $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$

function f(x) is continuous at x = 0

$\therefore$ $f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow \frac{0+1}{0-1} = \lim_{x\to 0}\left(\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}\right)$

$\Rightarrow -1 = \lim_{x\to 0}\left(\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}\right)\left(\frac{\sqrt{1+kx}+\sqrt{1-kx}}{\sqrt{1+kx}+\sqrt{1-kx}}\right)$

$\Rightarrow -1 = \lim_{x\to 0} \frac{(1+kx)-(1-kx)}{x[\sqrt{1+kx}+\sqrt{1-kx}]}$

$\Rightarrow -1 = \lim_{x\to 0} \frac{2k}{\sqrt{1+kx}+\sqrt{1-kx}}$

$\Rightarrow -1 = \frac{2k}{2} \quad \Rightarrow k = -1$

OR

$x = a\cos^3\theta$ and $y = a\sin^3\theta$

$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$ and $\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\Rightarrow \frac{dy}{dx} = -\tan\theta$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\sec^2\theta\cdot\frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\sec^2\theta\cdot\frac{1}{(-3a\cos^2\theta\cdot\sin\theta)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{3a}\sec^4\theta\cdot\text{cosec}\,\theta$$

$$\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{6}} = \frac{1}{3a}\left(\frac{2}{\sqrt{3}}\right)^4\cdot 2 = \frac{32}{27a}$$

17. Evaluate : **4**

$$\int\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx$$

OR

Evaluate :

$$\int\frac{x+2}{\sqrt{x^2+2x+3}}dx$$

Sol. $\int\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx$

$$= \int\frac{(2\cos^2 x - 1) - (2\cos^2\alpha - 1)}{\cos x - \cos\alpha}dx$$

$$= \int\frac{2(\cos x - \cos\alpha)(\cos x - \cos\alpha)}{(\cos x - \cos\alpha)}dx$$

$$= 2\int(\cos x + \cos\alpha)\,dx$$

$= 2(\sin x + x\cos\alpha) + c$

OR

$$I = \int\frac{x+2}{\sqrt{x^2+2x+3}}dx$$

$$I = \int\left\{\frac{(x+1)+1}{\sqrt{x^2+2x+3}}\right\}dx$$

$$I = \int\left\{\frac{(x+1)}{\sqrt{x^2+2x+3}}\right\}dx + \int\left\{\frac{1}{\sqrt{x^2+2x+3}}\right\}dx$$

$I = I_1 + I_2$

In I_1 let $x^2 + 2x + 3 = t^2$

$\therefore \quad (2x + 2)dx = 2t\, dt$

$\Rightarrow \quad (x + 1)dx = tdt$

$\therefore \quad I_1 = \int \frac{t \cdot dt}{t} = t$

$I_1 = \sqrt{x^2 + 2x + 3}$

Now in $I_2 = \int \frac{1}{\sqrt{x^2 + 2x + 3}} dx = \int \frac{dx}{\sqrt{(x+1)^2 + 2}}$

$I_2 = \log[(x + 1) + \sqrt{(x+1)^2 + 2}\,]$

Now $I = I_1 + I_2$

$\Rightarrow \quad I = \sqrt{x^2 + 2x + 3} + \log(x + 1 + \sqrt{x^2 + 2x + 3}\,) + c$

18. Evaluate : **4**

$$\int \frac{dx}{x(x^5 + 3)}$$

Sol. $I = \int \frac{dx}{x(x^5 + 3)}$

$I = \int \frac{x^4 dx}{x^5(x^5 + 3)}$

Let $x^5 = t \Rightarrow 5x^4\, dx = dt$

$I = \frac{1}{5}\int \frac{dt}{t(t+3)}$

$I = \frac{1}{5} \cdot \frac{1}{3} \int \left(\frac{1}{t} - \frac{1}{t+3}\right) dt$

$I = \frac{1}{15} \{\log t - \log(t + 3)\} + c$

$I = \frac{1}{15} \log\left(\frac{t}{t+3}\right) + c$

$I = \frac{1}{15} \log\left(\frac{x^5}{x^5 + 3}\right) + c$

19. Evaluate : **4**

$$\int_0^{2\pi} \frac{1}{1 + e^{\sin x}} dx$$

Sol. $I = \int_0^{2\pi} \frac{1}{1 + e^{\sin x}} dx \qquad .....(1)$

$I = \int_0^{2\pi} \frac{1}{1 + e^{\sin(2\pi - x)}} dx$

$I = \int_0^{2\pi} \frac{1}{1 + e^{-\sin x}} dx$

$I = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x}+1}dx$ .....(2)

Adding (1) & (2) we get

$\Rightarrow$ $2I = \int_0^{2\pi}\left(\frac{1+e^{\sin x}}{1+e^{\sin x}}\right)dx$

