EVERGREEN MOCK TEST PAPER (SOLVED)

CLASS - X

General Instructions

(i) Answer to this paper must be written on the paper provided separately. You will ***NOT*** *be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
(ii) The time given at the head of this paper is the time allowed for writing the answers.
(iii) Answer all questions from ***section A*** *and* ***any four*** *questions from* ***section B.***
(iv) ***All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in the loss of marks.***
(v) The intended marks for questions or parts of questions are given in brackets ( ).
(vi) Mathematical tables are provided.

SECTION : A (40 MARKS)

(Attempt ***all*** *Questions from this* ***section****)*

Q.1. (*a*) List price of a cooler is Rs 5940. The rate of sales tax is 10%. The customer requests the shopkeeper to allow a discount in the price of the cooler to such an extent that the price remains as Rs 5940 inclusive of sales tax. Find the discount in the price of the cooler.

(*b*) Find x from the equation : $\dfrac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}} = b$

(*c*) A sum amounts to Rs 2916 in 2 years and to Rs 3149·28 in 3 years at compound interest, find (*i*) the rate of interest per annum and (*ii*) the sum.

Sol. (*a*) We can use the formula; Amount payable = $x\left(1-\dfrac{d}{100}\right)\left(1+\dfrac{r}{100}\right)$

Where x = L.P, d = rate of discount, r = rate of sales tax

Here S.P. = Rs 5940, L.P. = Rs 5940, r = 10%

$$\therefore \quad 5940 = 5940\left(1-\frac{d}{100}\right)\left(1+\frac{10}{100}\right) \quad \Rightarrow \quad 1 = \frac{11}{10}\left(1-\frac{d}{100}\right)$$

$$\Rightarrow \quad 1-\frac{d}{100} = \frac{10}{11} \quad \Rightarrow \quad \frac{d}{100} = 1-\frac{10}{11} = \frac{1}{11}$$

$$\Rightarrow \quad d = \frac{100}{11} = 9\frac{1}{11}$$

$\therefore$ Rate of discount $= 9\frac{1}{11}\%$

Hence discount $= 9\frac{1}{11}\%$ of L. P

$$= \frac{100}{11} \times \frac{1}{100} \times 5940 = \text{Rs } 540$$

(*b*) Given

$$\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}} = \frac{b}{1}$$

$$\Rightarrow \frac{(\sqrt{a+x}+\sqrt{a-x})+(\sqrt{a+x}-\sqrt{a-x})}{(\sqrt{a+x}+\sqrt{a-x})-(\sqrt{a+x}-\sqrt{a-x})} = \frac{b+1}{b-1}$$ [By componendo and dividendo]

$$\Rightarrow \quad \frac{2\sqrt{a+x}}{2\sqrt{a-x}} = \frac{b+1}{b-1} \quad \Rightarrow \quad \frac{\sqrt{a+x}}{\sqrt{a-x}} = \frac{b+1}{b-1}$$

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$$\Rightarrow \quad \frac{a+x}{a-x} = \frac{b^2+2b+1}{b^2-2b+1} \quad \text{[On squaring both sides]}$$

$$\Rightarrow \quad \frac{(a+x)+(a-x)}{(a+x)-(a-x)} = \frac{(b^2+2b+1)+(b^2-2b+1)}{(b^2+2b+1)-(b^2-2b+1)} \quad \text{[By componendo and dividendo]}$$

$$\Rightarrow \quad \frac{2a}{2x} = \frac{2b^2+2}{4b} \quad \Rightarrow \quad \frac{a}{x} = \frac{b^2+1}{2b}$$

$$\Rightarrow \quad x = \frac{2ab}{b^2+1}$$ is the required value of x.

(*c*) Here A_1 = Rs 2916, n_1 = 2 years

A_2 = Rs 3149·28, n_2 = 3 years

Principal (P) = ?, Rate = R = ?

$$\text{Now } A_1 = P\left(1+\frac{R}{100}\right)^{n_1} \quad \text{Or} \quad 2916 = P\left(1+\frac{R}{100}\right)^2 \quad .... (i)$$

$$A_2 = P\left(1+\frac{R}{100}\right)^{n_2} \quad \text{Or} \quad 3149{\cdot}28 = P\left(1+\frac{R}{100}\right)^3 \quad .... (ii)$$

Dividing (*ii*) by (*i*)

$$\frac{3149{\cdot}28}{2916} = 1 + \frac{R}{100} \quad \Rightarrow \quad \frac{R}{100} = \frac{3149{\cdot}28}{2916} - 1$$

$$\Rightarrow \quad \frac{R}{100} = \frac{3149{\cdot}28 - 2916}{2916} = \frac{233{\cdot}28}{2916} \quad \Rightarrow \quad R = \frac{23328}{2916} = 8$$

$\therefore$ R = 8%

From equation (*i*)

$$2916 = P\left(1+\frac{8}{100}\right)^2 = P\left(\frac{27}{25}\right)^2$$

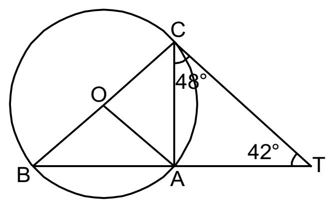
$$P = 2916 \times \frac{25}{27} \times \frac{25}{27} = 2500$$

$\therefore$ The sum = Rs 2500 and rate of interest = 8%.

