Comprehensive Test Series-03

CHAPTER-2(Inverse – Trigonometric Functions)

TIME: 1hr **MM:**

General Instructions:

- All Questions are compulsory.
- Marks are given alongwith the questions individually.
- Use of calculator is not permitted.

Q.1 Find the principal value of following:

(i) $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

(ii) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(iii) $\tan^{-1}\left(-\sqrt{3}\right)$

(iv) $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

(v) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Q.2 Find the value of following

(i) $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

(ii) $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2)$

Q.3 Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

Q.4 $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Q.5 Prove that $\tan^{-1} x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), |x| < \frac{1}{\sqrt{3}}$

Q.6 $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a > 0; \frac{-a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}$

Q.7 $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0$ and xy < 1

Q.8 If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)$=1, then find the value of x.

Q.9 If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the value of x.

Q. 10 Find the value $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.

Q. 11 $\tan^{-1}\sqrt{3}$ -$\cot^{-1}\left(-\sqrt{3}\right)$ is equal to

$(A)\pi$ $(B)-\frac{\pi}{2}$ $(C)0$ $(D)2\sqrt{3}$

Q.12 Simplify $\tan^{-1}\left[\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right], if\ \frac{a}{b}\tan x>-1$

Q.13 Show that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

Q.14 Find the value $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$.

Q.15 Prove that

(i)$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2},x\in\left(0,\frac{\pi}{4}\right)$

(ii) $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}}\le x\le 1$

(iii) $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

(iv) $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in[0,1]$

Q.16 solving the following equation: $2\tan^{-1}(\cos x)=\tan^{-1}(2\cos ecx)$