$\Rightarrow$ $2I = [x]_0^{2\pi}$

$\Rightarrow$ $2I = 2\pi$

$I = \pi$

20. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors. **4**

Sol. $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$

$\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

$\vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$

$\vec{a} - \vec{b} = -4\hat{i} - 0\hat{j} + (7-\lambda)\hat{k}$

given $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular

$\therefore$ $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$

$\{6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}\} \times \{-4\hat{i} - 0\hat{j} + (7-\lambda)\hat{k}\} = 0$

$6(-4) + 0(-2) + (7+\lambda)(7-\lambda) = 0$

$-24 + 49 - \lambda^2 = 0$

$\lambda^2 = 25 \Rightarrow \boxed{\lambda = \pm 5}$

21. Show that the lines **4**

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$;

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

are intersecting. Hence find their point of intersection.

OR

Find the vector equation of the plane through the points (2, 1, –1) and (–1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$.

Sol. If the given lines are intersecting then the shortest distance between the lines is zero and also they have same common point $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\Rightarrow$ $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2}$ $(= \lambda)$ (Let)

Let P is $(\lambda + 3, 2\lambda + 2, 2\lambda - 4)$

Also, $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

$\Rightarrow$ $\frac{x-5}{3} = \frac{y+2}{2} = \frac{z-0}{6}$ $(= \mu)$ (Let)

Let Q is $(3\mu + 5, 2\mu - 2, 6\mu)$

If lines are intersecting then P and Q will be same.

$\lambda + 3 = 3\mu + 5$ .....(1)

$2\lambda + 2 = 2\mu - 2$ .....(2)

$2\lambda - 4 = 6\mu$ .....(3)

Solve (2) & (3)

$\lambda + 1 = \mu - 1$

$\lambda - 2 = 3\mu$

$-\quad +\quad -$

$3 = -2\mu - 1$

$4 = -2\mu$

$\boxed{\mu = -2}$

Put $\mu = -2$ .....(3)

$2\lambda - 4 = 6(-2)$

$2\lambda = -12 + 4$

$2\lambda = -8$

$\boxed{\lambda = -4}$

Put μ & λ in (1)

$\lambda + 3 = 3\mu + 5$

$-4 + 3 = 3(-2) + 5$

$-1 = -1$

$\therefore$ from $\lambda = -4$ then P is $(-1, -6, -12)$

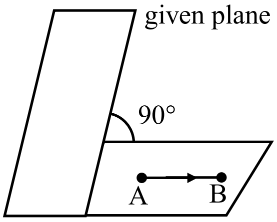
from $\mu = -2$ then Q is $(-1, -6, -12)$

as P and Q are same

$\therefore$ lines are intersecting lines and their point of intersection is $(-1, -6, -12)$.

OR

A(2, 1, –1) ; B(–1, 3, 4)



$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = -3\hat{i} + 2\hat{j} + 5\hat{k}$

given plane $x - 2y + 4z = 10$

$\therefore \quad \vec{n}_1 = \hat{i} - 2\hat{j} + 4\hat{k}$

The required plane is perpendicular to given plane.

Therefore $\vec{n}$ of required plane will be perpendicular to $\vec{n}_1$ and $\overrightarrow{AB}$.

$\therefore \quad \vec{n} \parallel (\vec{n}_1 \times \overrightarrow{AB})$

$\vec{n}_1 = \hat{i} - 2\hat{j} + 4\hat{k}$

$\overrightarrow{AB} = -3\hat{i} + 2\hat{j} + 5\hat{k}$

$\therefore \quad \vec{n}_1 \times \overrightarrow{AB} = -18\hat{i} - 17\hat{j} - 4\hat{k}$

$\therefore$ required plane is

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$\vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k}) = (2\hat{i} + \hat{j} - \hat{k}) \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k})$

$\vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k}) = -36 - 17 + 4$

$\vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k}) = -49$

$\boxed{18x + 17y + 4z = 49}$

22. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. Write at least one advantage of coming to school in time. **4**

Sol. If P(A come in school time) = 3/7

P(B come in school time) = 5/7

P(A not come in school time) = 4/7

P(B not come in school time) = 2/7

P(only one of them coming school in time)

$= P(A)\times P(\overline{B}) + P(\overline{A}) \cdot P(B)$

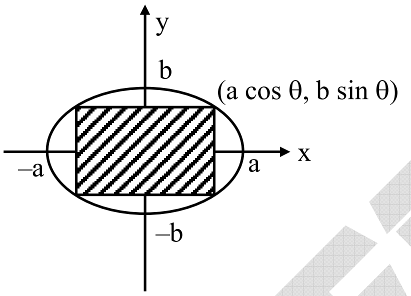
$= \frac{3}{7}\times\frac{2}{7}+\frac{5}{7}\times\frac{4}{7} = \frac{26}{49}$

23. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **6**

OR

Find the equations of tangents to the curve $3x^2 - y^2 = 8$, which pass through the point $\left(\frac{4}{3}, 0\right)$.