Q.2. (*a*) If $x + 2$ and $x - 3$ are the factors of $x^3 + ax + b$, then calculate the values of *a* and *b*, find the remaining factor.

(*b*) The marks of 15 students in a test are as follows : 11, 19, 13, 15, 16, 11, 14, 12, 16, 13, 20, 14, 15, 18, 15. Calculate (*i*) mean (*ii*) median (*iii*) mode.

(*c*) A, B and C are the points on a circle. The tangent at C meets BA produced in T. If ∠ATC = 42° and ∠ACT = 48°, then find ∠AOB.



Sol. (*a*) $(x + 2)$ and $(x - 3)$ are the factors of $f(x) = x^3 + ax + b$

$\therefore \quad f(-2) = 0$

$\Rightarrow \quad (-2)^3 + a(-2) + b = 0 \quad \Rightarrow \quad -8 - 2a + b = 0 \quad ... (i)$

Since $(x - 3)$ is factor of $f(x)$ $\quad \therefore \quad f(3) = 0$

$f(3) = (3)^3 + a(3) + b \quad \Rightarrow \quad 27 + 3a + b = 0 \quad ...(ii)$

Subtracting (*i*) from (*ii*)

$(27 + 3a + b) - (-8 - 2a + b) = 0 \quad \Rightarrow \quad 27 + 3a + b + 8 + 2a - b = 0$

$\Rightarrow \quad 35 + 5a = 0 \quad \Rightarrow \quad 5a = -35$

$$\Rightarrow \quad a = \frac{-35}{5} = -7$$

Put $a = -7$ in (*i*)

$-8 - 2(-7) + b = 0 \quad \Rightarrow \quad -8 + 14 + b = 0$

$\Rightarrow \quad 6 + b = 0 \quad \Rightarrow \quad b = -6$

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Putting the value of a and b in $f(x)$ then equation becomes $x^3 - 7x - 6$

$(x + 2)$ and $(x - 3)$ are factors of $x^3 - 7x - 6$ then other factor is

$$\frac{x^3 - 7x - 6}{(x+2)(x-3)} = \frac{x^3 - 7x - 6}{x^2 - x - 6}$$

$\therefore$ Other factor is $(x + 1)$.

$$x^2 - x - 6 \,)\, x^3 - 7x - 6 \,(\, x + 1$$
$$\underline{\overset{}{-}x^3 \overset{}{\mp} 6x \mp x^2}$$
$$- x + x^2 - 6$$
$$\underline{\mp x \pm x^2 \mp 6}$$
$$0$$

(*b*) **Mean** $= \dfrac{\text{Sum of observations}}{\text{No. of observations}}$

$$= \frac{11 + 19 + 13 + 15 + 16 + 11 + 14 + 12 + 16 + 13 + 20 + 14 + 15 + 18 + 15}{15} = \frac{222}{15} = 14{\cdot}8$$

Median : Arrange the data in increasing order

11, 11, 12, 13, 13, 14, 14, 15, 15, 15, 16, 16, 18, 19, 20

Number of observations is 15, which is odd.

$\therefore$ Median $= \left(\dfrac{n+1}{2}\right)^{th}$ term $= \left(\dfrac{15+1}{2}\right)^{th}$ term $=$ 8th term $= 15$

Mode : Clearly 15 occurs maximum number of time

$\therefore$ Mode $= 15$

(*c*) TC is the tangent at C.

$\therefore$ $\angle$TCA $= \angle$ABO (Angles in the alternate segment of a circle are equal)

$\Rightarrow$ $\angle$ABO $= 48°$.

Now, $\angle$ABO + $\angle$BTC + $\angle$BCT $= 180°$ (Sum of three angles of a triangle is 180°)

$\Rightarrow$ $48° + 42° + \angle$BCT $= 180°$ $\Rightarrow$ $\angle$BCT $= 90°$

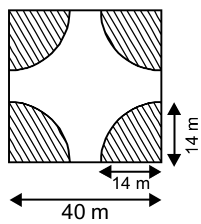
$\Rightarrow$ $\angle$BCA $+ 48° = 90°$ $\Rightarrow$ $\angle$BCA $= 42°$

Now, $\angle$AOB $= 2\,\angle$BCA (Angle subtended at the centre is double the angle subtended at the remaining part of the circle)

$\Rightarrow$ $\angle$AOB $= 2\,(42°) = 84°$.

Q.3. (*a*) Find the maturity value of a recurring deposit of Rs 75 per month for 48 months at the expiry periods, if the rate of interest is 9% p.*a*.

(*b*) At each corner of a square park of side 40 m, there is a flower bed in the form of a sector of radius 14 m as shown in the figure. Find the area of the remaining part of the park.



(*c*) A point P(x, y) is reflected in X-axis to P′(2, – 3). Write down the values of x and y. P″ is the image of P, when reflected in the Y-axis. Write down the co-ordinates of P″. Find the co-ordinates of P‴ when P is reflected in the line $x = 4$.

