

## UNIT 5

# LINES AND ANGLES

### (A) Main Concepts and Results

- An angle is formed when two lines or rays or line segments meet or intersect.
- When the sum of the measures of two angles is  $90^\circ$ , the angles are called complementary angles. Each of them is called complement of the other.
- When the sum of the measures of two angles is  $180^\circ$ , the angles are called supplementary angles. Each of them is called supplement of the other.
- Two angles are called adjacent angles, if they have a common vertex and a common arm but no common interior points.
- A linear pair is a pair of adjacent angles whose non-common sides are opposite rays.
- When two lines intersect, the vertically opposite angles so formed are equal.
- When two lines are intersected by a transversal, eight angles are formed. These angles can be classified as 4 interior angles, 4 exterior angles, 4 pairs of corresponding angles, 2 pairs of alternate interior angles, 2 pairs of alternate exterior angles and two pairs of interior angles on the same side of the transversal.
- If two parallel lines are intersected by a transversal,
  - (i) each pair of corresponding angles is equal.
  - (ii) each pair of alternate interior angles is equal.
  - (iii) each pair of interior angles on the same side of the transversal is supplementary.
- Converses of the above results are also true.

**(B) Solved Examples**

In each of the Examples 1 to 4, there are four options, out of which one option is correct. Write the correct one.

**Example 1:** The angles between North and East and North and West are

- (a) complementary angles
- (b) supplementary angles
- (c) both acute angles
- (d) both obtuse angles

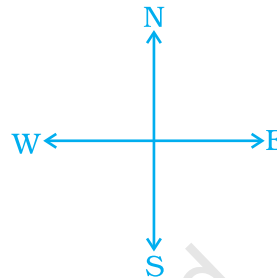


Fig. 5.1

**Solution:** Correct answer is (b).

**Example 2:** Which of the following pair of angles are supplementary?

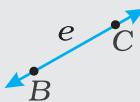
- (a)  $48^\circ, 42^\circ$
- (b)  $60^\circ, 60^\circ$
- (c)  $75^\circ, 105^\circ$
- (d)  $179^\circ, 2^\circ$

**A point** name a location.

$\bullet A$

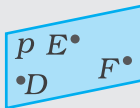
point  $A$

**A line** is perfectly straight and extends for ever in both directions.



line  $e$ , or  $\overleftrightarrow{BC}$

**A plane** is a perfectly flat surface that extends forever in all directions.



plane  $p$ , or plane  $DEF$

**A segment**, or line segment, is the part of a line between two points.



**A ray** is part of a line that starts at one point and extends for ever in one direction.



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**Solution:** Correct answer is (c).

**Example 3:** In Fig. 5.2, a pair of corresponding angles is

- (a)  $\angle 1, \angle 2$
- (b)  $\angle 3, \angle 6$
- (c)  $\angle 3, \angle 5$
- (d)  $\angle 3, \angle 7$

**Solution:** Correct answer is (d).

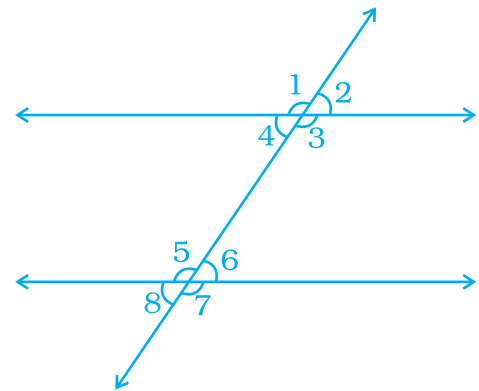


Fig. 5.2

**Example 4:** If two lines are intersected by a transversal, then the number of pairs of interior angles on the same side of the transversal is

- (a) 1
- (b) 2
- (c) 3
- (d) 4

**Solution:** Correct answer is (b).

An **angle** ( $\angle$ ) is formed by two rays with a common endpoint called the *vertex* (plural, *vertices*). Angles can be measured in degrees,  $m\angle 1$  means the measure of  $\angle 1$ . The angles can be named  $\angle XYZ$ ,  $\angle 1$ , or  $\angle Y$ . The vertex must be the middle letter.

**In Examples 5 to 7, fill in the blanks to make the statements true.**

**Example 5 :** Two lines in a plane which never meet at any point are called \_\_\_\_\_.

**Solution:** parallel lines

**Example 6:** Angles of a linear pair are \_\_\_\_\_ as well as \_\_\_\_\_.

**Solution:** adjacent, supplementary

**Example 7:** Adjacent angles have a common vertex, a common \_\_\_\_\_ and no-common \_\_\_\_\_.

**Solution:** arm, interior points

In Examples 8 to 11, state whether the statements are True or False.

**Example 8:** Sum of two complementary angles is  $180^\circ$ .

**Solution:** False

**Example 9:** Sum of two supplementary angles is  $180^\circ$ .

**Solution:** True

**Example 10:** Sum of interior angles on the same side of a transversal with two parallel lines is  $90^\circ$ .

**Solution:** False

**Example 11:** Vertically opposite angles are equal.

**Solution:** True

**Example 12:** In Fig. 5.3, four line segments PQ, QR, RS and ST are making the letter W,  $PQ \parallel RS$  and  $QR \parallel ST$ . If angle between PQ and QR is  $39^\circ$ , find the values of  $x$  and  $y$ .

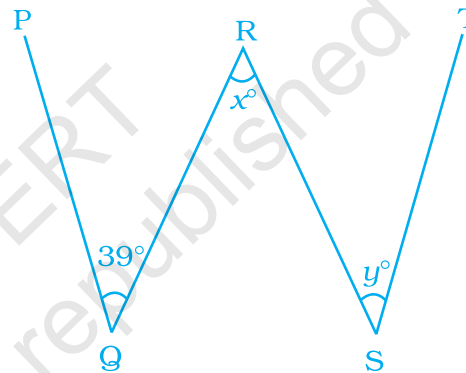


Fig. 5.3

**Solution:** Since  $PQ \parallel RS$  and QR is transversal, so

$$x = 39^\circ \quad [\text{Alternate interior angles}]$$

Again  $QR \parallel ST$  and RS is a transversal.

$$\text{Therefore, } y = x \quad [\text{Alternate interior angles}]$$

$$\text{or } y = 39^\circ$$

**Example 13:** In Fig. 5.4, are the angles 1 and 2 of the letter N forming a pair of adjacent angles? Give reasons.

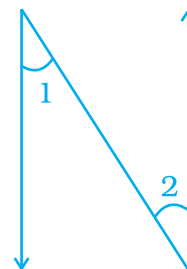


Fig. 5.4

**Solution:** No,  $\angle 1$  and  $\angle 2$  are not forming a pair of adjacent angles as they do not have a common vertex.

**Example 14:** In Fig. 5.5, the points A, O and B are collinear. Ray  $OC \perp$  ray  $OD$ . Check whether

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- (i)  $\angle AOD$  and  $\angle BOC$  are complementary,  
 (ii)  $\angle AOC$  and  $\angle BOC$  are supplementary.

**Solution:**

Since points A, O and B are collinear (Given), therefore AB is a straight line.

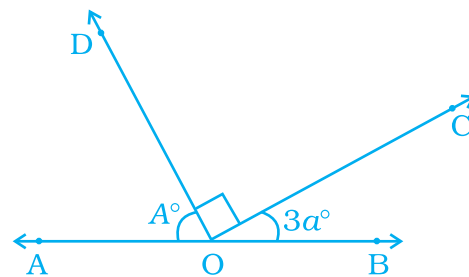


Fig. 5.5

- (i) As O is a point on the line AB, therefore  $\angle AOD + \angle DOC + \angle BOC = 180^\circ$   
 or,  $\angle AOD + \angle BOC + 90^\circ = 180^\circ$   
 or,  $\angle AOD + \angle BOC = 90^\circ$   
 So,  $\angle AOD$  and  $\angle BOC$  are complementary angles.
- (ii) Also,  $\angle AOC$  and  $\angle BOC$  are supplementary as  $\angle AOC + \angle BOC = 180^\circ$

A **right angle** measures  $90^\circ$ . An **acute angle** measures greater than  $0^\circ$  and less than  $90^\circ$ . An **obtuse angle** measures greater than  $90^\circ$  and less than  $180^\circ$ . **Complementary angles** are two angles whose measures add to  $90^\circ$ . **Supplementary angles** are two angles whose measures add to  $180^\circ$ .

**Example 15:** In Fig. 5.6  $AB \parallel EF$ ,  $ED \parallel CB$  and  $\angle APE$  is  $39^\circ$ . Find  $\angle CQF$ .

**Solution:**

Since  $ED \parallel BC$  and AB is a transversal, so

so  $\angle QBP = \angle APE$   
 [Corresponding angles]

or  $\angle QBP = 39^\circ$

Now,  $AB \parallel EF$  and BC is a transversal.

