Comprehensive Test Series-05
(Application of Derivatives)
XII

TIME: 1.5hr. **MM: 60**

General Instructions:

- All Questions are compulsory.
- Use of calculator is not permitted.

Using differentials, find the approximate value of the each of the following up to 3 places of decimals.

Q.1 $\sqrt{0.037}$, $(26)^{\frac{1}{3}}$, $(82)^{\frac{1}{4}}$, $(0.0037)^{\frac{1}{2}}$, $(3.968)^{\frac{3}{2}}$

Q 2 Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

Q.3 Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1 %.

Q.4 If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

Q.5 Find point on the curve $\frac{x^2}{4}+\frac{y^2}{25}=1$ at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

Q.6 Find the point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

Q.7 For the curve $y = 4x^3 - 2x^5$, Find all the points at which the tangent passes through the origin.

Q.8 Find the point on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.

Q.9 Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

Q.10 Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

Q.11 Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point (x_0, y_0).

Q.12 A ladder 5m long is learning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

Q.13 A particle moves along the curve $6y = x^3 + 2$. Find the point on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Q.14 A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

Q.15 Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm?

Q.16 The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue cost when $x = 7$.