Sol. (*a*) Monthly instalment = Rs 75

Rate of interest = 9 %

Here $x = 48$

$$\text{Interest} = \left[75 \times \frac{48(48+1)}{2} \times \frac{1}{12} \times \frac{9}{100}\right] = \text{Rs } 661{\cdot}50$$

Monthly amount = Rs (75 × 48 + 661·50) = Rs 4261·50

$\therefore$ Maturity value = Rs 4261·50

(*b*) Each side of square park = 40 m

Area of square park of side 40 m = 40m × 40m = 1600m^2

Radius of the flower bed sector = 14 m

Area of flower bed of one sector $= \dfrac{1}{4}\pi r^2 = \dfrac{1}{4} \times \dfrac{22}{7} \times 14 \times 14$

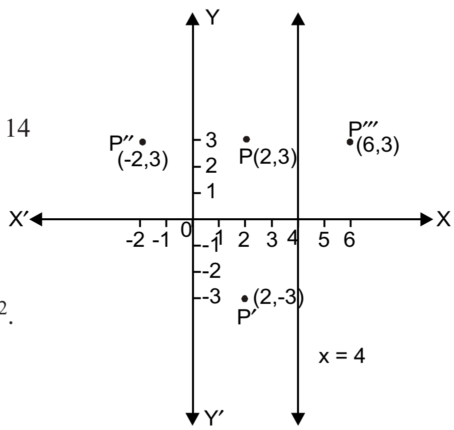
$= 154$ m^2

Area of the 4 flower be sectors = 154m^2 × 4

$= 616$ m^2

Hence area of the remaining part of the park

$= (1600 - 616)$m$^2 = 984$ m^2.



(*c*) Clearly, the values of x and y are : $x = 2$, $y = 3$

Coordinates of P″ are (–2, 3).

The coordinate of P‴ are (6, 3).

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Q.4. (*a*) If 2 sin A – 1 = 0 then show that : sin 3A = 3 sin A – 4 $\sin^3$ A.

(*b*) Given that $x \in$ N, solve the following inequality and graph the solution on the number line $-12 \le 3 - 4x \le 11$.

(*c*) Find the mean of the following distribution :

Age (in Years)	**5 – 15**	**15 – 25**	**25 – 35**	**35 – 45**	**45 – 55**	**55 – 65**	**65 – 75**
Frequency	**6**	**10**	**15**	**13**	**24**	**8**	**7**

Sol. (*a*) $2 \sin A - 1 = 0 \Rightarrow 2 \sin A = 1$

$\Rightarrow \sin A = \frac{1}{2} \Rightarrow A = 30°$

L.H.S. $= \sin 3A = \sin 3(30°) = \sin 90° = 1$

R.H.S. $= 3 \sin A - 4 \sin^3 A = 3 \sin 30° - 4 \sin^3 30°$

$$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3 = \frac{3}{2} - \frac{4}{8} = \frac{3}{2} - \frac{1}{2} = 1$$

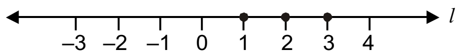
$\therefore$ L.H.S. = R.H.S.

(*b*) The given inequality is : $-12 \le 3 - 4x \le 11$

$\Rightarrow -12 - 3 \le -4x \le 11 - 3 \Rightarrow -15 \le -4x \le 8$

$\Rightarrow \frac{15}{4} \ge x \ge -2$ Or $3\frac{3}{4} \ge x \ge -2$

Or $-2 \le x \le 3\frac{3}{4}$



Since $x \in$ N $\therefore x = \{1, 2, 3\}$.

(*c*) Arrange the distribution as under :

Class interval	Frequency (f_i)	Class marks (x_i)	$f_i x_i$
5 – 15	6	10	60
15 – 25	10	20	200
25 – 35	15	30	450
35 – 45	13	40	520
45 – 55	24	50	1200
55 – 65	8	60	480
65 – 75	7	70	490
	$\sum f_i = 83$		$\sum f_i x_i = 3400$

$$\therefore \quad \text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3400}{83} = 40{\cdot}96 \text{ years}$$

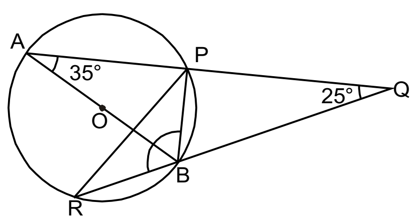
SECTION : B (40 MARKS)

*(Attempt **any four** questions from this **section**)*

Q.5. (*a*) A tent of height 33 m is in the form of a right circular cylinder of diameter 120 m and height 22 m surmounted by a right circular cone of the same diameter. Calculate the total surface area of the tent.

(*b*) Prove that : $\frac{1}{\sec\theta + \tan\theta} + \frac{1}{\sec\theta - \tan\theta} = 2 \sec\theta$

(*c*) In the given figure, AB is a diameter of the given circle. APQ and, RBQ are two straight lines through P and B, If ∠BAP = 35° and ∠BQP = 25° then find (*i*) ∠PRB (*ii*) ∠PBR (*iii*) ∠BPR



Sol. (*a*) Height of the cone = (33 –22) m = 11 m.