Therefore,  $\angle FQB = \angle QBP$   
 [Alternate interior angles]

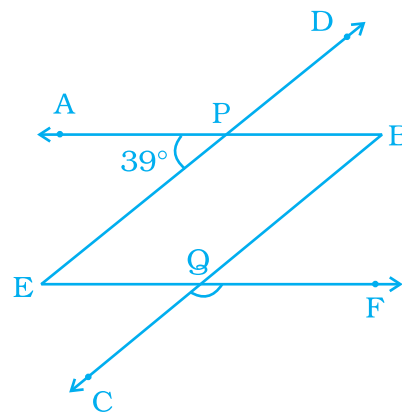


Fig. 5.6

$$\text{or } \angle FQB = 39^\circ$$

$$\text{Also, } \angle CQF + \angle FQB = 180^\circ \quad [\text{Linear pair}]$$

$$\text{So } \angle CQF + 39^\circ = 180^\circ$$

$$\text{or } \angle CQF = 180^\circ - 39^\circ$$

$$\text{or } \angle CQF = 141^\circ$$

**Example 16:** Out of a pair of complementary angles, one is two-third of the other. Find the angles.

**Solution:** Let one angle be  $x$ .

$$\text{So, other angle} = 90^\circ - x$$

$$\text{Thus, } \frac{2}{3} \times x = 90^\circ - x$$

$$\text{or } 2x = 270^\circ - 3x$$

$$\text{or } 2x + 3x = 270^\circ$$

$$\text{or } 5x = 270^\circ$$

$$\text{or } x = \frac{270^\circ}{5} = 54^\circ$$

$$\text{So, one angle} = 54^\circ \text{ and the other angle} = 90^\circ - 54^\circ = 36^\circ.$$

**Congruent** figures have the same size and same shape.

- Segments that have the same length are congruent.
- Angles that have the same measure are congruent.
- The symbol for congruence is  $\cong$ , which is read as “is congruent to.”

**Example 17:** In Fig. 5.7, CD intersects the line AB at F,  $\angle CFB = 50^\circ$  and  $\angle EFA = \angle AFD$ . Find the measure of  $\angle EFC$ .

**Solution:** Let  $\angle EFA = x$ .

$$\text{Then } \angle AFD = x.$$

It is given that CD intersects line AB at F.

Therefore,  $\angle CFB = \angle AFD$   
(Vertically opposite angles)

$$\text{So, } x = 50^\circ$$

$$\text{But } \angle EFA = \angle AFD \text{ which gives } \angle EFA = 50^\circ$$

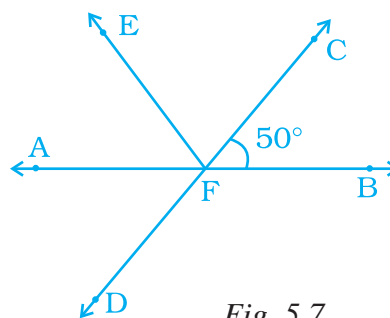


Fig. 5.7

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Now  $\angle CFB + \angle EFA + \angle EFC = 180^\circ$  [As AB is a straight line].

$$\text{or, } 50^\circ + 50^\circ + \angle EFC = 180^\circ$$

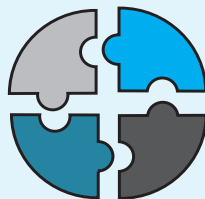
$$\text{or, } \angle EFC = 180^\circ - 100^\circ$$

Thus,  $\angle EFC = 80^\circ$ .

### Think and Discuss

- 1. Tell** which statements are correct: If  $\angle X$  and  $\angle Y$  are congruent,  
**a.**  $\angle X = \angle Y$  **b.**  $m\angle X = m\angle Y$  **c.**  $\angle X \cong \angle Y$ .
- 2. Explain** why vertically opposite angles must always be congruent.

### Application on Problem Solving Strategy



#### Example 18

In the given figure, find out which pair of lines are parallel.

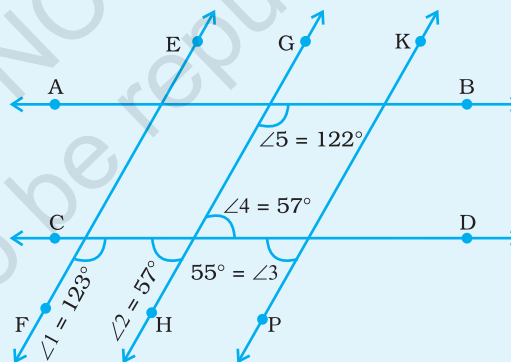


Fig. 5.8

**Solution:**



#### Understand and Explore the Problem

- What information is given in the question?

Lines AB and CD are intersecting three lines EF, GH and KP at distinct points forming angles  $\angle 1 = 123^\circ$ ,  $\angle 2 = 57^\circ$ ,  $\angle 3 = 55^\circ$  and  $\angle 5 = 122^\circ$ .

- What are you trying to find?

We are trying to find

(a)  $EF \parallel GH$  or not

- (b)  $GH \parallel KP$  or not
- (c)  $EF \parallel KP$  or not
- (d)  $AB \parallel CD$  or not



### Plan a Strategy

- (a) Since we want to find whether the lines are parallel or not, therefore recall the conditions when the lines are parallel.

The lines are parallel if it satisfies any one of the following,

- (1) when corresponding angles are equal
- (2) when alternate interior angles are equal
- (3) when the sum of interior angles on the same side of the transversal is  $180^\circ$ .

- (b) Find out what type of angles are formed by lines EF, GH, KP taking AB or CD as transversal.



### Solve

- For lines EF and GH, taking CD as transversal,  $\angle 1$  and  $\angle 2$  are interior angles on the same side of the transversal. Therefore, we check whether the sum of  $\angle 1$  and  $\angle 2$  is  $180^\circ$  or not.

$$\angle 1 = 123^\circ, \angle 2 = 57^\circ, \angle 1 + \angle 2 = 123^\circ + 57^\circ = 180^\circ$$

Since the sum of interior  $\angle$ 's on the same side of the transversal is  $180^\circ$ , therefore  $EF \parallel GH$ .

- For lines GH and KP, taking CD as transversal,  $\angle 2$  and  $\angle 3$  are corresponding  $\angle$ 's. If these angles are equal, then lines are parallel.

$$\angle 2 = 57^\circ, \angle 3 = 55^\circ$$

$\angle 2 \neq \angle 3$ . Since corresponding angles are not equal, therefore, GH is not parallel to KP.

- Similarly, for lines EF and KP, taking CD as transversal  $\angle 1$  and  $\angle 3$  are interior angles on the same side of the transversal.

$$\angle 1 = 123^\circ, \angle 3 = 55^\circ, \angle 1 + \angle 3 = 123^\circ + 55^\circ = 178^\circ. \text{ Since the sum is not equal to } 180^\circ,$$

therefore EF is not parallel to KP.

- For lines AB and CD, taking GH as a transversal  $\angle 2 = \angle 4 = 57^\circ$  (vertically opp.  $\angle$ 's).

$\angle 5$  and  $\angle 4$  are interior angles on the same side of the

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transversal and  $\angle 5 + \angle 4 = 122^\circ + 57^\circ = 179^\circ \neq 180^\circ$ .  
Therefore, AB is not parallel to CD.



### Revise

- $EF \parallel GH$ , since sum of interior  $\angle$ 's on the same side of transversal is  $180^\circ$ .
- GH is not parallel to KP, since corresponding angles formed are not equal.
- EF is not parallel to KP, since the sum of interior  $\angle$ 's on the same side of the transversal is not equal to  $180^\circ$ .
- AB is not parallel to CD, since the sum of interior  $\angle$ 's on the same side of the transversal is not equal to  $180^\circ$ .

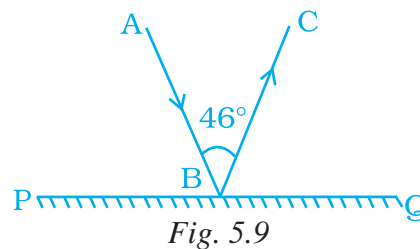
## Think and Discuss

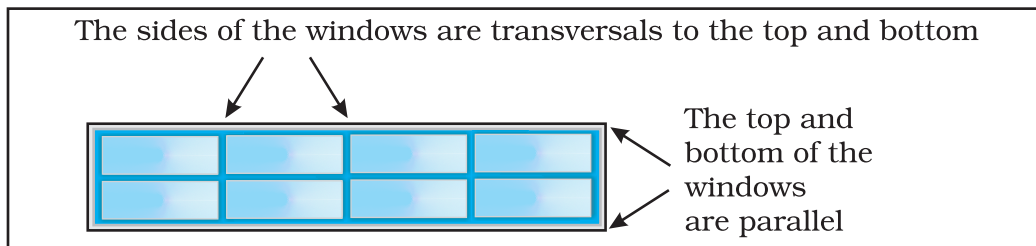
1. **Can** you find whether the lines EF, GH, KP, AB and CD are parallel or not by using other conditions of parallel lines?
2. **Discuss** with your classmates regarding their method towards this problem.

