Comprehensive Test Series-8

[Applied Problems in Maxima and Minima]

TIME: 1.5hr **MM: 48**

General Instructions:

- All Questions are compulsory.
- Use of calculator is not permitted.

Q1. Show that of all the rectangles inscribed in a given circle, the square has the maximum area.

Q2. A wire of length 36 m is to be cut into two pieces. One of the pieces is toe be made into a square and the other into a circle. What should be the length of the two pieces, so that he combined area of the square and the circle is minimum?

Q3. A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also find this maximum volume.

Q4. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimension in order that the area may be maximum.

Q5. Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm.

Q6. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$.

Q7. Show that the semi-vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$

Q8. Show that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

Q9. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle $\frac{4}{27}\pi h^3 \tan^2\alpha$.

Q10. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^2}{6\sqrt{3}}$ cubic units.

Q11. The combined resistance R of two resistors R_1 and R_2 ($R_1, R_2 > 0$) is given by $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$

If $R_1 + R_2 = C$ (a constant) , show that maximum resistance R is obtained by choosing $R_1 = R_2$

Q12. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, 1).