$\therefore$ Slant height of the cone

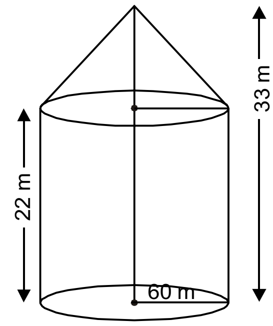
$$= \sqrt{60^2 + 11^2} \text{ m} = \sqrt{3721} \text{ m} = 61\text{m}.$$

$\therefore$ Total surface area of the tent

$$= 2\pi rh + \pi rl = \pi r(2h + l)$$

$$= \frac{22}{7} \times 60\ (44 + 61)$$

$$= \frac{22}{7} \times 60 \times 105\ m^2 = 19800\ \text{m}^2$$



(*b*) $\dfrac{1}{\sec\theta + \tan\theta} + \dfrac{1}{\sec\theta - \tan\theta} = 2\sec\theta$

$$\text{L.H.S.} = \frac{1}{\sec\theta + \tan\theta} + \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{\sec\theta - \tan\theta + \sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta} = \frac{\sec\theta + \sec\theta}{\sec^2\theta - \tan^2\theta}$$

$= 2\sec\theta$ = R.H.S. $\quad (\because \sec^2\theta - \tan^2\theta = 1)$

(*c*) $\angle PRB = \angle PAB$ (Angles in the same segment of a circle are equal)

$\angle PRB = 35°$

$\because$ AB is the diameter of the circle

$\therefore$ $\angle APB = 90°$ (Angle in the semi-circle is 90°)

Also, $\angle PRB + \angle PQB + \angle RPQ = 180°$

$\Rightarrow$ $35° + 25° + \angle RPQ = 180°$

$\Rightarrow$ $\angle RPQ = 120°$

And, $\angle QPB = 180° - 90°$ (since $\angle APB = 90°$)

$\Rightarrow$ $\angle QPB = 90°$.

Now, $\angle BPR = \angle RPQ - \angle QPB$ (see fig.)

$\Rightarrow$ $\angle BPR = 120° - 90° = 30°$.

Now, $\angle PBR = 180° - (\angle PRB + \angle BPR)$ $[\because \angle PBR + \angle PRB + \angle BPR = 180]$

$\Rightarrow$ $\angle PBR = 180° - 35° - 30°$

$\Rightarrow$ $\angle PBR = 115°$

Q.6. (*a*) A page from Neha's passbook is given below :

Date	Particulars	Withdrawals	Deposits	Balance
1.10.05	BF	—	—	5000
9.11.05	By cash	—	8000	13000
8.12.05	By cash	—	8000	21000
20.12.05	To cheque no. 0483	9000	—	12000
25.1.06	By cash	—	8000	20000
16.2.06	By cash	—	8000	28000
27.2.06	To cheque no. 0484	19000	—	9000
7.3.06	By cash	—	8000	17000
4.4.06	By cash	—	8000	25000
18.4.06	By cash	—	2000	27000
27.5.06	By cash	—	8000	35000
14.6.06	To cheque no. 0485	10000	—	25000

Neha closes the account finally on 24.06.2006. Find the interest she gets from October, 2005 to the day of closing of the account at 3.5% p.a.

(*b*) Find x, y, z and w, if $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} + \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}$

(*c*) Write down the equation of the line passing through (–3, 2) and perpendicular to the line $x + 3y - 5 = 0$

Sol. (*a*) Minimum balance for the month of October = Rs 5000

November = Rs 13000

December = Rs 12000

January = Rs 12000

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$$\begin{aligned} \text{February} &= \text{Rs } 9000 \\ \text{March} &= \text{Rs } 17000 \\ \text{April} &= \text{Rs } 25000 \\ \text{May} &= \underline{\text{Rs } 27000} \\ & \quad \underline{\text{Rs } 1,20,000} \end{aligned}$$

Now, $\quad \text{interest} = \dfrac{1,20,000 \times 3{\cdot}5 \times 1}{100 \times 12} = \text{Rs } 350.$

(*b*) $\quad 3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} + \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} + \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} 4+x & x+y+6 \\ z+w-1 & 3+2w \end{bmatrix}$

Now compare corresponding elements

$3x = 4 + x \quad \Rightarrow \quad 3x - x = 4$

$\Rightarrow \quad 2x = 4 \quad \therefore \quad x = 2$

Also, $\quad 3y = x + y + 6$

$\Rightarrow \quad 3y - y = x + 6 \quad \Rightarrow \quad 2y = 2 + 6 = 8 \quad$ (Put $x = 2$)

$\Rightarrow \quad y = \dfrac{8}{2} = 4 \quad \therefore \quad y = 4$

Also, $\quad 3w = 3 + 2w \quad \Rightarrow \quad 3w - 2w = 3$

$\therefore \quad w = 3$

and $\quad 3z = z + w - 1 \quad$ and $\Rightarrow \quad 3z - z = 3 - 1. \quad$ (Put $w = 3$)

$\Rightarrow \quad 2z = 2 \quad\quad z = 1$

Now, $x = 2, y = 4, z = 1, w = 3$

(*c*) The given equation is $x + 3y - 5 = 0$

$\Rightarrow \quad 3y = -x + 5 \quad \Rightarrow \quad y = \left(\dfrac{-1}{3}\right)x + \dfrac{5}{3}$

$\therefore \quad$ Slope of this line $= -\dfrac{1}{3}$.