## (C) Exercise

**In questions 1 to 41, there are four options out of which one is correct. Write the correct one.**

1. The angles between North and West and South and East are
  - (a) complementary
  - (b) supplementary
  - (c) both are acute
  - (d) both are obtuse
2. Angles between South and West and South and East are
  - (a) vertically opposite angles
  - (b) complementary angles
  - (c) making a linear pair
  - (d) adjacent but not supplementary
3. In Fig. 5.9, PQ is a mirror, AB is the incident ray and BC is the reflected ray. If  $\angle ABC = 46^\circ$ , then  $\angle ABP$  is equal to
  - (a)  $44^\circ$
  - (b)  $67^\circ$
  - (c)  $13^\circ$
  - (d)  $62^\circ$





4. If the complement of an angle is  $79^\circ$ , then the angle will be of  
 (a)  $1^\circ$  (b)  $11^\circ$  (c)  $79^\circ$  (d)  $101^\circ$
5. Angles which are both supplementary and vertically opposite are  
 (a)  $95^\circ, 85^\circ$  (b)  $90^\circ, 90^\circ$  (c)  $100^\circ, 80^\circ$  (d)  $45^\circ, 45^\circ$
6. The angle which makes a linear pair with an angle of  $61^\circ$  is of  
 (a)  $29^\circ$  (b)  $61^\circ$  (c)  $122^\circ$  (d)  $119^\circ$
7. The angles  $x$  and  $90^\circ - x$  are  
 (a) supplementary (b) complementary  
 (c) vertically opposite (d) making a linear pair
8. The angles  $x - 10^\circ$  and  $190^\circ - x$  are  
 (a) interior angles on the same side of the transversal  
 (b) making a linear pair  
 (c) complementary  
 (d) supplementary
9. In Fig. 5.10, the value of  $x$  is  
 (a)  $110^\circ$  (b)  $46^\circ$   
 (c)  $64^\circ$  (d)  $150^\circ$
10. In Fig. 5.11, if  $AB \parallel CD$ ,  $\angle APQ = 50^\circ$  and  $\angle PRD = 130^\circ$ , then  $\angle QPR$  is  
 (a)  $130^\circ$  (b)  $50^\circ$   
 (c)  $80^\circ$  (d)  $30^\circ$

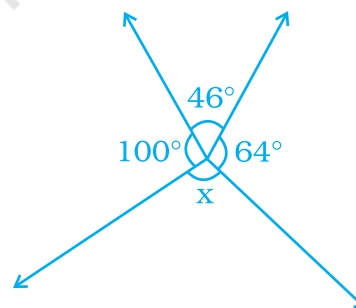


Fig. 5.10

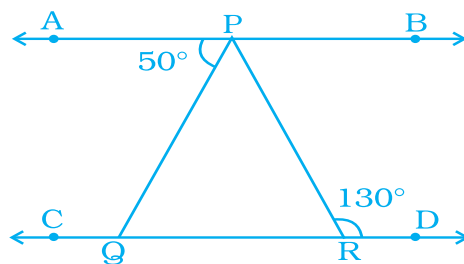


Fig. 5.11

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11. In Fig. 5.12, lines  $l$  and  $m$  intersect each other at a point. Which of the following is false?

- (a)  $\angle a = \angle b$  (b)  $\angle d = \angle c$   
(c)  $\angle a + \angle d = 180^\circ$  (d)  $\angle a = \angle d$

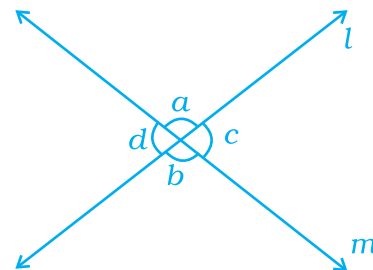


Fig. 5.12

12. If angle P and angle Q are supplementary and the measure of angle P is  $60^\circ$ , then the measure of angle Q is

- (a)  $120^\circ$  (b)  $60^\circ$   
(c)  $30^\circ$  (d)  $20^\circ$

13. In Fig. 5.13, POR is a line. The value of  $a$  is

- (a)  $40^\circ$  (b)  $45^\circ$   
(c)  $55^\circ$  (d)  $60^\circ$

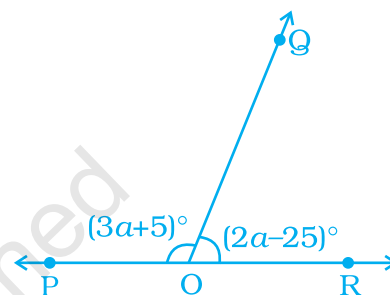


Fig. 5.13

14. In Fig. 5.14, POQ is a line. If  $x = 30^\circ$ , then  $\angle QOR$  is

- (a)  $90^\circ$  (b)  $30^\circ$   
(c)  $150^\circ$  (d)  $60^\circ$

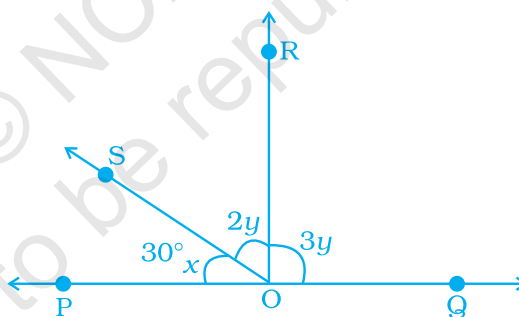


Fig. 5.14

## PROPERTIES OF TRANSVERSALS TO PARALLEL LINES

If two parallel lines are intersected by a transversal, corresponding angles are congruent,

- alternate interior angles are congruent
- and alternate exterior angles are congruent.

If the transversal is perpendicular to the parallel lines, all of the angles formed are congruent to  $90^\circ$  angles.

15. The measure of an angle which is four times its supplement is

(a)  $36^\circ$  (b)  $144^\circ$   
(c)  $16^\circ$  (d)  $64^\circ$

16. In Fig. 5.15, the value of  $y$  is

(a)  $30^\circ$  (b)  $15^\circ$   
(c)  $20^\circ$  (d)  $22.5^\circ$

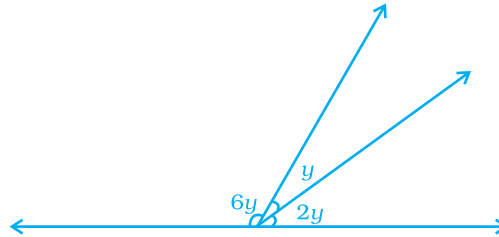


Fig. 5.15

17. In Fig. 5.16,  $PA \parallel BC \parallel DT$  and  $AB \parallel DC$ . Then, the values of  $a$  and  $b$  are respectively.

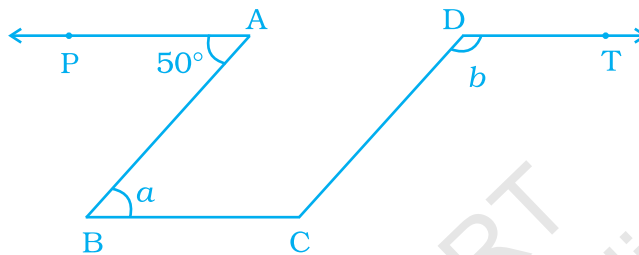


Fig. 5.16

- (a)  $60^\circ, 120^\circ$  (b)  $50^\circ, 130^\circ$  (c)  $70^\circ, 110^\circ$  (d)  $80^\circ, 100^\circ$
18. The difference of two complementary angles is  $30^\circ$ . Then, the angles are
- (a)  $60^\circ, 30^\circ$  (b)  $70^\circ, 40^\circ$   
(c)  $20^\circ, 50^\circ$  (d)  $105^\circ, 75^\circ$
19. In Fig. 5.17,  $PQ \parallel SR$  and  $SP \parallel RQ$ . Then, angles  $a$  and  $b$  are respectively
- (a)  $20^\circ, 50^\circ$  (b)  $50^\circ, 20^\circ$   
(c)  $30^\circ, 50^\circ$  (d)  $45^\circ, 35^\circ$

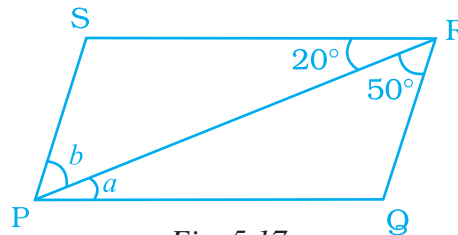


Fig. 5.17

20. In Fig. 5.18,  $a$  and  $b$  are
- (a) alternate exterior angles  
(b) corresponding angles  
(c) alternate interior angles  
(d) vertically opposite angles

21. If two supplementary angles are in the ratio  $1 : 2$ , then the bigger angle is

(a)  $120^\circ$  (b)  $125^\circ$   
(c)  $110^\circ$  (d)  $90^\circ$

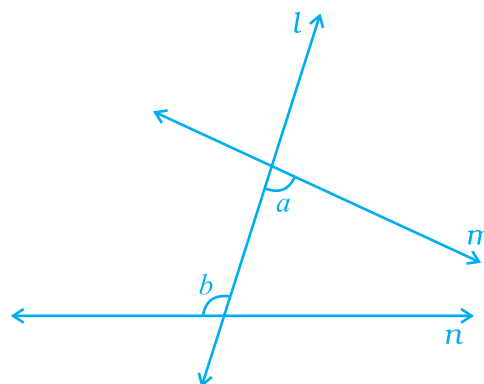


Fig. 5.18

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22. In Fig. 5.19,  $\angle ROS$  is a right angle and  $\angle POR$  and  $\angle QOS$  are in the ratio 1 : 5. Then,  $\angle QOS$  measures

- (a)  $150^\circ$                       (b)  $75^\circ$   
(c)  $45^\circ$                         (d)  $60^\circ$

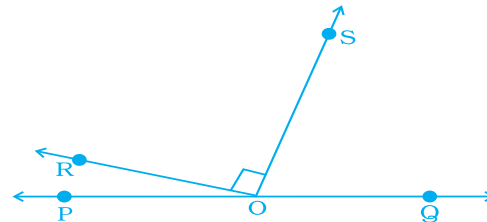


Fig. 5.19

23. Statements **a** and **b** are as given below:

- a** : If two lines intersect, then the vertically opposite angles are equal.  
**b** : If a transversal intersects, two other lines, then the sum of two interior angles on the same side of the transversal is  $180^\circ$ .

