Comprehensive Test Series-04
(Matrices)
XII

TIME: 1.5hr   **MM: 50**

General Instructions:

- ➢ All Questions are compulsory.
- ➢ Use of calculator is not permitted.

Q. 1   Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by

$$a_{ij} = \begin{cases} i - j, \text{if } i \geq j. \\ i + j \text{ if } i < j \end{cases}$$

Q.2   If $A = \begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix}$, find A^{16}.

Q.3   If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove by induction that $(aI + bA)^n = a^n I + na^{n-1} bA$

Where I is the unity matrix of order 2 and n is a positive integer.

Q.4 Find the matrix such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

Q. 5 If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$, or all positive integers in .

Q. 6 Find the integral value of x if $[x \quad 4 \quad -1] \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix} [x \quad 4 \quad -1]^t = 0.$

Q. 7 Find the inverse of the following metrics, if it exist using elementary row operations.

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Q.8 If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, find a, b so that $A^2 = aA + b1$.

Q 9. Find a and b if $\left\{ 3 \begin{bmatrix} 2 & 1 & -3 \\ 1 & 4 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & 3 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}.$

Q.10 Given $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, find the matrix X such that

AX = 3B + 2C.