Now the slope of the required line = 3.

Also, required line passes through (–3, 2)

$\therefore \quad y - 2 = 3(x + 3) \quad\quad [\because y - y_1 = m(x - x_1)]$

$\Rightarrow \quad y - 2 = 3x + 9 \quad \Rightarrow \quad y - 3x - 11 = 0$

Which is the required equation of the line.

Q.7. (*a*) The details of income and investments of Mr. Mathur for a particular year are as given below :

Annual Salary	**Rs 250000.**
LIC Premium	**Rs 18000 p.a.**
Provident Fund	**Rs 3500 per month**
Tax deducted at source for first 11 months	**Rs 2600 per month**

Calculate the tax payable in the last month of the year.You may use the following:

Tax Slab	***Income Tax***
Upto Rs 50000	**No Tax**
Rs 50001 – Rs 60000	**10% of the amount exceeding Rs 50000**
Rs 60001 – Rs 150000	**Rs 1000 + 20% of the amount exceeding Rs 60000**
Rs 150001 and above	**Rs 19000 + 30% of the amount exceeding Rs 150000**
Standard deduction	**1/3rd of the total annual income subject to a maximum of Rs 20000.**
Rebate in tax	**20% of the total savings subject to maximum of Rs 12000.**
Education cess	**2% of the tax payable after rebate.**

(***b***) **From the top of the cliff 200 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60°, find the height of the tower.**

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Sol. (*a*) Computation of taxable income :

Gross annual income = Rs 250000

Standard deduction = Rs 20000

∴ Taxable income = 230000

Computation of income tax.

As the taxable income is above Rs 150000

∴ Income tax = Rs 19000 + 30% of the amount exceeding Rs 150000

$$= \text{Rs } 19000 + \frac{30}{100} \times 80{,}000 = \text{Rs } (19000 + 24000) = \text{Rs } 43000$$

Computation of rebate

L.I.C. Premium = Rs 18000

P.F. = Rs 3500 × 12 = Rs 42000

Total Rs = Rs 60000

∴ Tax rebate = 20% of Rs 60, 000 subject to maximum of Rs 12, 000

$$= \text{Rs } \frac{20}{100} \times 60000 = \text{Rs } 12000$$

Computation of income tax payable = 43000 – Rs 12000 = Rs 31000

$$\text{Education cess} = \text{Rs } 2\% \text{ of the income tax} = \text{Rs } \frac{2}{100} \times 31000 = \text{Rs } 620$$

Net income tax = Rs (31000 + 620) = Rs 31620

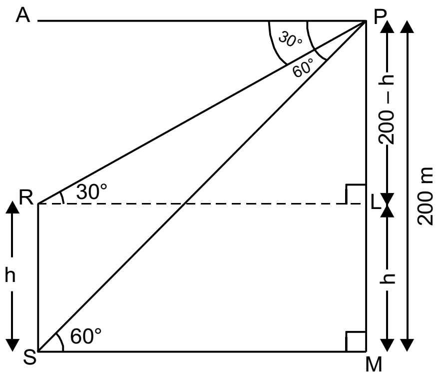
Advance tax paid for 11 months.

= Rs 2600 × 11 = Rs 28600

Hence, tax payable in the last month

= Rs 31620 – Rs 28600 = Rs 3020

(*b*) Let P be the top of the cliff MP = 200 m.



Let SR = h be the tower.

From R, draw RL ⊥ on MP.

Since $\angle APR = 30°$

∴ $\angle LRP = 30°$ (Alternate angles)

Since $\angle APS = 60°$

∴ $\angle MSP = 60°$ (Alternate angles)

Now RL = SM ...(*i*)

LP = MP – ML = MP – SR = 200 – *h*

In in rt ∠d Δ PLR

$$\frac{RL}{LP} = \cot 30°$$

$$\Rightarrow \quad \frac{RL}{200 - h} = \sqrt{3} \qquad \Rightarrow \qquad RL = (200 - h)\sqrt{3} \qquad ...(ii)$$

In rt ∠d ΔPMS

$$\frac{SM}{MP} = \cot 60°$$

$$\Rightarrow \quad \frac{SM}{200} = \frac{1}{\sqrt{3}} \qquad \Rightarrow \qquad SM = \frac{200}{\sqrt{3}} \qquad ....(iii)$$

From (*i*) an (*iii*) we get

$$RL = \frac{200}{\sqrt{3}} \qquad ...(iv)$$

From (*iv*) and (*ii*) we get

$$\frac{200}{\sqrt{3}} = (200 - h)\sqrt{3} \qquad \Rightarrow \qquad 200 = (200 - h)\,3$$

$$\Rightarrow \quad 200 = 600 - 3h. \qquad \Rightarrow \qquad 3h = 600 - 200 = 400 \text{ m.}$$

$$\Rightarrow \quad h = \frac{400}{3} = 133{\cdot}33\text{m.}$$

Thus, required height of the tower = 133·33m

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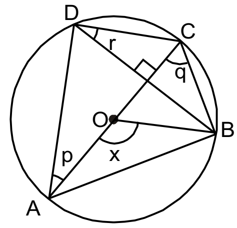
Q.8. (*a*) In the adjoining figure, AC is the diameter of circle with centre O. Chord BD is perpendicular to AC. Write down the angles *p,q,r* in terms of *x*.



