SAMPLE QUESTION PAPER

CLASS-XII (2016-17)

MATHEMATICS (041)

Time allowed: **3** hours Maximum Marks: **100**

General Instructions:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

SECTION-A

Questions from 1 to 4 are of 1 mark each.

1. What is the principal value of $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$?

2. A and B are square matrices of order 3 each, $|A| = 2$ and $|B| = 3$. Find $|3AB|$

3. What is the distance of the point (p, q, r) from the x-axis?

4. Let f : R → R be defined by $f(x) = 3x^2 – 5$ and g : R → R be defined by $g(x) = \frac{x}{x^2+1}$. Find gof

SECTION-B

Questions from 5 to 12 are of 2 marks each.

5. How many equivalence relations on the set {1,2,3} containing (1,2) and (2,1) are there in all ? Justify your answer.

6. Let l_i, m_i, n_i ; i = 1, 2, 3 be the direction cosines of three mutually perpendicular vectors in space. Show that $AA' = I_3$, where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.

7. If $e^y (x + 1) = 1$, show that $\frac{dy}{dx} = -e^y$

8. Find the sum of the order and the degree of the following differential equations:

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1 + x) = 0$$

9. Find the Cartesian and Vector equations of the line which passes through the point (–2, 4,–5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$

10. Solve the following Linear Programming Problem graphically:

Maximize Z = 3x + 4y

subject to

$x + y \leq 4, x \geq 0$ and $y \geq 0$

11. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.

12. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

SECTION-C

Questions from 13 to 23 are of 4 marks each.

13. If A + B + C = π, then find the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

OR

Using properties of determinant, prove that

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

14. It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ Rolle's theorem holds in [1, 3] with c = $2 + \frac{1}{\sqrt{3}}$. Find the values of 'a' and 'b'

15. Determine for what values of x, the function $f(x) = x^3 + \frac{1}{x^3}$ ($x \neq 0$) is strictly increasing or strictly decreasing

OR

Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$

16. Evaluate $\int_0^2 (x^2 + 3)\, dx$ as limit of sums.

17. Find the area of the region bounded by the y-axis, y = cos x and y = sinx, $0 \leq x \leq \frac{\pi}{2}$

18. Can $y = ax + \frac{b}{a}$ be a solution of the following differential equation?

$$y = x\frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \quad ....(*)$$

If no, find the solution of the D.E.(*).

OR

Check whether the following differential equation is homogeneous or not

$$x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$$

Find the general solution of the differential equation using substitution y=vx.

19. If the vectors $\vec{p} = a\hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{q} = \hat{\imath} + b\hat{\jmath} + \hat{k}$ and $\vec{r} = \hat{\imath} + \hat{\jmath} + \widehat{CK}$ are coplanar, then for a, b, c ≠ 1 show that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

20. A plane meets the coordinate axes in A, B and C such that the centroid of Δ ABC is the point (α, β,γ) . Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

21. If a 20 year old girl drives her car at 25 km/h, she has to spend Rs 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs 5/km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. Write any one value reflected in the problem.

22. The random variable X has a probability distribution P(X) of the following form, where k is some number:

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k (ii) Find P(X <2) (iii) Find $P(X \leq 2)$ (iv) Find $P(X \geq 2)$

23. A bag contains (2n +1) coins. It is known that 'n' of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, find the value of 'n'.

SECTION-D

Questions from 24 to 29 are of 6 marks each

24. Using properties of integral, evaluate $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$

OR

Find: $\int \frac{\sin x}{\sin^3 x+\cos^3 x}\,dx$

25. Does the following trigonometric equation have any solutions? If Yes, obtain the solution(s):

$$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = -\tan^{-1} 7$$

OR

Determine whether the operation ∗ define below on $\mathbb{Q}$ is binary operation or not.

$$a * b = ab+1$$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in $\mathbb{Q}$.

