

## SAMPLE PAPER: MATHEMATICS

### CLASS-XII: 2014-15

#### TYPOLOGY

	VSA (1 M)	LA-I (4 M)	LA-II (6 M)	100
Remembering	2, 5	11, 15, 19	24	20
Understanding	1, 4	8, 12	23	16
Applications	6	14, 18, 13	21, 26	25
HOTS	3	10, 17	20, 22	21
Evaluation & MD	-	7, 9, 16	25	18

**SECTION-A**

Question number 1 to 6 carry 1 mark each.

1. The position vectors of points A and B are  $\vec{a}$  and  $\vec{b}$  respectively.  
P divides AB in the ratio 3 : 1 and Q is mid-point of AP. Find the position vector of Q. 1
2. Find the area of the parallelogram, whose diagonals are  $\vec{d}_1=5\hat{i}$  and  $\vec{d}_2=2\hat{j}$  1
3. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane. 1
4. If  $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$ , Write the cofactor of  $a_{32}$  (the element of third row and 2<sup>nd</sup> column). 1
5. If m and n are the order and degree, respectively of the differential equation  $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$ , then write the value of m+n. 1
6. Write the differential equation representing the curve  $y^2 = 4ax$ , where  $a$  is an arbitrary constant. 1

**SECTION-B**

Question numbers 7 to 19 carry 4 marks each.

7. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at the rate of Rs. 20, Rs.15 and Rs. 5 per unit respectively. School A sold 25 paper-bags 12 scrap-books and 34 pastel sheets. School B sold 22 paper-bags, 15 scrapbooks and 28 pastel-sheets while school C sold 26 paper-bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school.  
By such exhibition, which values are inculcated in the students? 4
8. Let  $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$ , then show that  $A^2 - 4A + 7I = O$ .

Using this result calculate  $A^3$  also.

**OR**

If  $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix}$ , find  $A^{-1}$ , using elementary row operations. 4

9. If  $x, y, z$  are in GP, then using properties of determinants, show that

$$\begin{vmatrix} px + y & x & y \\ py + z & y & z \\ 0 & px + y & py + z \end{vmatrix} = 0, \text{ where } x \neq y \neq z \text{ and } p \text{ is any real number.} \quad 4$$

10. Evaluate :  $\int_{-1}^1 |x \cos \pi x| dx$ . 4

11. Evaluate :  $\int \frac{1+\sin 2x}{1+\cos 2x} \cdot e^{2x} dx$ . 4

**OR**

Evaluate :  $\int \frac{x^4}{(x-1)(x^2+1)} dx$

12. Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 3' given that 'there is at least one head'. 4

**OR**

How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

13. For three vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  if  $\vec{a} \times \vec{b} = \vec{c}$  and  $\vec{a} \times \vec{c} = \vec{b}$ , then prove that  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are mutually perpendicular vectors,  $|\vec{b}| = |\vec{a}|$  and  $|\vec{a}| = 1$  4

14. Find the equation of the line through the point  $(1,-1,1)$  and perpendicular to the lines joining the points  $(4,3,2)$ ,  $(1,-1,0)$  and  $(1,2,-1)$ ,  $(2,1,1)$  4

**OR**

Find the position vector of the foot of perpendicular drawn from the point P(1,8,4) to the line joining A(O,-1,3) and B(5,4,4). Also find the length of this perpendicular.

15. Solve for  $x$ :  $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$

OR

Prove that:  $2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$  4

16. If  $x = \sin t$ ,  $y = \sin kt$ , show that

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + k^2y = 0$$
 4

17. If  $y^x + x^y + x^x = a^b$ , find  $\frac{dy}{dx}$  4

18. It is given that for the function  $f(x) = x^3 + bx^2 + ax + 5$  on  $[1, 3]$ , Rolle's theorem holds with  $c = 2 + \frac{1}{\sqrt{3}}$ .

Find values of  $a$  and  $b$ . 4

19. Evaluate :  $\int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$  4

### SECTION-C

Question numbers 20 to 26 carry 6 marks each.

20. Let  $A = \{1, 2, 3, \dots, 9\}$  and  $R$  be the relation in  $A \times A$  defined by  $(a, b) R (c, d)$  if  $a+d = b+c$  for  $a, b, c, d \in A$ .

Prove that  $R$  is an equivalence relation. Also obtain the equivalence class  $[(2, 5)]$ . 6

OR

Let  $f: \mathbb{N} \rightarrow \mathbb{R}$  be a function defined as  $f(x) = 4x^2 + 12x + 15$ .

Show that  $f: \mathbb{N} \rightarrow S$  is invertible, where  $S$  is the range of  $f$ . Hence find inverse of  $f$ .