(*b*) A lady holds 1800 Rs 100 shares of company that pays 15% dividend annually. Calculate her annual dividend. If she had bought these shares at 40% premium, what percentage return does she get on her investment. Give your answer to the nearest integer.

(*c*) If the point (*x, y*) be equidistant from the points (*a + b, b – a*) and (*b – a, a + b*), prove that *bx = ay.*

Sol. (*a*) Now $q = \frac{x}{2}$ (∵ Angle at the centre is double as compare to the circumference)

Clearly, $q + \angle CBD + 90° = 180°$ (∵ Sum of three angles of Δ is 180°)

⇒ $\frac{x}{2} + 90° + \angle CBD = 180°$

Also, $\angle CBD = p$ (Angles in the same segment are equal)

⇒ $p = 90° - \frac{x}{2}$

Now, $\angle BOC = 180° - x$ (∵ $\angle BOC + x = 180°$)

⇒ $r = \angle BDC = \frac{1}{2}(180° - x)$ (∵ $\angle BDC = \frac{1}{2}\angle BOC$)

∴ $r = 90° - \frac{x}{2}$

Hence, $p = 90° - \frac{x}{2}$, $q = \frac{x}{2}$ and $r = 90° - \frac{x}{2}$.

(*b*) Number of shares = 1800.

Face value of 1 share = Rs 100

Dividend = 15%

∴ Annual dividend = Rs (15% of 1800 × 100) = $\frac{15}{100} \times 1800 \times 100$ = Rs 27000

Total investment in purchasing 1800 shares at 40% premium.

= Rs $\frac{140}{100} \times 100 \times 1800$ = Rs 2,52,000

∴ Return % = $\frac{27000}{252000} \times 100 = 10{\cdot}71\% = 11\%$

(*c*) Let the point P (*x*, *y*) and A (*a* + *b*, *b* – *a*) and B (*a* – *b*, *a* + *b*)

Then PA = PB (∵ P is equidistant from A and B)

$$\sqrt{(x-(a+b))^2+(y-(b-a))^2} = \sqrt{(x-(a-b))^2+(y-(a+b))^2}$$

Solving $\sqrt{(x-(a+b))^2+(y-(b-a))^2}$

$$= \sqrt{x^2+(a+b)^2-2x(a+b)+y^2+(b-a)^2-2y(b-a)}$$

$$= \sqrt{x^2+a^2+b^2+2ab-2ax-2bx+y^2+b^2+a^2-2ab-2by+2ya}$$

Solving $\sqrt{(x-(a-b))^2+(y-(a+b))^2}$

$$= \sqrt{x^2+(a-b)^2-2x(a-b)+y^2+(b+a)^2-2y(b+a)}$$

Now we have $= \sqrt{x^2+a^2+b^2-2ab-2ax+2bx+y^2+b^2+a^2+2ab-2by-2ay}$

Squaring both sides and simplify the equations.

$x^2 + a^2 + b^2 + 2ab - 2ax - 2bx + y^2 + b^2 + a^2 - 2ab - 2by + 2ay$.

$= x^2 + a^2 + b^2 - 2ab - 2ax + 2bx + y^2 + a^2 + b^2 + 2ab - 2ay - 2by$

⇒ $-2bx - 2bx = -2ay - 2ay$

$-4bx = -4ay$

$bx = ay.$

e-mail : evergren@del3.vsnl.net.in

Q.9. (*a*) A train covers a distance of 600 km at x km/hr. Had the speed been (x + 20) km/hr, the time taken to cover the distance would have been reduced by 5 hours. Write down an equation in x and solve it to determine x.

(*b*) If $f(x) = \begin{cases} 3x-2 &, \text{ when } x<0 \\ -2 &, \text{ when } x=0 \\ x+11 &, \text{ when } x>0 \end{cases}$

find $f(-1)$ and $f(3)$

(*c*) A cylinderical can whose base is horizontal and of radius 3·5 cm contains sufficient water so that when a sphere is placed in the can the water just covers the sphere. Given that the sphere just fits into the can, calculate :

(*i*) the total surface area of the can in contact with water when the sphere is in it.

(*ii*) the depth of water in the can before the sphere was put into the can. Take $\pi = \frac{22}{7}$ and give your answers in proper fractions.