26. Find the value of x, y and z, if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies A' = A^{-1}

OR

Verify: A(adj A) = (adj A)A = $|A||\mathrm{I}$ for matrix A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

27. Find $\frac{dy}{dx}$, if y = $e^{\sin^2 x}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

28. Find the shortest distance between the line x – y + 1 = 0 and the curve $y^2 = x$

29. Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:

$\vec{r}$ = (8 + 3λ) $\hat{\imath}$ – (9 + 16λ) $\hat{\jmath}$ +(10 + 7λ) $\hat{k}$

$\vec{r}$ = 15 $\hat{\imath}$ + 29 $\hat{\jmath}$ +5 $\hat{k}$ + μ (3 $\hat{\imath}$ + 8 $\hat{\jmath}$ – 5 $\hat{k}$)

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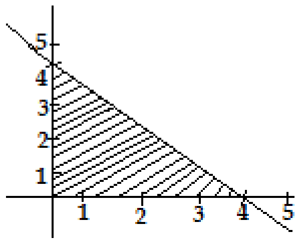
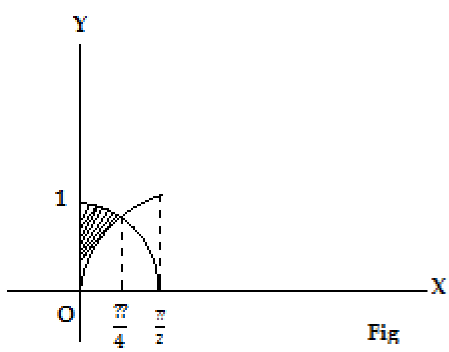
Marking Scheme

1.	$\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\left(-\tan\frac{\pi}{3}\right) = -\frac{\pi}{3}$	1
2.	$\lvert 3AB\rvert = 3^3\,\lvert A\rvert\,\lvert B\rvert = 27\times 2\times 3 = 162$	1
3.	Distance of the point (p, q, r) from the x-axis = Distance of the point (p, q, r) from the point (p,0,0) $= \sqrt{q^2 + r^2}$	1
4.	gof(x) = g{ f(x)} = g($3x^2 - 5$) = $\frac{3x^2-5}{(3x^2-5)^2+1} = \frac{3x^2-5}{9x^4-30x^2+26}$	1
5.	Equivalence relations could be the following: { (1,1), (2,2), (3,3), (1,2), (2,1)} and (1) { (1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2)} (1) So, only two equivalence relations.(Ans.)	2
6.	AA' = $\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}\begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = I_3 $\longrightarrow$ (1) because $l_i^2 + m_i^2 + n_i^2 = 1$, for each i = 1, 2, 3 $\longrightarrow$ $1/2$ $l_i l_j + m_i m_j + n_i n_j = 0$ (i ≠j) for each i, j = 1, 2, 3 $\longrightarrow$ $1/2$	2
7.	On differentiating $e^y(x + 1) = 1$ w.r.t. x, we get $e^y + (x+1)\,e^y\frac{dy}{dx} = 0$ $\longrightarrow$ (1) $\Rightarrow\ e^y + \frac{dy}{dx} = 0$ $\Rightarrow\ \frac{dy}{dx} = -e^y$ $\longrightarrow$ (1)	2
8.	Here, $\left\{\frac{d^2y}{dx^2} + (1+x)\right\}^3 = -\frac{dy}{dx}$ $\longrightarrow$ (1) Thus, order is 2 and degree is 3. So, the sum is 5 $\longrightarrow$ (1)	2
9.	Here, $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$ is same as $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{6}$ Cartesian equation of the line is $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$ $\longrightarrow$ (1) Vector equation of the line is $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$ $\longrightarrow$ (1)	2

10.	The feasible region is a triangle with vertices O(0,0), A(4,0) and B(0,4) $Z_o = 3\times0 + 4\times0 = 0$ $Z_A = 3\times4 + 4\times0 = 12$ $Z_B = 3\times0 + 4\times4 = 16$ (1) Thus, maximum of Z is at B(0,4) and the maximum value is 16 $\longrightarrow$ $^1/_2$  $\longrightarrow$ $^1/_2$	2
11.	Sample space = { B_1B_2, B_1G_2, G_1B_2, G_1G_2} , B_1 and G_1 are the older boy and girl respectively. Let E_1 = both the children are boys; E_2 = one of the children is a boy ; E_3 = the older child is a boy Then, (i) $P(E_1/E_2) = P\left(\frac{E_1 \cap E_2}{E_2}\right) = \frac{1/4}{3/4} = \frac{1}{3}$ $\longrightarrow$ (1) (ii) $P(E_1/E_3) = P\left(\frac{E_1 \cap E_3}{E_3}\right) = \frac{1/4}{2/4} = \frac{1}{2}$ $\longrightarrow$ (1)	2
12.	Here, Area(A) = $\frac{\sqrt{3}}{4}x^2$, where 'x' is the side of the equilateral triangle $\longrightarrow$ $^1/_2$ So. $\frac{dA}{dt} = \frac{\sqrt{3}}{2} \times \frac{dx}{dt}$ $\longrightarrow$ (1) $= \frac{\sqrt{3}}{2}(10)(2) = 10\sqrt{3}$ cm²/sec $\longrightarrow$ $^1/_2$	2
13.	As A + B + C = π, $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix}$ $\longrightarrow$ (2) $= 0 \times \begin{vmatrix} 0 & \tan A \\ -\tan A & 0 \end{vmatrix} - \sin B \times \begin{vmatrix} -\sin B & \tan A \\ -\cos C & 0 \end{vmatrix} + \cos C \times \begin{vmatrix} -\sin B & 0 \\ -\cos C & -\tan A \end{vmatrix}$ = 0 – sin B tan A cos C + cos C sin B tan A = 0 (Ans.) $\longrightarrow$ (2) **OR**	4