21. Compute, using integration, the area bounded by the lines

$$x+2y = 2, \quad y-x=1 \quad \text{and} \quad 2x+y=7$$
 6

22. Find the particular solution of the differential equation 6

$$xe^{\frac{y}{x}} - y \sin\left(\frac{y}{x}\right) + x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) = 0, \text{ given that}$$

$$y = 0, \text{ when } x = 1$$

OR

Obtain the differential equation of all circles of radius  $r$ .

23. Show that the lines  $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda (-3\hat{i} + \hat{j} + 5\hat{k})$  and  $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu (-\hat{i} + 2\hat{j} + 5\hat{k})$  are coplanar. Also, find the equation of the plane containing these lines. 6
24. 40% students of a college reside in hostel and the remaining reside outside. At the end of year, 50% of the hosteliars got A grade while from outside students, only 30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hostelier? 6
25. A man rides his motorcycle at the speed of 50km/h. He has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 80km/h, the petrol cost increases to Rs. 3 per km. He has atmost Rs. 120 to spend on petrol and one hour's time. Using LPP find the maximum distance he can travel. 6
26. A jet of enemy is flying along the curve  $y = x^2+2$  and a soldier is placed at the point (3, 2). Find the minimum distance between the soldier and the jet. 6

**MARKING SCHEME****SAMPLE PAPER****SECTION-A**

1.  $\frac{1}{8} (5\vec{a} + 3\vec{b})$  1
2. 5 sq. units 1
3.  $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 29$  1
4. -14 1
5.  $m + n = 4$  1
6.  $2x \frac{dy}{dx} - y = 0$  1

**SECTION-B**

7. Sale matrix for A, B and C is  $\begin{pmatrix} 25 & 12 & 34 \\ 22 & 15 & 28 \\ 26 & 18 & 36 \end{pmatrix}$  ½

Price matrix is  $\begin{pmatrix} 20 \\ 15 \\ 5 \end{pmatrix}$  ½

$$\therefore \begin{pmatrix} 25 & 12 & 34 \\ 22 & 15 & 28 \\ 26 & 18 & 36 \end{pmatrix} \begin{pmatrix} 20 \\ 15 \\ 5 \end{pmatrix} = \begin{pmatrix} 500 + 180 + 170 \\ 440 + 225 + 140 \\ 520 + 270 + 180 \end{pmatrix} \quad \frac{1}{2}$$

$$\therefore \text{Amount raised by} = \begin{pmatrix} 850 \\ 805 \\ 970 \end{pmatrix} \quad \frac{1}{2}$$

School A = Rs 850, school B = Rs 805, school C = Rs 970

**Values**

- Helping the orphans 1
  - Use of recycled paper 1
8.  $A^2 = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 12 \\ -4 & 1 \end{pmatrix}$  1

$$\therefore A^2 - 4A + 7I = \begin{pmatrix} 1 & 12 \\ -4 & 1 \end{pmatrix} + \begin{pmatrix} -8 & -12 \\ 4 & -8 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad 2$$

$$A^2 = 4A - 7I \Rightarrow A^3 = 4A^2 - 7A = 4(4A - 7I) - 7A$$

$$= 9A - 28I = \begin{pmatrix} 18 & 27 \\ -9 & 18 \end{pmatrix} + \begin{pmatrix} -28 & 0 \\ 0 & -28 \end{pmatrix}$$

$$= \begin{pmatrix} -10 & 27 \\ -9 & -10 \end{pmatrix} \quad 1$$

OR

$$\text{Write } A = IA \text{ we get } \begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A \quad \frac{1}{2}$$

$$R_2 \rightarrow R_2 - 2R_1 \Rightarrow \begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad 1$$

$$R_2 \rightarrow R_2 - 3R_3 \Rightarrow \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} A \quad 1$$

$$\begin{array}{l} R_1 \rightarrow R_1 + R_2 \\ R_3 \rightarrow R_3 - 2R_2 \end{array} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix} A \quad 1$$

$$\therefore A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix} \quad \frac{1}{2}$$

$$9. \quad \Delta = \begin{vmatrix} px + y & x & y \\ py + z & y & z \\ 0 & px + y & py + z \end{vmatrix}$$

$$C_1 \rightarrow C_1 - pC_2 - C_3, \Delta = \begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -p^2x - py - py - z & px + y & py + z \end{vmatrix} \quad 1\frac{1}{2}$$