Sol. (*a*)

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \Rightarrow \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\Rightarrow \frac{600}{x} - \frac{600}{x+20} = 5 \quad (\because T_2 - T_1 = 5) \quad \Rightarrow 600\left(\frac{1}{x} - \frac{1}{x+20}\right) = 5$$

$$\Rightarrow 600\left(\frac{20}{x(x+20)}\right) = 5 \quad \Rightarrow \frac{600\times 20}{5} = x^2 + 20x$$

$$\Rightarrow x^2 + 20x - 2400 = 0 \quad \Rightarrow x^2 + 60x - 40x - 2400 = 0$$

$$\Rightarrow x(x+60) - 40(x+60) = 0 \quad \Rightarrow (x+60)(x-40) = 0$$

$$\Rightarrow x = 40 \quad \therefore x = 40$$

(*b*) To find $f(-1)$, we use $f(x) = 3x - 2$

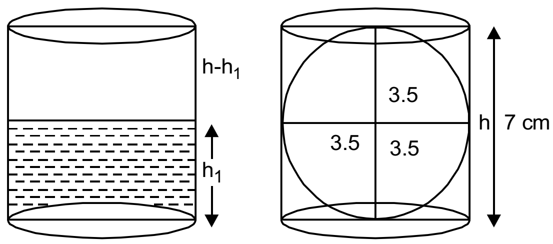
$\therefore \quad f(-1) = 3(-1) - 2 = -3 - 2 = -5$

To find $f(3)$, we use $f(x) = x + 1$

$\therefore \quad f(3) = 3 + 1 = 4$

(*c*) Diameter of cylinder = h of cylinder

$\therefore \quad r = 3·5$ cm; $h = 7$ cm



Before immersion After immersion

(*i*) The total surface area in contact with water

$= $ base area + curved surface area

$= \pi r^2 + 2\pi rh = \pi r(r + 2h)$

$$= \frac{22}{7} \times \frac{35}{10}\left(\frac{35}{10} + 2\times 7\right)\text{cm}^2 = 11\left(\frac{7}{2} + 14\right)\text{cm}^2 = 11\left(\frac{7+28}{2}\right)\text{cm}^2$$

$$= 11 \times \left(\frac{35}{2}\right)\text{cm}^2 = 192·5 \text{ cm}^2$$

(*ii*) Volume of water in the can before the sphere was placed in it = volume of cylinder – vol. of sphere

$$\pi r^2 h_1 = \pi r^2 h - \frac{4}{3}\pi r^3$$

$$\Rightarrow \pi r^2 h_1 = \pi r^2\left(h - \frac{4}{3}r\right)$$

$$\Rightarrow h_1 = \left(7 - \frac{4}{3}\times\frac{7}{2}\right)$$

$$\Rightarrow h_1 = \frac{21-14}{3} = \frac{7}{3} \text{ cm}$$

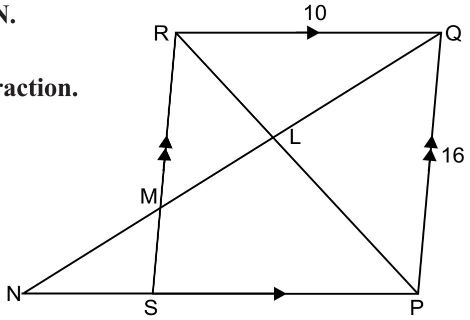
So depth is $= \frac{7}{3}$ cm.

e-mail : evergren@del3.vsnl.net.in

Q.10. (*a*) In the given figure, PQRS is a parallelogram, PQ = 16 cm, QR = 10 cm, L is a point on PR, such that RL: LP = 2 : 3, QL produced to meet RS in M and PS in N.

(*i*) Prove that ΔRLQ ~ ΔPLN, hence find PN.

(*ii*) Name a triangle similar to ΔRLM. Evaluate RM as a fraction.



(*b*) Attempt this questions on a graph paper. The table shows the distribution of marks gained by a group of students in an examination :

Marks less than	10	20	30	40	50	60	70	80	90	100
Number of students	5	10	30	60	105	180	270	355	390	400

Plot these values and draw a smooth curve through the points. Estimate from the graph (*i*) the median marks (*ii*) the quartile marks.

Sol. (*a*) (*i*) In a Δs RLQ and PLN, we have :

$\angle RLQ = \angle PLN$ (vert. opp. ∠s).

$\angle RQL = \angle PNL$ (RQ || NP these are a pair of alt. int. ∠s)

$\therefore \quad \Delta RLQ \sim \Delta PLN$ (By A.A. similarity)

$$\Rightarrow \quad \frac{RL}{PL} = \frac{RQ}{PN}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{10}{PN} \qquad \left(\because \frac{RP}{PL} = \frac{2}{3} \text{ \& } RQ = 10\,\text{cm given}\right)$$

$\Rightarrow \quad 2PN = 30$

$\Rightarrow \quad PN = 15$ cm.

(*ii*) Now, $\angle RLM = \angle PLQ$ (vert. opp. ∠s)

Also, $\angle MRL = \angle LPQ$ (alternate int. ∠s)