	Let $\Delta = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$ Applying $C_1 \to C_1 + C_3$, we get $\Delta = (a+b+c)\begin{vmatrix} 1 & a-b & a \\ 1 & b-c & b \\ 1 & c-a & c \end{vmatrix}$ $\longrightarrow$ **(1)** Applying $R_2 \to R_2 - R_1$, and $R_3 \to R_3 - R_1$, we get $\Delta = (a+b+c)\begin{vmatrix} 1 & a-b & a \\ 0 & 2b-a-c & b-a \\ 0 & 2a+b+c & c-a \end{vmatrix}$ $\longrightarrow$ **(1)** Expanding Δ along first column, we have the result $\longrightarrow$ (2)	4
14.	Since Rolle's theorem holds true, $f(1) = f(3)$ i.e., $(1)^3 - 6(1)^2 + a(1) + b = (3)^3 - 6(3)^2 + a(3) + b$ i.e., $a + b + 22 = 3a + b$ $\Longrightarrow a = 11$ $\longrightarrow$ (2) Also, $f'(x) = 3x^2 - 12x + a$ or $3x^2 - 12x + 11$ As $f'(c) = 0$, we have $3(2 + \frac{1}{\sqrt{3}})^2 - 12(2 + \frac{1}{\sqrt{3}}) + 11 = 0$ As it is independent of b, b is arbitrary. $\longrightarrow$ (2)	4
15.	Here, $f'(x) = 3x^2 - 3x^{-4} = \frac{3(x^6 - 1)}{x^4}$ $\longrightarrow$ (1) $= \frac{3(x^4 + x^2 + 1)}{x^4}(x+1)(x-1)$ Critical points are -1 and 1 $\longrightarrow$ (1) $\Longrightarrow f'(x) > 0$ if $x > 1$ or $x < -1$; and $f'(x) < 0$ if $-1 < x < 1$ $\{\because \frac{3(x^4 + x^2 + 1)}{x^4}$ always + ive$\}$ Hence, f(x) is strictly increasing for $x > 1$ $\longrightarrow$ (1) or $x < -1$; and strictly decreasing for (-1,0)u(0,1) [1] $\longrightarrow$ (1) **OR** Here, $\frac{dy}{dx} = 3x^2 - 11$ $\longrightarrow$ $^1/_2$ So, slope of the tangent is $3x^2 - 11$	4 4