Then

- (a) Both **a** and **b** are true                      (b) **a** is true and **b** is false  
(c) **a** is false and **b** is true                      (d) both **a** and **b** are false

24. For Fig. 5.20, statements **p** and **q** are given below:

**p** :  $a$  and  $b$  are forming a linear pair.

**q** :  $a$  and  $b$  are forming a pair of adjacent angles.

Then,

- (a) both **p** and **q** are true  
(b) **p** is true and **q** is false  
(c) **p** is false and **q** is true  
(d) both **p** and **q** are false

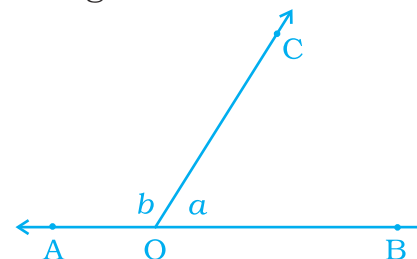
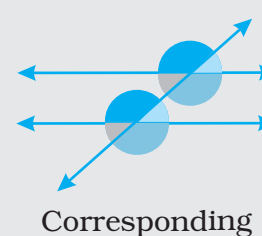
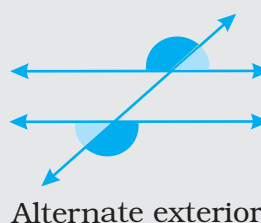
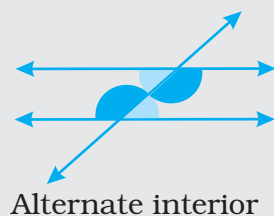


Fig. 5.20

**A transversal** is a line that intersects two or more lines that lie in the same plane in distinct points. Transversals to parallel lines form angles with special properties.



25. In Fig. 5.21,  $\angle AOC$  and  $\angle BOC$  form a pair of

- (a) vertically opposite angles
- (b) complementary angles
- (c) alternate interior angles
- (d) supplementary angles

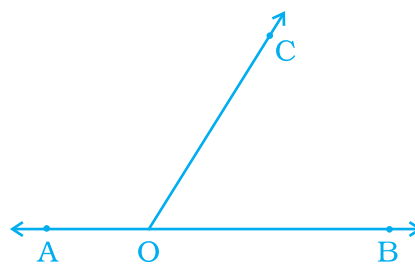


Fig. 5.21

26. In Fig. 5.22, the value of  $a$  is

- (a)  $20^\circ$
- (b)  $15^\circ$
- (c)  $5^\circ$
- (d)  $10^\circ$

27. In Fig. 5.23, if  $QP \parallel SR$ , the value of  $a$  is

- (a)  $40^\circ$
- (b)  $30^\circ$
- (c)  $90^\circ$
- (d)  $80^\circ$

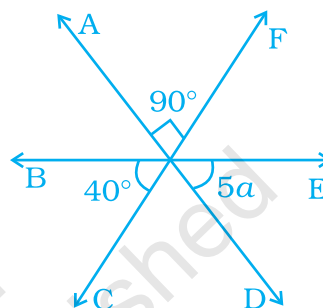


Fig. 5.22

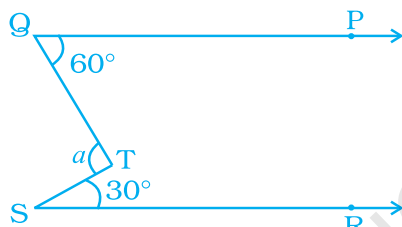


Fig. 5.23

28. In which of the following figures,  $a$  and  $b$  are forming a pair of adjacent angles?

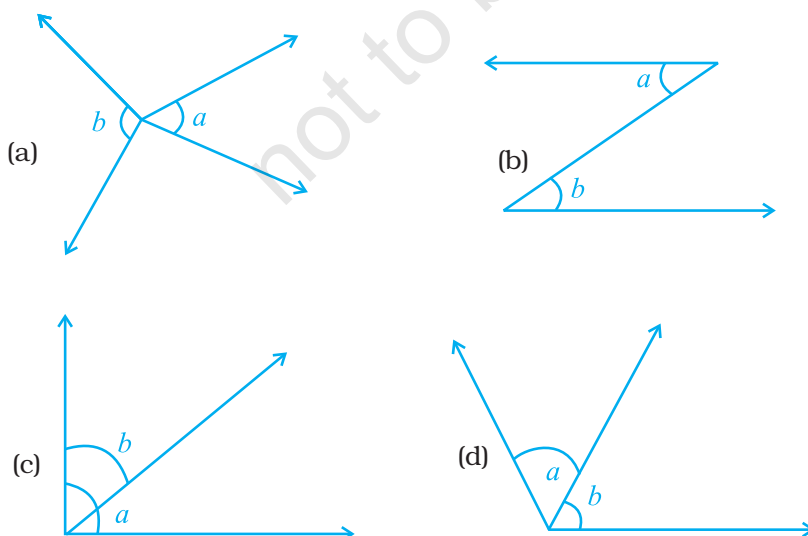


Fig. 5.24

## MATHEMATICS

## Think and Discuss

- Tell** how many different angles would be formed by a transversal intersecting three parallel lines. How many different angle measures would there be?
- Explain** how a transversal could intersect two other lines so that corresponding angles are not congruent.

**29.** In a pair of adjacent angles, (i) vertex is always common, (ii) one arm is always common, and (iii) uncommon arms are always opposite rays

Then

- All (i), (ii) and (iii) are true
- (iii) is false
- (i) is false but (ii) and (iii) are true
- (ii) is false

**30.** In Fig. 5.25, lines PQ and ST intersect at O. If  $\angle POR = 90^\circ$  and  $x : y = 3 : 2$ , then  $z$  is equal to

- $126^\circ$
- $144^\circ$
- $136^\circ$
- $154^\circ$

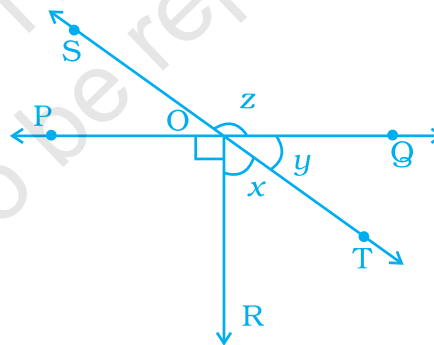
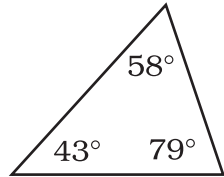
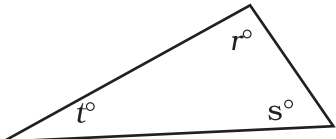


Fig. 5.25

## TRIANGLE SUM THEOREM

| Words                                                 | Numbers                                                                                                                           | Algebra                                                                                                                         |
|-------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| The angle measures of a triangle add to $180^\circ$ . |  $43^\circ + 58^\circ + 79^\circ = 180^\circ$ |  $r^\circ + s^\circ + t^\circ = 180^\circ$ |

31. In Fig. 5.26, POQ is a line, then  $a$  is equal to

(a)  $35^\circ$  (b)  $100^\circ$   
(c)  $80^\circ$  (d)  $135^\circ$

32. Vertically opposite angles are always

(a) supplementary  
(b) complementary  
(c) adjacent  
(d) equal

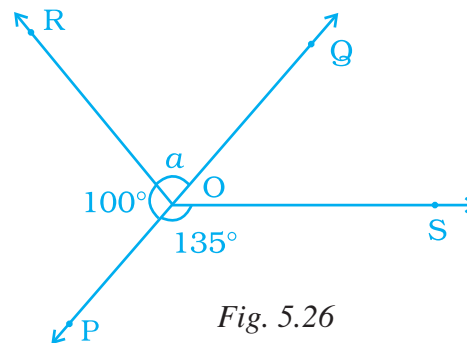


Fig. 5.26

33. In Fig. 5.27,  $a = 40^\circ$ . The value of  $b$  is

(a)  $20^\circ$  (b)  $24^\circ$   
(c)  $36^\circ$  (d)  $120^\circ$

34. If an angle is  $60^\circ$  less than two times of its supplement, then the greater angle is

(a)  $100^\circ$  (b)  $80^\circ$   
(c)  $60^\circ$  (d)  $120^\circ$

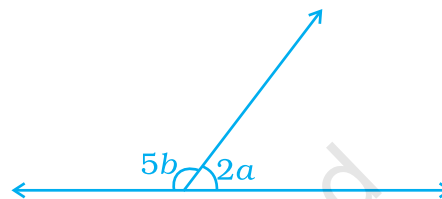


Fig. 5.27

35. In Fig. 5.28,  $PQ \parallel RS$ .

If  $\angle 1 = (2a + b)^\circ$  and  $\angle 6 = (3a - b)^\circ$ , then the measure of  $\angle 2$  in terms of  $b$  is

(a)  $(2 + b)^\circ$  (b)  $(3 - b)^\circ$   
(c)  $(108 - b)^\circ$  (d)  $(180 - b)^\circ$

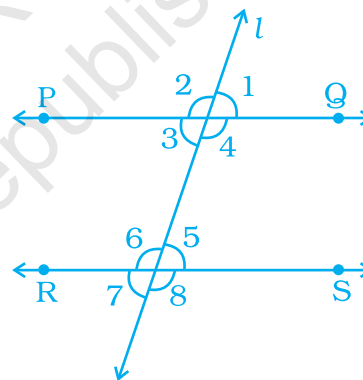


Fig. 5.28

36. In Fig. 5.29,  $PQ \parallel RS$  and  $a : b = 3 : 2$ .

Then,  $f$  is equal to

(a)  $36^\circ$  (b)  $108^\circ$   
(c)  $72^\circ$  (d)  $144^\circ$

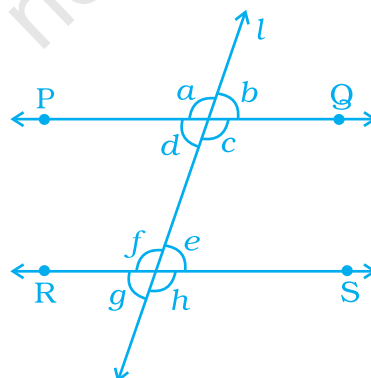


Fig. 5.29

## MATHEMATICS

An **acute triangle** has 3 acute angles. A **right triangle** has 1 right angle, An **obtuse triangle** has 1 obtuse angle.