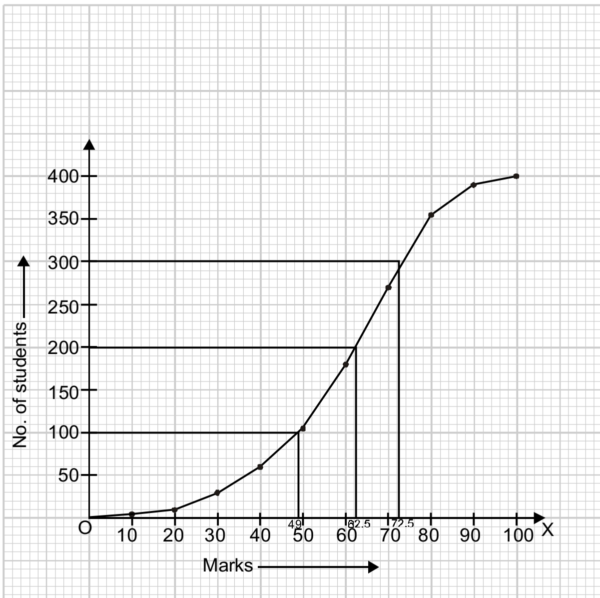
$\therefore \quad \Delta RLM \sim \Delta PLQ$

$$\Rightarrow \quad \frac{RL}{PL} = \frac{RM}{PQ} \quad \Rightarrow \quad \frac{2}{3} = \frac{RM}{16}$$

$$\Rightarrow \quad RM = \frac{32}{3} \text{ cm.}$$

(*b*)

Marks less than	No. of students
Less than 10	5
Less than 20	10
Less than 30	30
Less than 40	60
Less than 50	105
Less than 60	180
Less than 70	270
Less than 80	355
Less than 90	390
Less than 100	400



(*i*) Median mark = 62·5

(*ii*) (*a*) Lower quartile = 49

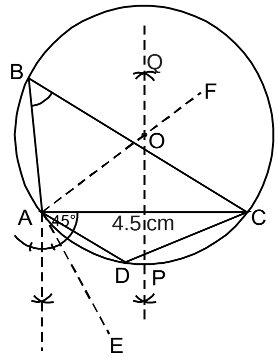
(*b*) Upper quartile = 72·5

e-mail : evergren@del3.vsnl.net.in

Q.11. (*a*) Construct a cyclic quadrilateral ABCD in which AC = 4·5 cm, ∠ABC = 45°, AB = 3cm and AD = 2·3 cm

(*b*) A (2, – 4), B (3, 3) and C(– 1, 5) are the vertices of ΔABC. Find the equation of the median AD from the vertex A to the opposite side BC. Also find the length AD.

(*c*) Solve the equation : $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$; $x \neq -1, -2, -4$.



Sol. (*a*) **Steps of Construction**

(*i*) Draw the line segment AC = 4·5 cm.

(*ii*) Construct ∠CAE = 45°.

(*iii*) Draw the right bisector PQ of AC.

(*iv*) Draw FA ⊥ AE, intersecting PQ in O.

(*v*) With O as centre and radius equal to OA, draw a circle.

(*vi*) Taking A as centre and radius AB = 3 cm draw an arc, cutting the major arc CA at B.

(*vii*) With A as centre and radius AD = 2·3 cm, draw an arc to cut the minor arc AC at D.

(*viii*) Join BA, BC, DA and DC. Then, ABCD is the required cyclic quadrilateral.

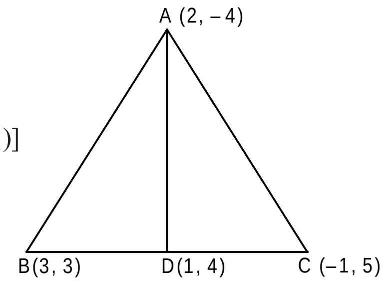
(*b*) Let the given triangle is ABC

The coordinates of A (2, – 4), B (3, 3) and C (– 1, 5). and A, B, C are vertices of ΔABC.

AD is the median from the vertex A on the opposite side BC.

The co-ordinates of D are

$$D = \left(\frac{3-1}{2}, \frac{3+5}{2}\right) = (1, 4)$$



So the equation of AD

$$y + 4 = \frac{4-(-4)}{1-2}(x-2) \quad \left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$\Rightarrow \quad y + 4 = \frac{8}{-1}(x-2)$$

$$\Rightarrow \quad -y - 4 = 8x - 16$$

$$\Rightarrow \quad 8x + y - 12 = 0$$

(*c*) The given equation is

$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4} \quad \Rightarrow \quad \frac{(x+2)+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$$

$$\Rightarrow \quad (x+4)(3x+4) = 4(x+1)(x+2) \quad \Rightarrow \quad 3x^2 + 12x + 4x + 16 = 4(x^2 + 2x + x + 2)$$

$$\Rightarrow \quad 3x^2 + 16x + 16 = 4(x^2 + 3x + 2) \quad \Rightarrow \quad 3x^2 + 16x + 16 = 4x^2 + 12x + 8$$

$$\Rightarrow \quad -x^2 + 4x + 8 = 0 \quad \Rightarrow \quad x^2 - 4x - 8 = 0$$

Comparing with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -4$, $c = -8$

$$\therefore \quad x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot (-8)}}{2 \times 1} \quad \Rightarrow \quad \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm \sqrt{16 \times 3}}{2}$$

$$\Rightarrow \quad \frac{4 \pm 4\sqrt{3}}{2} = \frac{2(2 \pm 2\sqrt{3})}{2} \quad \Rightarrow \quad = 2 \pm 2\sqrt{3}$$

∴ The roots of the given equation are $2 \pm 2\sqrt{3}$

e-mail : evergren@del3.vsnl.net.in