	Slope of the given tangent line is 1. Thus, $3x^2 - 11 = 1$ $\longrightarrow$ (1) that gives x = ± 2 When x = 2, y = 2 – 11 = -9 When x = – 2, y = $-2 - 11 = -13$ Out of the two points (2, – 9) and (–2, –13) $\longrightarrow$ (2) only the point (2, – 9) lies on the curve Thus, the required point is (2, –9) $\longrightarrow$ $^1/_2$		
16.	Here, f(x) = $x^2 + 3$, a = 0 , b = 2 and nh = b – a = 2 $\longrightarrow$ (1) $\int_0^2 (x^2 + 1)\,dx = \lim_{h\to 0} h[\,f(a) + f(a+h) + f(a+2h) + ... + f(a + \overline{n-1}\,h)]$ $\longrightarrow$ (1) $= \lim_{h\to 0} h[\,3 + 1^2h^2 + 3 + 2^2h^2 + 3 + \cdots ... + (n-1)^2h^2 + 3]$ $= \lim_{h\to 0} h[\,3n + h^2\,\{1^2 + 2^2 + 3^2 + \cdots ...\ (n-1)^2\}]$ $= \lim_{h\to 0} [\,3nh + h^3\,\{\frac{(n-1)n\,(2n-1)}{6}\}]$ $= \lim_{h\to 0} [\,3nh + \{\frac{(nh-h)nh\,(2nh-h)}{6}\}]$ $\longrightarrow$ (1) $= \lim_{h\to 0} [\,3 \times 2 + \{\frac{(2-h)2\,(4-h)}{6}\}]$ $= 6 + \frac{16}{6}$,i.e., $\frac{26}{3}$ $\longrightarrow$ **(1)**	4	
17.	The rough sketch of the bounded region is shown on the right. $\longrightarrow$ (1) Required area = $\int_0^{\pi/4} \cos x\,dx - \int_0^{\pi/4} \sin x\,dx$ $\longrightarrow$ (1) $= (\sin x + \cos x)]_0^{\pi/4}$ $\longrightarrow$ (1) $= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \sin 0 - \cos 0$ $= \frac{2}{\sqrt{2}} - 1$, i.e, $(\sqrt{2} - 1)$ sq units $\longrightarrow$ (1) 	4	
18.	y = ax + $\frac{b}{a}$ .... (1) gives $\frac{dy}{dx}$ = a $\longrightarrow$ $\left(1\frac{1}{2}\right)$ Substituting this value of ‘a’ in (1), we get	4	

	$y = x\frac{dy}{dx} + \frac{b}{\frac{dy}{dx}}$ $\longrightarrow$ $\left(1\frac{1}{2}\right)$ **Thus, $y = ax + \frac{b}{a}$ is a solution of the following differential equation $y = x\frac{dy}{dx} + \frac{b}{\frac{dy}{dx}}$.** $\longrightarrow$ **1** **OR** Given differential equation can be written as $\frac{dy}{dx} = \frac{1+xy+\cos\left(\frac{y}{x}\right)}{x^2} = \frac{y}{x} + [\frac{1+\cos\left(\frac{y}{x}\right)}{x^2}]$ ......(1) Let $F(x,y) = \frac{y}{x} + [\frac{1+\cos\left(\frac{y}{x}\right)}{x^2}]$. Then $F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} + [\frac{1+\cos\left(\frac{\lambda y}{\lambda x}\right)}{(\lambda x)^2}]$ $= \frac{y}{x} + [\frac{1+\cos\left(\frac{y}{x}\right)}{\lambda^2 x^2}] \neq F(x,y)$ Hence, the given D.E. is not a homogeneous equation. $\longrightarrow$ (1) Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (1), we get $v + x\frac{dv}{dx} = v + \frac{1+\cos v}{x^2}$ $\Rightarrow \frac{dv}{1+\cos v} = \frac{1}{x^3}\mathrm{dx}$ $\Rightarrow \boldsymbol{sec^2}\left(\frac{v}{2}\right)dv = \frac{2}{x^3}\mathrm{dx}$ $\longrightarrow$ (1) Integrating both sides, we get $2\tan\frac{v}{2} = -\frac{1}{x^2} + C$ $\longrightarrow$ $1\frac{1}{2}$ or $2\tan\frac{y}{2x} = -\frac{1}{x^2} + C$ $\longrightarrow$ $\frac{1}{2}$	4
19.	Since the vector $\vec{p}$, $\vec{q}$ and $\vec{r}$ are coplanar $\therefore [\vec{p}, \vec{q}, \vec{r}] = 0$ $[\vec{p}\ \ \vec{q}\ \ \vec{r}] = 0$ $\longrightarrow$ (1) i.e., $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$ $R_2 \rightarrow R_2 - R_1$ $R_3 \rightarrow R_3 - R_1$ $\longrightarrow$ (1) or $\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$	4