An **equilateral triangle** has 3 congruent sides and 3 congruent angles. An **isosceles triangle** has at least 2 congruent sides and 2 congruent angles. A **scalene triangle** has no congruent sides and no congruent angles.

**37.** In Fig. 5.30, line  $l$  intersects two parallel lines  $PQ$  and  $RS$ . Then, which one of the following is not true?

- (a)  $\angle 1 = \angle 3$       (b)  $\angle 2 = \angle 4$   
(c)  $\angle 6 = \angle 7$       (d)  $\angle 4 = \angle 8$

**38.** In Fig. 5.30, which one of the following is not true?

- (a)  $\angle 1 + \angle 5 = 180^\circ$   
(b)  $\angle 2 + \angle 5 = 180^\circ$   
(c)  $\angle 3 + \angle 8 = 180^\circ$   
(d)  $\angle 2 + \angle 3 = 180^\circ$

**39.** In Fig. 5.30, which of the following is true?

- (a)  $\angle 1 = \angle 5$       (b)  $\angle 4 = \angle 8$       (c)  $\angle 5 = \angle 8$       (d)  $\angle 3 = \angle 7$

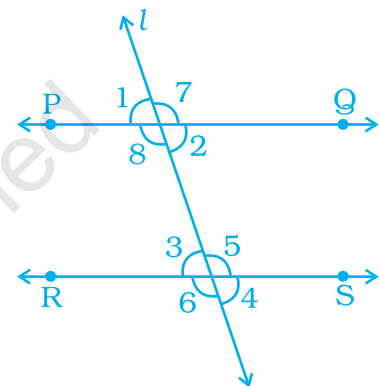


Fig. 5.30

### Think and Discuss

- 1. Explain** whether a right triangle can be equilateral. Can it be isosceles? scalene?
- 2. Explain** whether a triangle can have two right angles. Can it have two obtuse angles?

**40.** In Fig. 5.31,  $PQ \parallel ST$ . Then, the value of  $x + y$  is

- (a)  $125^\circ$       (b)  $135^\circ$   
(c)  $145^\circ$       (d)  $120^\circ$

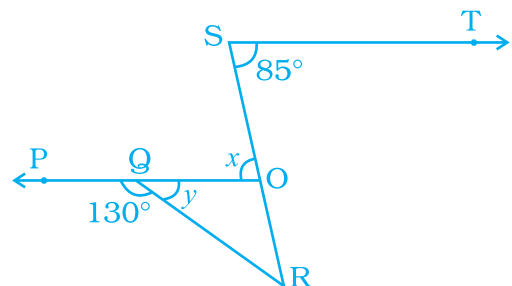


Fig. 5.31

41. In Fig. 5.32, if  $PQ \parallel RS$  and  $QR \parallel TS$ , then the value  $a$  is  
 (a)  $95^\circ$  (b)  $90^\circ$  (c)  $85^\circ$  (d)  $75^\circ$

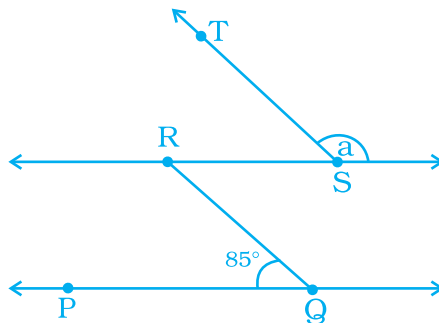


Fig. 5.32

In questions 42 to 56, fill in the blanks to make the statements true.

42. If sum of measures of two angles is  $90^\circ$ , then the angles are \_\_\_\_\_.  
 43. If the sum of measures of two angles is  $180^\circ$ , then they are \_\_\_\_\_.  
 44. A transversal intersects two or more than two lines at \_\_\_\_\_ points.

If a transversal intersects two parallel lines, then (Q. 45 to 48).

45. sum of interior angles on the same side of a transversal is \_\_\_\_\_.  
 46. alternate interior angles have one common \_\_\_\_\_.  
 47. corresponding angles are on the \_\_\_\_\_ side of the transversal.  
 48. alternate interior angles are on the \_\_\_\_\_ side of the transversal.  
 49. Two lines in a plane which do not meet at a point anywhere are called \_\_\_\_\_ lines.  
 50. Two angles forming a \_\_\_\_\_ pair are supplementary.  
 51. The supplement of an acute is always \_\_\_\_\_ angle.  
 52. The supplement of a right angle is always \_\_\_\_\_ angle.  
 53. The supplement of an obtuse angle is always \_\_\_\_\_ angle.  
 54. In a pair of complementary angles, each angle cannot be more than \_\_\_\_\_  $90^\circ$ .  
 55. An angle is  $45^\circ$ . Its complementary angle will be \_\_\_\_\_.  
 56. An angle which is half of its supplement is of \_\_\_\_\_.

In questions 57 to 71, state whether the statements are True or False.

57. Two right angles are complementary to each other.

## MATHEMATICS

58. One obtuse angle and one acute angle can make a pair of complementary angles.
59. Two supplementary angles are always obtuse angles.
60. Two right angles are always supplementary to each other.
61. One obtuse angle and one acute angle can make a pair of supplementary angles.
62. Both angles of a pair of supplementary angles can never be acute angles.
63. Two supplementary angles always form a linear pair.
64. Two angles making a linear pair are always supplementary.
65. Two angles making a linear pair are always adjacent angles.
66. Vertically opposite angles form a linear pair.
67. Interior angles on the same side of a transversal with two distinct parallel lines are complementary angles.
68. Vertically opposite angles are either both acute angles or both obtuse angles.
69. A linear pair may have two acute angles.
70. An angle is more than  $45^\circ$ . Its complementary angle must be less than  $45^\circ$ .
71. Two adjacent angles always form a linear pair.
72. Write down each pair of adjacent angles shown in the following figures:

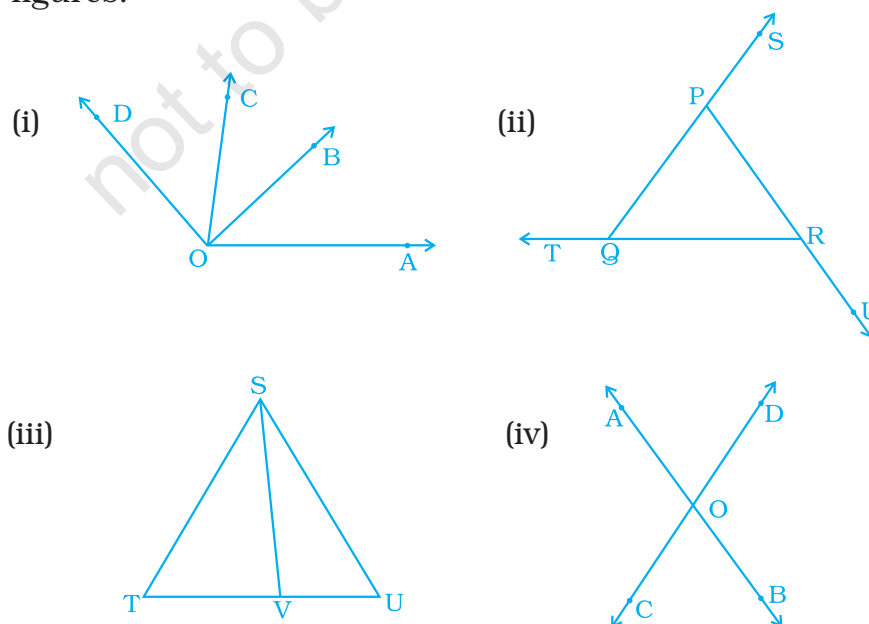


Fig. 5.33

- 73.** In each of the following figures, write, if any, (i) each pair of vertically opposite angles, and (ii) each linear pair.

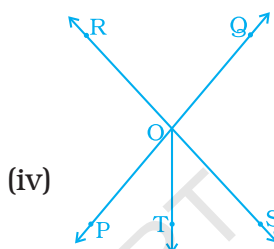
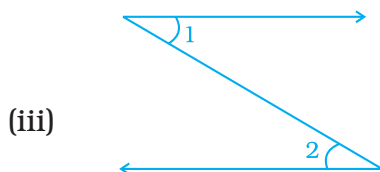
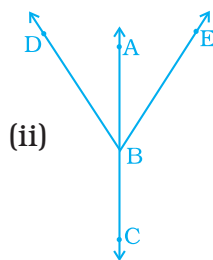
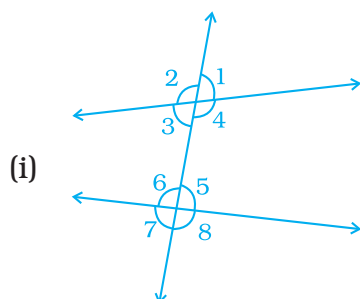


Fig. 5.34

- 74.** Name the pairs of supplementary angles in the following figures:

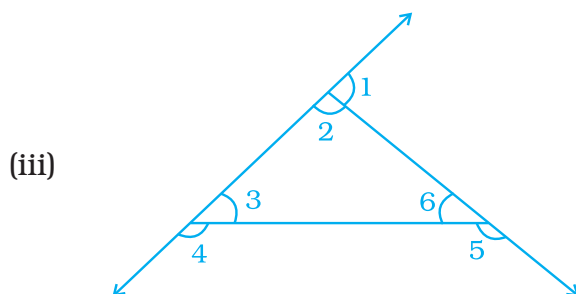
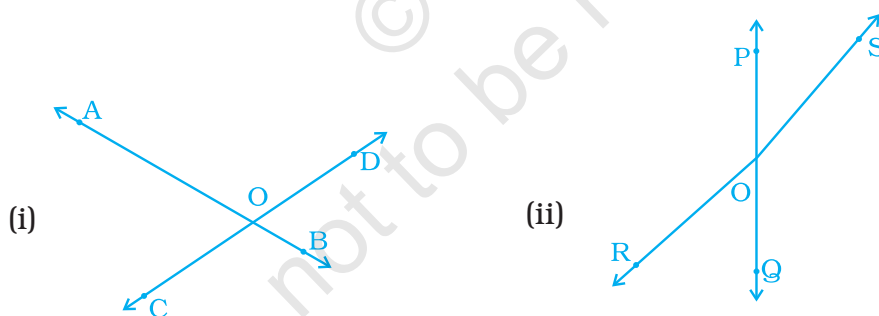


Fig. 5.35

## MATHEMATICS

75. In Fig. 5.36,  $PQ \parallel RS$ ,  $TR \parallel QU$  and  $\angle PTR = 42^\circ$ . Find  $\angle QUR$ .

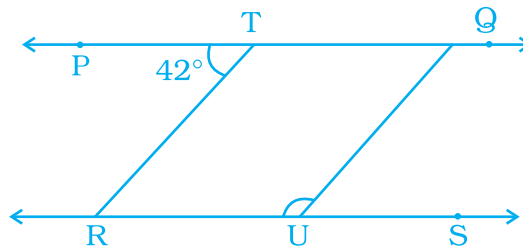
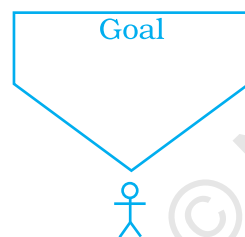
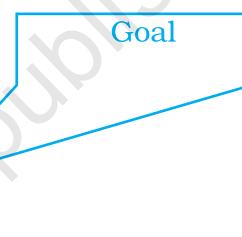


Fig. 5.36

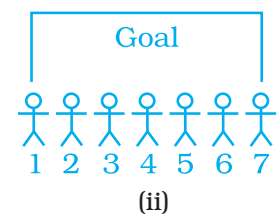
76. The drawings below (Fig. 5.37), show angles formed by the goalposts at different positions of a football player. The greater the angle, the better chance the player has of scoring a goal. For example, the player has a better chance of scoring a goal from Position A than from Position B.



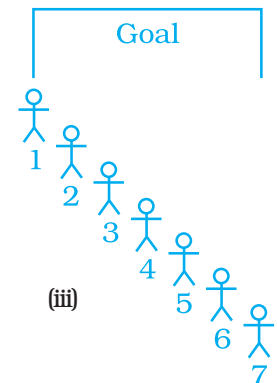
(i) Player in Position A



Player in Position B



(ii)



(iii)

Fig. 5.37

In Parts (a) and (b) given below it may help to trace the diagrams and draw and measure angles.

- Seven football players are practicing their kicks. They are lined up in a straight line in front of the goalpost [Fig.(ii)]. Which player has the best (the greatest) kicking angle?
  - Now the players are lined up as shown in Fig. (iii). Which player has the best kicking angle?
  - Estimate atleast two situations such that the angles formed by different positions of two players are complement to each other.
77. The sum of two vertically opposite angles is  $166^\circ$ . Find each of the angles.

78. In Fig. 5.38,  $l \parallel m \parallel n$ .

$\angle QPS = 35^\circ$  and  $\angle QRT = 55^\circ$ . Find  $\angle PQR$ .

79. In Fig. 5.39, P, Q and R are collinear points and  $TQ \perp PR$ ,

Name; (a) pair of complementary angles

(b) two pairs of supplementary angles.

(c) four pairs of adjacent angles.

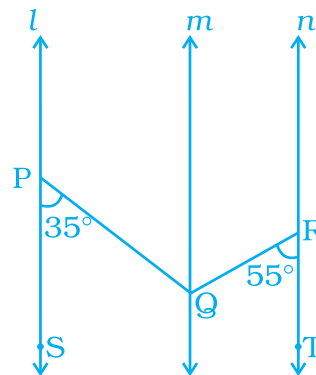


Fig. 5.38

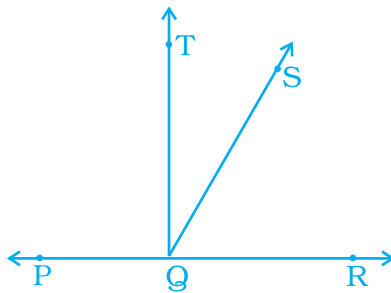


Fig. 5.39

80. In Fig. 5.40,  $OR \perp OP$ .

(i) Name all the pairs of adjacent angles.

(ii) Name all the pairs of complementary angles.

81. If two angles have a common vertex and their arms form opposite rays (Fig. 5.41), Then,

(a) how many angles are formed?

(b) how many types of angles are formed?

(c) write all the pairs of vertically opposite angles.

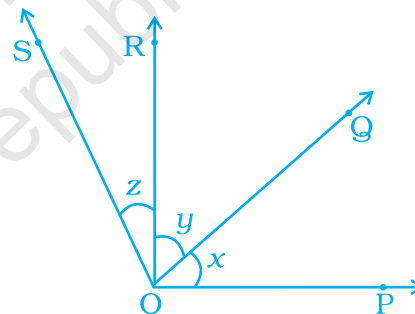


Fig. 5.40

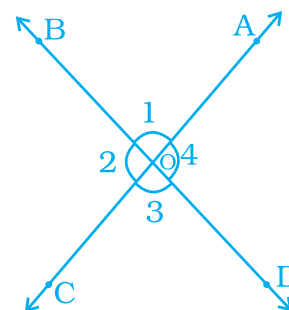


Fig. 5.41

82. In (Fig 5.42) are the following pairs of angles adjacent? Justify your answer.

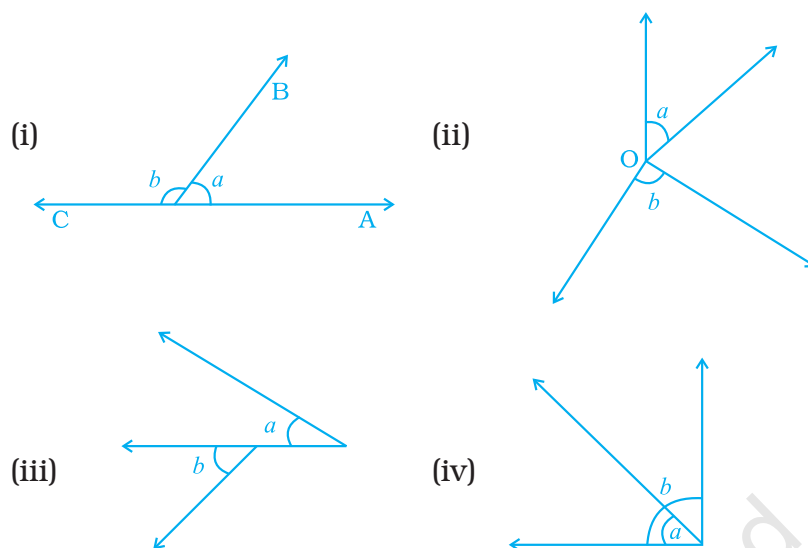


Fig. 5.42

**83.** In Fig. 5.43, write all the pairs of supplementary angles.

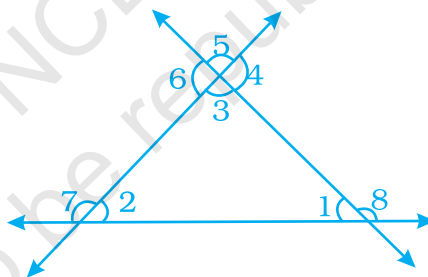


Fig. 5.43

- 84.** What is the type of other angle of a linear pair if
- one of its angle is acute?
  - one of its angles is obtuse?
  - one of its angles is right?
- 85.** Can two acute angles form a pair of supplementary angles? Give reason in support of your answer.
- 86.** Two lines AB and CD intersect at O (Fig. 5.44). Write all the pairs of adjacent angles by taking angles 1, 2, 3, and 4 only.

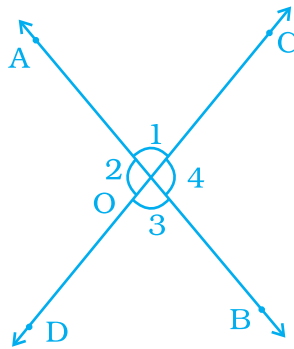
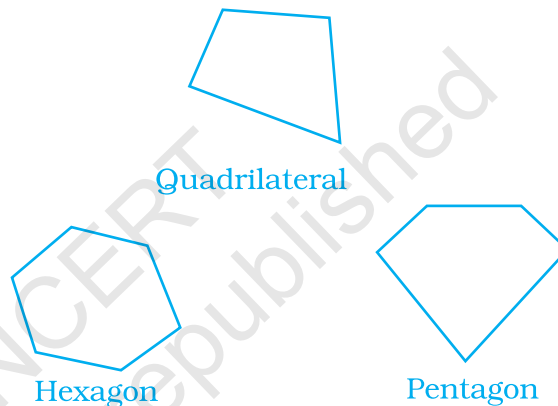


Fig. 5.44

| Polygon       | Number of Sides |
|---------------|-----------------|
| Triangle      | 3               |
| Quadrilateral | 4               |
| Pentagon      | 5               |
| Hexagon       | 6               |
| Heptagon      | 7               |
| Octagon       | 8               |
| $n$ -gon      | $n$             |



87. If the complement of an angle is  $62^\circ$ , then find its supplement.
88. A road crosses a railway line at an angle of  $30^\circ$  as shown in Fig. 5.45. Find the values of  $a$ ,  $b$  and  $c$ .

