Comprehensive Test Series-10
(Application of Derivatives)
XII

TIME: 1hr.

MM: 50

General Instructions:

- All Questions are compulsory.
- Use of calculator is not permitted.

Q.1 An open box with a square base is to be made out of given quantity of card board of area C^2 square units. Show that the maximum volume of box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Q.2 Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of 2 cm^2/sec in its surface area through a tiny hole at the vertex in the bottom. When the slant height of the water is 4cm, find the rate of decrease of the slant height of the water.

Q.3 Determine the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is decreasing or increasing?

Q.4 Find the maximum and minimum values of $f(x) = x + \sin 2x$ in the interval $[0, 2\pi]$.

Q5. Show that the curves $4x = y^2$ and $4xy = k$ cut at right angles if $k^2 = 512$

Q.6 Use differentials find the approximate value.

$$\left(\frac{17}{81}\right)^{\frac{1}{4}}$$

Q.7 A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs. 70 per sq. metres for the base and Rs 45 per m^2 for sides. What is the cost of least expensive tank?

Q.8 A window is in the form of a rectangular surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light though the whole opening.

Q.9 Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and having semi-vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

Q.10 Show that the normal at any point θ to the curve

$x = a\cos\theta + a\theta\sin\theta$

$y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

Q.11 Find the both the maximum and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 1$ on the interval [1,4]

Q.12 Find the equations of tangent and normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at (x_0, y_0)