	$\Rightarrow$ a(b -1)(c-1) -1 (1-a)(c -1) - 1(1-a)(b – 1) = 0 i.e., a(1 - b)(1- c) + (1-a)(1 - c) + (1- a)(1 – b) = 0 $\longrightarrow$ (1) Dividing both the sides by (1-a)(1 - b)(1- c), we get $\frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$ i.e., $-\left(1-\frac{1}{1-a}\right)+\frac{1}{1-b}+\frac{1}{1-c}=0$ i.e., $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$ $\longrightarrow$ (1)	
20.	We know that the equation of the plane having intercepts a, b and c on the three coordinate axes is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ $\longrightarrow$ (1) Here, the coordinates of A, B and C are (a,0,0), (0,b,0) and (0,0,c) respectively. The centroid of Δ ABC is ($\frac{a}{3},\frac{b}{3},\frac{c}{3}$). $\longrightarrow$ (1) Equating ($\frac{a}{3},\frac{b}{3},\frac{c}{3}$) to (α, β,γ), we get a = 3α, b = 3β and c = 3γ $\longrightarrow$ (1) Thus, the equation of the plane is $\frac{x}{3\alpha}+\frac{y}{3\beta}+\frac{z}{3\gamma}=1$ or $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$ $\longrightarrow$ **(1)**	4
21.	Let the distance covered with speed of 25 km/h = x km and the distance covered with speed of 40 km/h = y km (½) Total distance covered = z km The L.P.P. of the above problem, therefore, is $\longrightarrow$ (1) Maximize z = x + y subject to constraints $4x + 5y \leq 200$ $\longrightarrow$ (1) $\frac{x}{25}+\frac{y}{40}\leq 1$ $x\geq 0, y \geq 0$ **(1)** Any one value $\longrightarrow$ (½)	4
22.	Here, X 0 1 2 P(X) k 2k 3k (i) Since P(0) + P(1) + P(2)= 1, we have	4

	$k + 2k + 3k = 1$ i.e., $6k = 1$, or $k = \frac{1}{6}$ $\longrightarrow$ (1) (ii) $P(X < 2) = P(0) + P(1) = k + 2k = 3k = \frac{1}{2}$; $\longrightarrow$ (1) (iii) $P(X \leq 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 1$ $\longrightarrow$ (1) (iv) $P(X \geq 2) = P(2) = 3k = \frac{1}{2}$ $\longrightarrow$ (1)	
23.	Let the events be described as follows: E_1 : a coin having head on both sides is selected. E_2 : a fair coin is selected. A : head comes up in tossing a selected coin $P(E_1) = \frac{n}{2n+1}$; $P(E_2) = \frac{n+1}{2n+1}$; $P(A/E_1) = 1$; $P(A/E_2) = \frac{1}{2}$ $\longrightarrow$ (2) It is given that $P(A) = \frac{31}{42}$. So, $P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) = \frac{31}{42}$ $\Rightarrow \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2} = \frac{31}{42}$ $\longrightarrow$ (1) $\Rightarrow \frac{1}{2n+1}\left[n + \frac{n+1}{2}\right] = \frac{31}{42}$ $\Rightarrow 42(3n+1) = 62(2n+1)$ $\Rightarrow 2n = 20, \ or \ n = 10$ $\longrightarrow$ (1)	4
24.	$I = \int_0^{\pi} \frac{x}{1+\sin x}dx = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}dx$ (1) $= \pi \int_0^{\pi} \frac{1}{1+\sin x}dx - \int_0^{\pi} \frac{x}{1+\sin x}dx$ $\Rightarrow 2I = \pi \int_0^{\pi} \frac{1}{1+\sin x}dx$ (1) $\Rightarrow \frac{\pi}{2} \int_0^{\pi} \frac{1}{1+\cos\left(\frac{\pi}{2} - x\right)}dx$ $\Rightarrow \frac{\pi}{2} \int_0^{\pi} \frac{1}{2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}dx$ $\Rightarrow \frac{\pi}{4} \int_0^{\pi} sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)dx$ (1) $\Rightarrow I = \frac{\pi}{4}\left[-2\tan\left[\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]\right]_0^{\pi}$ (2) $\Rightarrow I = \frac{\pi}{4}[2 - (-2)] = \pi$ (1) **OR**	6

Let I = $\int \frac{\sin x}{\sin^3 x+\cos^3 x}\,dx = \int \frac{\tan x \sec^2 x}{\tan^3 x+1}\,dx$ (½)

On substituting tan x = t and $\sec^2 x\,dx = dt$, we get (1)

I = $\int \frac{t}{t^3+1}\,dt = \int \frac{t}{(t+1)(t^2-t+1)}\,dt$ (½)

$= -\frac{1}{3}\int \frac{1}{t+1}\,dt + \frac{1}{3}\int \frac{t+1}{t^2-t+1}\,dt$

$= -\frac{1}{3}\log|t+1| + \frac{1}{6}\int \frac{(2t-1)+3}{t^2-t+1}\,dt$ (1)

$= -\frac{1}{3}\log|t+1| + \frac{1}{6}\int \frac{2t-1}{t^2-t+1}\,dt + \frac{1}{2}\int \frac{1}{t^2-t+1}\,dt$

$= -\frac{1}{3}\log|t+1| + \frac{1}{6}\log|t^2-t+1| + \frac{1}{2}\int \frac{1}{\left(t-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dt$

$= -\frac{1}{3}\log|t+1| + \frac{1}{6}\log|t^2-t+1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)$ (2)

$= -\frac{1}{3}\log|\tan x+1| + \frac{1}{6}\log|\tan^2 x-\tan x+1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right) + c$ (1)

6

25.