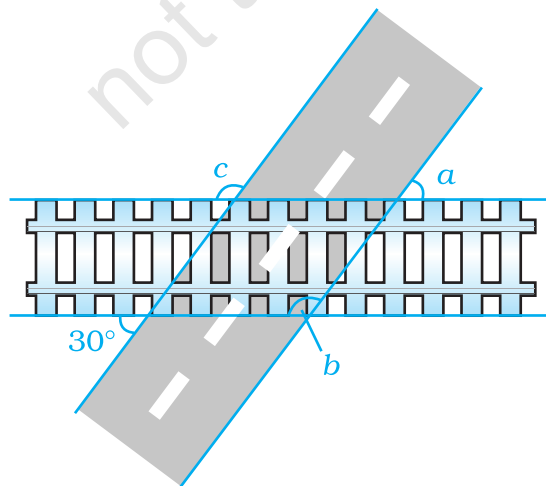


Fig. 5.45

## MATHEMATICS

89. The legs of a stool make an angle of  $35^\circ$  with the floor as shown in Fig. 5.46. Find the angles  $x$  and  $y$ .

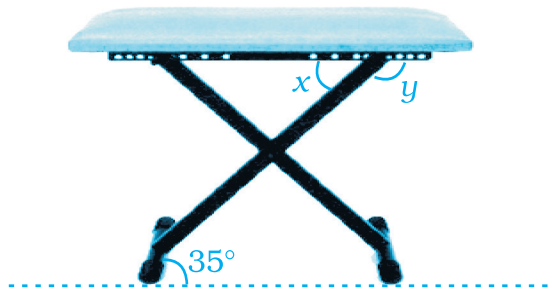


Fig. 5.46

90. Iron rods  $a, b, c, d, e$  and  $f$  are making a design in a bridge as shown in Fig. 5.47, in which  $a \parallel b, c \parallel d, e \parallel f$ . Find the marked angles between
- $b$  and  $c$
  - $d$  and  $e$
  - $d$  and  $f$
  - $c$  and  $f$

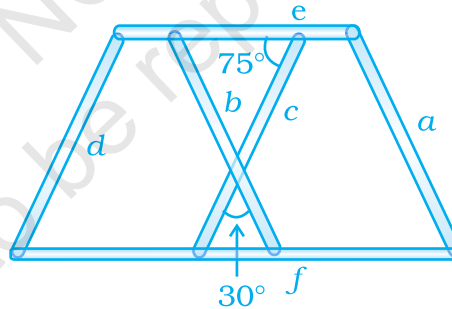


Fig. 5.47

91. Amisha makes a star with the help of line segments  $a, b, c, d, e$  and  $f$ , in which  $a \parallel d, b \parallel e$  and  $c \parallel f$ . Chhaya marks an angle as  $120^\circ$  as shown in Fig. 5.48 and asks Amisha to find the  $\angle x, \angle y$  and  $\angle z$ . Help Amisha in finding the angles.

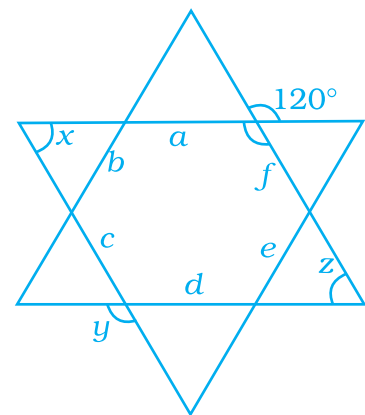


Fig. 5.48

92. In Fig. 5.49,  $AB \parallel CD$ ,  $AF \parallel ED$ ,  $\angle AFC = 68^\circ$  and  $\angle FED = 42^\circ$ . Find  $\angle EFD$ .

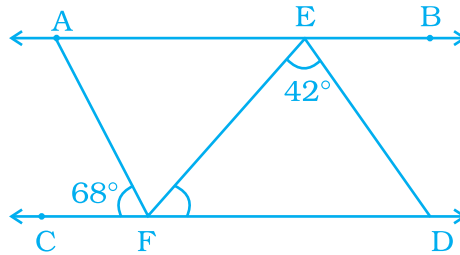


Fig. 5.49

93. In Fig. 5.50, OB is perpendicular to OA and  $\angle BOC = 49^\circ$ . Find  $\angle AOD$ .

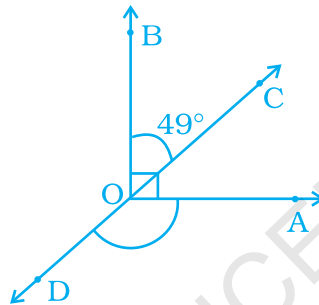


Fig. 5.50

94. Three lines AB, CD and EF intersect each other at O. If  $\angle AOE = 30^\circ$  and  $\angle DOB = 40^\circ$  (Fig. 5.51), find  $\angle COF$ .

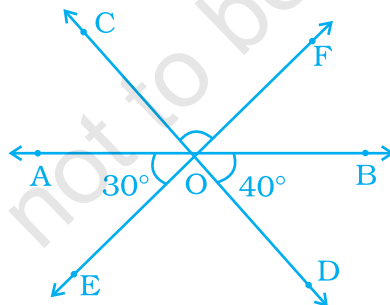


Fig. 5.51

95. Measures (in degrees) of two complementary angles are two consecutive even integers. Find the angles.
96. If a transversal intersects two parallel lines, and the difference of two interior angles on the same side of a transversal is  $20^\circ$ , find the angles.

## MATHEMATICS

97. Two angles are making a linear pair. If one of them is one-third of the other, find the angles.
98. Measures (in degrees) of two supplementary angles are consecutive odd integers. Find the angles.
99. In Fig. 5.52,  $AE \parallel GF \parallel BD$ ,  $AB \parallel CG \parallel DF$  and  $\angle CHE = 120^\circ$ . Find  $\angle ABC$  and  $\angle CDE$ .

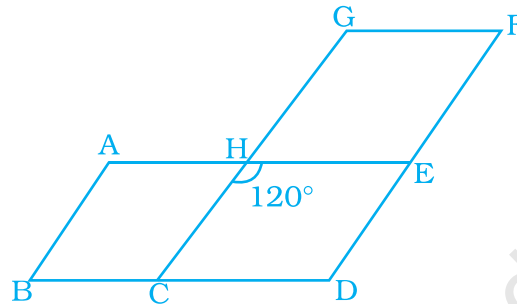


Fig. 5.52

100. In Fig. 5.53, find the value of  $\angle BOC$ , if points A, O and B are collinear.

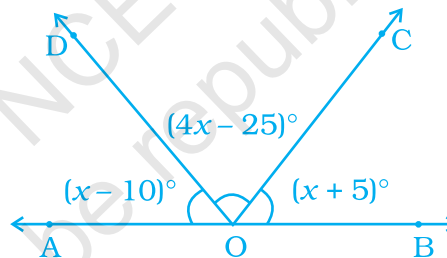


Fig. 5.53

101. In Fig. 5.54, if  $l \parallel m$ , find the values of  $a$  and  $b$ .

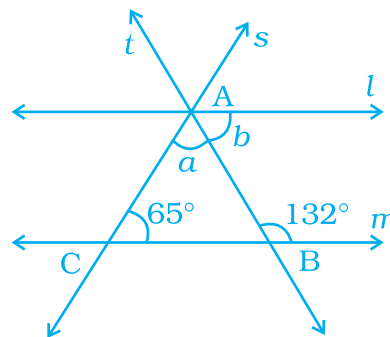


Fig. 5.54

- 102.** In Fig. 5.55,  $l \parallel m$  and a line  $t$  intersects these lines at P and Q, respectively. Find the sum  $2a + b$ .

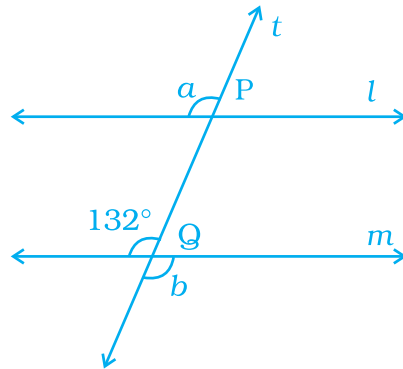


Fig. 5.55

- 103.** In Fig. 5.56,  $QP \parallel RS$ . Find the values of  $a$  and  $b$ .

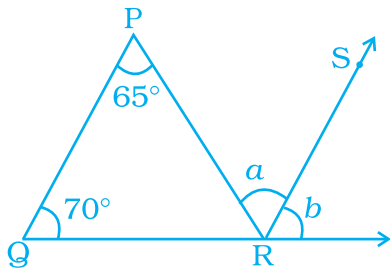


Fig. 5.56

- 104.** In Fig. 5.57,  $PQ \parallel RT$ . Find the value of  $a + b$ .

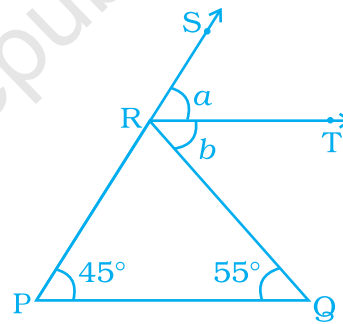


Fig. 5.57

- 105.** In Fig 5.58, PQ, RS and UT are parallel lines.