Sol. Given ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



Area of rectangle

$A = 2a\cos\theta \cdot 2b\sin\theta$

$A = 2ab \cdot \sin 2\theta$

$\therefore\ A_{max.} = 2ab$

OR

Let a point (x_1, y_1)

$3x^2 - y^2 = 8 \Rightarrow 6x - 2y\cdot y' = 0$

$\Rightarrow y' = \frac{3x}{y}$

$\therefore$ Tangent $y - y_1 = \frac{3x_1}{y_1}(x - x_1)$

It passing through $\left(\frac{4}{3}, 0\right)$.

$-y_1 = \frac{3x_1}{y_1}\left(\frac{4}{3} - x_1\right)$

$\Rightarrow -y_1^2 = 4x_1 - 3x_1^2 \quad\Rightarrow\quad y_1^2 = 3x_1^2 - 4x_1$

$\Rightarrow 3x_1^2 - 8 = 3x_1^2 - 4x_1$

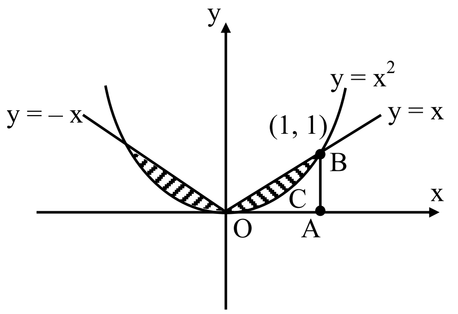
$\therefore\ x_1 = 2$

So $12 - y^2 = 8$

$\Rightarrow y^2 = 4 \Rightarrow y_1 = \pm 2$

24. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$. **6**

Sol.



Required area = 2[area of ΔOAB – Area of curve OCBA]

$$A = 2\left[\frac{1}{2}(1)(1) - \int_0^1 x^2 dx\right]$$

$$A = 2\left[\frac{1}{2} - \frac{1}{3}\right] \Rightarrow A = 2\left[\frac{1}{6}\right] = \frac{1}{3}$$

25. Find the particular solution of the differential equation $(\tan^{-1}y - x)dy = (1 + y^2)dx$, given that when x = 0, y = 0. **6**

Sol. $(\tan^{-1}y - x)dy = (1 + y^2)dx$

$$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$IF = e^{\int \frac{1}{1+y^2}dy}$$

$$IF = e^{\tan^{-1}y}$$

$$x.\ IF = \int Q.IF dy + c$$

$$\Rightarrow x.\ e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{(1+y^2)} \cdot e^{\tan^{-1}y} dy + c$$

Put $\tan^{-1}y = t$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \int \underset{I}{t} \cdot \underset{II}{e^t} \cdot dt + c$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = (t \cdot e^t) - (e^t) + c$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \tan^{-1}y \cdot e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$$

26. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity. **6**

OR

Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the planes $\vec{r} \cdot (\hat{i} - \hat{j} - 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

Sol. P_1 is $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$

P_1 is $x + 3y - 6 = 0$

P_2 is $\vec{r}\cdot(3\hat{i}-\hat{j}-4\hat{k})=0$

P_2 is $3x - y - 4z = 0$

Equation of plane passing through intersection of P_1 and P_2 is $P_1 + \lambda P_2 = 0$

$(x + 3y - 6) + \lambda(3x - y - 4z) = 0$

$(1 + 3\lambda)x + (3 - \lambda)y + (-4\lambda)z + (-6) = 0$

Its distance from (0, 0, 0) is 1.

$$\left|\frac{0+0+0-6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}\right| = 1$$

$36 = (1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2$

$36 = 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2$

$36 = 26\lambda^2 + 10 \Rightarrow 26\lambda^2 = 26 \Rightarrow \lambda^2 = 1 \Rightarrow \boxed{\lambda = \pm 1}$

Hence required plane is

For $\lambda = 1$, $(x + 3y - 6) + 1\,(3x - y - 4z) = 0$

$4x + 2y - 4z - 6 = 0$

For $\lambda = -1$, $(x + 3y - 6) - 1\,(3x - y - 4z) = 0$

$-2x + 4y + 4z - 6 = 0$

OR

P_1 is $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=5$

$\therefore\ \vec{n}_1 = \hat{i}-\hat{j}+2\hat{k}$

P_2 is $\vec{r}\cdot(3\hat{i}+\hat{j}+\hat{k})=6$

$\vec{n}_2 = 3\hat{i}+\hat{j}+\hat{k}$

The line parallel to plane P_1 & P_2 will be perpendicular to $\vec{n}_1$ & $\vec{n}_2$