$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = -\tan^{-1} 7$

$\Rightarrow \tan^{-1}\left[\frac{\left(\frac{x+1}{x-1}\right)+\left(\frac{x-1}{x}\right)}{1-\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right] = -\tan^{-1} 7$, if $\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right) < 1$ .....(*) (2)

$\Rightarrow \tan^{-1}\left[\frac{x(x+1)+(x-1)^2}{(x-1)x-(x+1)(x-1)}\right] = -\tan^{-1} 7$

$\Rightarrow \frac{(x^2+x)+(x^2+1-2x)}{(x^2-x)-(x^2-1)} = \tan\left[-\tan^{-1} 7\right]$

$\Rightarrow \frac{2x^2-x+1}{-x+1} = -7$ (1)

$\Rightarrow 2x^2 - 8x + 8 = 0$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x = 2$ (1)

Let us now verify whether x = 2 satisfies the condition (*)

For x = 2,

$\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right) = 3 \times \frac{1}{2} = \frac{3}{2}$ which is not less than 1

Hence, this value does not satisfy the condition (*) (1)

i.e., there is no solution to the given trigonometric equation. (1)

OR

Given $*$ on $\mathbb{Q}$, defined by a$*$b = ab+1

Let, a$\in\mathbb{Q}$, b$\in\mathbb{Q}$ then

ab$\in\mathbb{Q}$

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	and (ab+1)∈$\mathbb{Q}$ $\Rightarrow$ a∗b=ab+1 is defined on $\mathbb{Q}$ ∴ ∗ is a binary operation on $\mathbb{Q}$ (1) **Commutative**: a∗b = ab+1 b∗a = ba+1 =ab+1 (∵ ba = ab in $\mathbb{Q}$) $\Rightarrow$ a∗b =b∗a So ∗ is commutative on $\mathbb{Q}$ (1) **Associative**: (a∗b)∗c= (ab+1)∗c =(ab+1)c+1 = abc+c+1 a∗(b∗c)=a∗(bc+1) = a(bc+1)+1 = abc+a+1 ∴ (a∗b)∗c ≠ a∗(b∗c) So ∗ is not associative on $\mathbb{Q}$ (1) **Identity Element :** Let e ∈$\mathbb{Q}$ be the identity element, then for every a ∈$\mathbb{Q}$ a∗e=a and e∗a=a ae+1=a and ea+1=a $\Rightarrow$ e= $\frac{a-1}{a}$ and e= $\frac{a-1}{a}$ (1) e is not unique as it depend on \`a' ,hence identity element does not exist for ∗ (1) **Inverse**: since there is no identity element hence, there is no inverse. (1)	6
26.	The relation A' = A^{-1} gives A'A = A^{-1}A = I (1) Thus, $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\left(1\frac{1}{2}\right)$ $\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (2) $\Rightarrow 2x^2 = 1;\ 6y^2 = 1$ and $3z^2 = 1$ $\Rightarrow$ x = $\pm\frac{1}{\sqrt{2}}$; y = $\pm\frac{1}{\sqrt{6}}$; z = $\pm\frac{1}{\sqrt{3}}$ $\left(1\frac{1}{2}\right)$ **OR**	6

Here, $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$ = 1(0+0) +1(9+2) +2(0– 0) =11 (1)

$\Rightarrow |A|I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ ....(1) (½)

adj A $= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$ (2)

Now, A(adj A) = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ (1)

and (adj A)A = $\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ (1)

Thus, it is verified that A(adj A) = (adj A)A = $|A|I$ (½)

Marks: 6

27. Putting x = cos 2Θ in $\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$, we get (1)

$2\tan^{-1}\sqrt{\frac{1-\cos 2\Theta}{1+\cos 2\Theta}}$

i.e., $2\tan^{-1}\sqrt{\frac{2\sin^2\Theta}{2\cos^2\Theta}} = 2\tan^{-1}(\tan\Theta) = 2\Theta = \cos^{-1}x$ (2)