(i) If  $c = 57^\circ$  and  $a = \frac{c}{3}$ , find the value of  $d$ .

(ii) If  $c = 75^\circ$  and  $a = \frac{2}{5}c$ , find  $b$ .

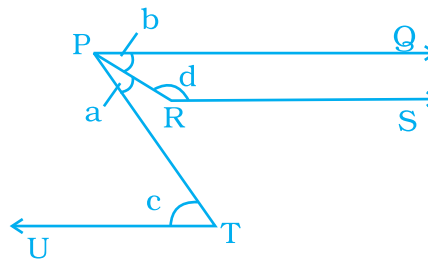


Fig. 5.58

## MATHEMATICS

106. In Fig. 5.59,  $AB \parallel CD$ . Find the reflex  $\angle EFG$ .

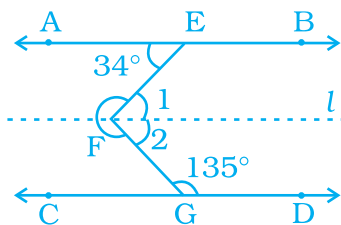
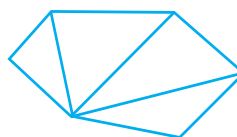


Fig. 5.59

**Look for a pattern** between the number of sides and the number of triangles.



Heptagon  
7 sides  
5 triangles



Hexagon  
6 sides  
4 triangles

107. In Fig. 5.60, two parallel lines  $l$  and  $m$  are cut by two transversals  $n$  and  $p$ . Find the values of  $x$  and  $y$ .

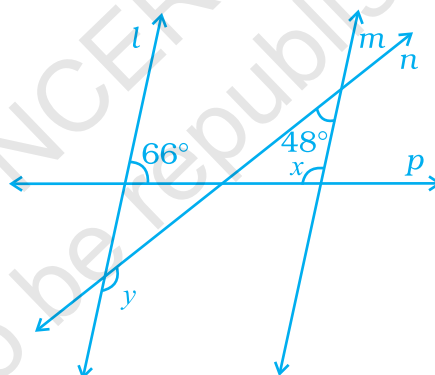


Fig. 5.60

108. In Fig. 5.61,  $l$ ,  $m$  and  $n$  are parallel lines, and the lines  $p$  and  $q$  are also parallel. Find the values of  $a$ ,  $b$  and  $c$ .

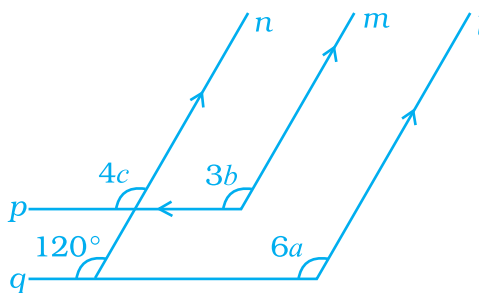


Fig. 5.61

**109.** In Fig. 5.62, state which pair of lines are parallel. Give reason.

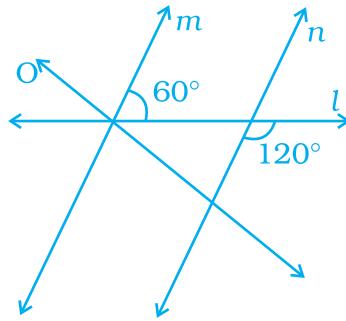


Fig. 5.62

**110.** In Fig. 5.63, examine whether the following pairs of lines are parallel or not:

(i) EF and GH

(ii) AB and CD

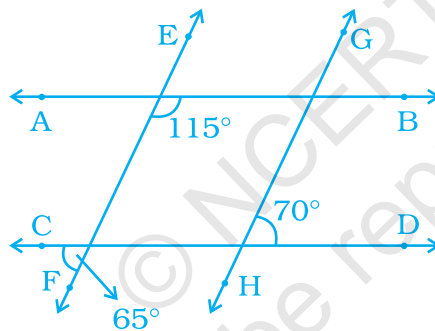


Fig. 5.63

**111.** In Fig. 5.64, find out which pair of lines are parallel:

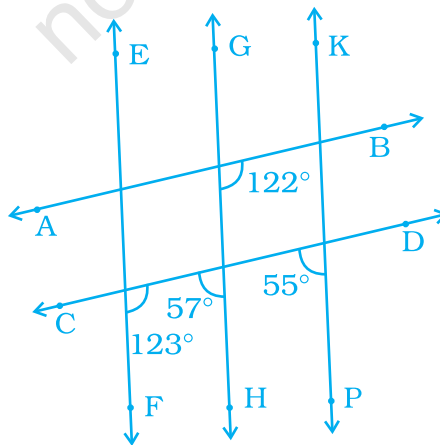


Fig. 5.64

## MATHEMATICS

**112.** In Fig. 5.65, show that

- (i)  $AB \parallel CD$
- (ii)  $EF \parallel GH$

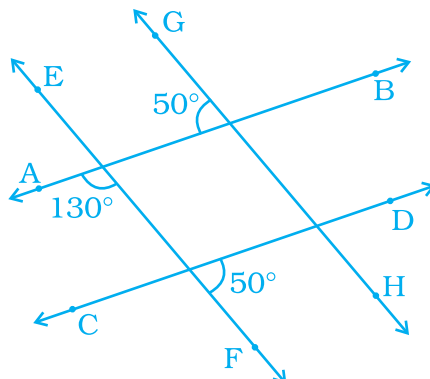


Fig. 5.65

**113.** In Fig. 5.66, two parallel lines  $l$  and  $m$  are cut by two transversals  $p$  and  $q$ . Determine the values of  $x$  and  $y$ .

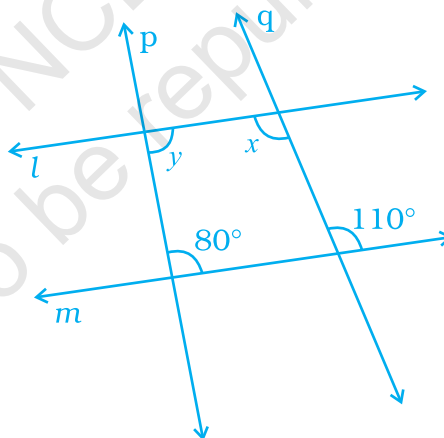


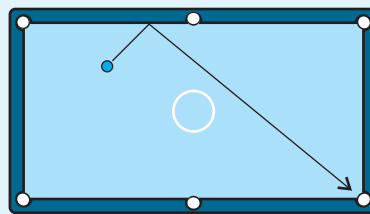
Fig. 5.66

### (D) Applications

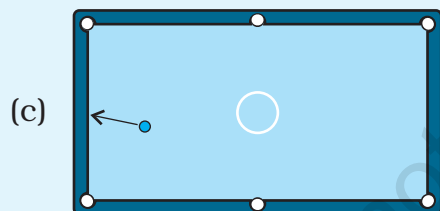
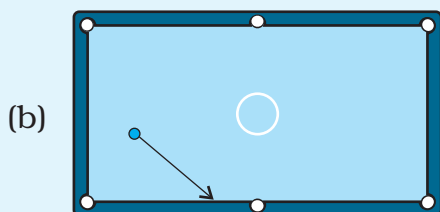
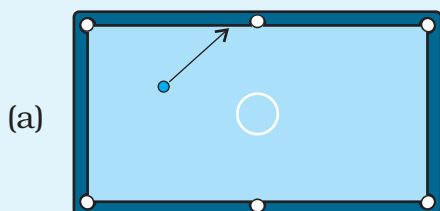
- The game pool belongs to billiard sports and generally played with a cue stick which is used to strike billiard balls, moving them around a cloth-covered billiards table with six pocket bounded by rubber cushions.

The angle at which a pool ball hits the side of a table has the same

measure as the angle at which it bounces off the side. This is shown in the drawing at the right. The marked angles have the same measure, and the arrow shows the ball's path.

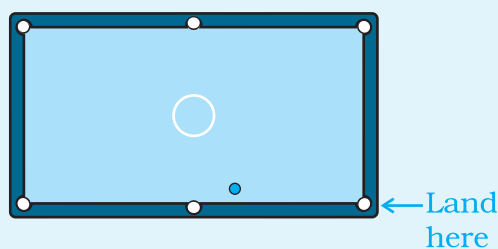


In Parts (a)–(c), trace the drawing. Then use your protractor to find the path the ball will take when it bounces off the side. Tell whether the ball will go into a pocket or hit another side. (Draw just one bounce.)



*When light hits a mirror, it behaves in the same way as a pool ball hitting the side of a table. If light hits a mirror at an angle, it bounces off at the same angle. In physics, this law is often stated as “the angle of incidence = the angle of reflection.”*

- (d) Try to trace this drawing. Draw a path for which the ball will bounce off a side and land in the lower-right pocket.



**MATHEMATICS****2. Crossword Puzzle**

Fill the crossword puzzle with the help of following clues:

**Across**

- Two lines in a plane which do not intersect each other.
- A pair of adjacent angles having their non common arms opposite rays.
- A pair of angles having a common vertex, a common arm and their interiors do not overlap.
- The two lines are intersected by a line at distinct points.
- The sum of two angles is  $90^\circ$ .

**Down**

- Sum of two angles is  $180^\circ$ .
- The two lines in a plane intersect each other at one and only one point are called \_\_\_\_\_.
- When two parallel lines intersected by a transversal at two distinct points then the \_\_\_\_\_ angles are equal.