$\therefore\ \vec{b} \parallel (\vec{n}_1 \times \vec{n}_2)$

$\vec{n}_1 = \hat{i}-\hat{j}+2\hat{k}$

$\vec{n}_2 = 3\hat{i}-\hat{j}+\hat{k}$

$\vec{n}_1\times\vec{n}_2 = -3\hat{i}+5\hat{j}+4\hat{k}$

$\therefore\ \vec{b} = -3\hat{i}+5\hat{j}+4\hat{k}$

Point is (1, 2, 3)

$\therefore\ \vec{a} = \hat{i}+2\hat{j}+3\hat{k}$

$\therefore$ required line is $\vec{r} = \vec{a}+\lambda\vec{b}$

$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(-3\hat{i}+5\hat{j}+4\hat{k})$

27. In a hockey match, both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not. **6**

Sol. P(6 get) = 1/6

P(6 not get) = $P(\overline{6\,\text{get}})$ = 5/6

P(A win) = P(A get 6) + $P(\overline{6\,\text{get}})\cdot P(\overline{6\,\text{get}})$ P(6 get) + $P(\overline{6\,\text{get}})\cdot P(\overline{6\,\text{get}}) \cdot P(\overline{6\,\text{get}})\cdot P(\overline{6\,\text{get}})$ P(6 get) + ...+ ∞

$$\text{P(A win)} = \frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+....+\infty$$

$$= \frac{1}{6}+\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}+....+\infty$$

$\because \; S_\infty = \dfrac{a}{1-r}$

$$= \frac{\left(\frac{1}{6}\right)}{\left(1-\frac{25}{36}\right)} = \frac{36}{11\times 6} = \frac{6}{11}$$

Similarly winning for B

P(B win) = 1 – P(A win)

$$= 1 - \frac{6}{11} = \frac{5}{11}$$

28. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B. If A and B are priced at ₹ 100 and ₹ 120 per unit respectively, how should he use his resources to maximise the total revenue ? Form the above as an LPP and solve graphically.

Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate ? **6**

Sol. if $z_{max.} = 100x + 120y$

	type A	type B	
worker	2	3	30
capital	3	1	17

Subject to,

$2x + 3y \le 30$

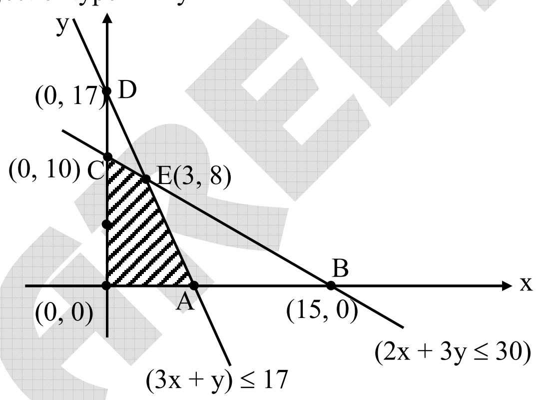
$3x + y \le 17$

$x \ge 0$

$y \ge 0$

Let object of type A = x

Object of type B = y



pts.	Coordinate	$Z^{max.} = 100x + 120y$
O	(0, 0)	Z = 0
A	$\left(\frac{17}{3}, 0\right)$	$Z = \frac{1700}{3}$
E	(3, 8)	Z = 300 + 960 = 1260
C	(0, 10)	Z = 1200

maximum revenue = 1260.

29. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some other (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these value, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards. **6**

Sol. Given $x + y + z = 12$ …..(1)

$3(y + z) + 2x = 33$ …..(2)

$(x + z) = 2y$ …..(3)

$x + y + z = 12$

$2x + 3y + 3z = 33$

$x - 2y + z = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$AX = B$

$A^{-1}(AX) = A^{-1}(B)$

$I \cdot X = A^{-1} \cdot B$

$X = A^{-1}.B$

$$X = \frac{(Adj.A) \cdot B}{|A|}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{vmatrix}$$

$|A| = 1(3 + 6) - 1(2 - 3) + 1(-4 - 3)$

$|A| = 9 + 1 - 7 = 3$

$|A| \neq 0$

$$(Adj.\ A) = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$(Adj\ A) \cdot B = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}_{3\times 3}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}_{3\times 1}$$

$$(Adj.) \cdot B = \begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix}$$

$$\therefore\ X = \frac{(Adj.A) \cdot B}{|A|}$$

$$X = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$x = 3, y = 4, z = 5.$