Hence, y = $e^{\sin^2 x}\cos^{-1}x$

$\Rightarrow \log y = \sin^2 x + \log(\cos^{-1}x)$

$\Rightarrow \frac{1}{y}\times\frac{dy}{dx} = 2\sin x\cos x + \frac{1}{\cos^{-1}x}\times\frac{-1}{\sqrt{1-x^2}} = \sin 2x - \frac{1}{\cos^{-1}x\sqrt{1-x^2}}$ (2)

$\Rightarrow \frac{dy}{dx} = e^{\sin^2 x}\cos^{-1}x\left[\sin 2x - \frac{1}{\cos^{-1}x\sqrt{1-x^2}}\right]$ (1)

Marks: 6

28. Let (t^2, t) be any point on the curve $y^2 = x$. Its distance (S) from the

line x – y + 1 = 0 is given by $^1/_2$

S = $\left|\frac{t-t^2-1}{\sqrt{1+1}}\right|$ $^1/_2$

$= \frac{t^2-t+1}{\sqrt{2}}$ $\{\because t^2 - t + 1 = \left(t-\frac{1}{2}\right)^2 + \frac{3}{4} > 0\}$ (1)

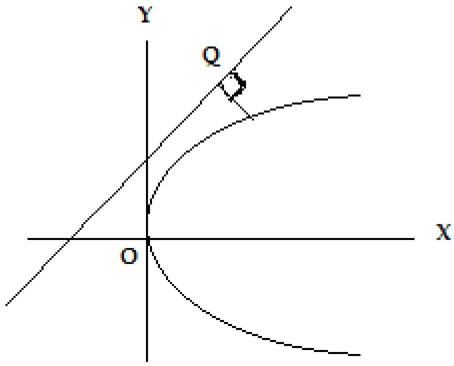
$\Rightarrow \frac{dS}{dt} = \frac{1}{\sqrt{2}}(2t-1)$ (1)

and $\frac{d^2S}{dt^2} = \sqrt{2} > 0$ (1)

Now, $\frac{dS}{dt} = 0 \Rightarrow \frac{1}{\sqrt{2}}(2t-1) = 0$,i.e., $t = \frac{1}{2}$ (1)

Thus, S is minimum at $t = \frac{1}{2}$

Marks: 6

	So, the required shortest distance is $\frac{(\frac{1}{2})^2-(\frac{1}{2})+1}{\sqrt{2}} = \frac{3}{4\sqrt{2}}$, or $\frac{3\sqrt{2}}{8}$ (1)  Fig. 1	
29.	1) the line which are neither intersecting nor parallel. (1) 2) The given equations are $\vec{r} = 8\hat{\imath} - 9\hat{\jmath} + 10\hat{k} + \mu(3\hat{\imath} - 16\hat{\jmath} + 7\hat{k})$ ....(1) (½) $\vec{r} = 15\hat{\imath} + 29\hat{\jmath} + 5\hat{k} + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$ ....(2) Here, $\overrightarrow{a_1} = 8\hat{\imath} - 9\hat{\jmath} + 10\hat{k}$; $\overrightarrow{a_2} = 15\hat{\imath} + 29\hat{\jmath} + 5\hat{k}$ $\overrightarrow{b_1} = 3\hat{\imath} - 16\hat{\jmath} + 7\hat{k}$; $\overrightarrow{b_2} = 3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$ Now, $\overrightarrow{a_2} - \overrightarrow{a_1} = (15-8)\hat{\imath} + (29+9)\hat{\jmath} + (5-10)\hat{k} = 7\hat{\imath} + 38\hat{\jmath} - 5\hat{k}$ (½) and $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}$ (1) $\Rightarrow (\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1}) = (24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}).(7\hat{\imath} + 38\hat{\jmath} - 5\hat{k}) = 1176$ (1) Shortest distance = $\left\lvert \frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{\lvert \overrightarrow{b_1} \times \overrightarrow{b_2} \rvert} \right\rvert$ (1) $= \left\lvert \frac{1176}{\sqrt{24^2+36^2+72^2}} \right\rvert = \frac{1176}{\sqrt{7056}} = \frac{1176}{84} = \frac{98}{7}$ (1)	

--0-0-0--