Expanding by  $R_3$ 

$$\Delta = (-p^2x - 2py - z)(xz - y^2) \quad 1$$

Since  $x, y, z$  are in GP,  $\therefore y^2 = xz$  or  $y^2 - xz = 0$  1

$\therefore \Delta = 0$  1/2

10.  $\int_{-1}^1 |x \cdot \cos \pi x| dx = 2 \int_0^1 |x \cos \pi x| dx$  1

$= 2 \int_0^{1/2} (x \cos \pi x) dx + 2 \int_{1/2}^1 -(x \cos \pi x) dx$  1

$= 2 \left[ \frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2} \right]_0^{1/2} - 2 \left[ \frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2} \right]_{1/2}^1$  1

$= 2 \left[ \frac{1}{2\pi} - \frac{1}{\pi^2} \right] - 2 \left[ \frac{-1}{\pi^2} - \frac{1}{2\pi} \right] = \frac{2}{\pi}$  1

11.  $I = \int \frac{1+\sin 2x}{1+\cos 2x} \cdot e^{2x} dx = \frac{1}{2} \int \frac{1+\sin t}{1+\cos t} \cdot e^t dt$  (where  $2x=t$ ) 1/2

$= \frac{1}{2} \int \left( \frac{1}{2\cos^2 t/2} + \frac{2 \sin t/2 \cos t/2}{2 \cos^2 t/2} \right) e^t dt$  1

$= \frac{1}{2} \int \left( \frac{1}{2} \sec^2 t/2 + \tan t/2 \right) e^t dt$  1

$\tan t/2 = f(t)$  then  $f'(t) = \frac{1}{2} \sec^2 t/2$

Using  $\int (f(t) + f'(t)) e^t dt = f(t) e^t + C$ , we get 1/2

$I = \frac{1}{2} \tan t/2 \cdot e^t + C = \frac{1}{2} \tan x \cdot e^{2x} + C$  1

OR

We have

$$\begin{aligned} \frac{x^4}{(x-1)(x^2+1)} &= (x+1) + \frac{1}{x^3-x^2+x-1} \\ &= (x+1) + \frac{1}{(x-1)(x^2+1)} \end{aligned} \quad \dots\dots (1) \quad 1$$

Now express  $\frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)}$  \dots\dots (2)



So,

$$1 = A(x^2 + 1) + (Bx + C)(x - 1)$$

$$= (A + B)x^2 + (C - B)x + A - C$$

Equating coefficients,  $A + B = 0$ ,  $C - B = 0$  and  $A - C = 1$ ,

Which give  $A = \frac{1}{2}$ ,  $B = C = -\frac{1}{2}$ . Substituting values of  $A$ ,  $B$ , and  $C$  in (2), we get

$$\frac{1}{(x-1)(x^2+1)} = \frac{1}{2(x-1)} - \frac{1}{2} \frac{x}{(x^2+1)} - \frac{1}{2(x^2+1)} \quad \dots\dots\dots (3) \quad 1$$

Again, substituting (3) in (1), we have

$$\frac{x^4}{(x-1)(x^2+1)} = (x + 1) + \frac{1}{2(x-1)} - \frac{1}{2} \frac{x}{(x^2+1)} - \frac{1}{2(x^2+1)}$$

Therefore

$$\int \frac{x^4}{(x-1)(x^2+1)} dx = \frac{x^2}{2} + x + \frac{1}{2} \log |x - 1| - \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \tan^{-1} x + C \quad 1+1$$

12. Let  $E$  : Die shows a number  $> 3$

$$E : \{H4, H5, H6\} \quad 1/2$$

and  $F$  : there is atleast one head.

$$\therefore F : \{HT, H1, H2, H3, H4, H5, H6\} \quad 1/2$$

$$P(F) = 1 - \frac{1}{4} = \frac{3}{4} \quad 1$$

$$P(E \cap F) = \frac{3}{12} = \frac{1}{4} \quad 1$$

$$\therefore P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \quad 1$$

OR

$p = \frac{1}{2}, q = \frac{1}{2}$ , let the coin be tossed  $n$  times

$$\therefore P(r \geq 1) > \frac{90}{100} \quad \frac{1}{2}$$

$$\text{or } 1 - P(r=0) > \frac{90}{100} \quad \frac{1}{2}$$

$$P(r=0) < 1 - \frac{9}{10} = \frac{1}{10} \quad \frac{1}{2}$$

$${}^n C_0 \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^0 < \frac{1}{10} \Rightarrow \frac{1}{2^n} < \frac{1}{10} \quad 1\frac{1}{2}$$

$$\Rightarrow 2^n > 10, \therefore n = 4 \quad 1$$

$$13. \left. \begin{array}{l} \vec{a} \times \vec{b} = \vec{c} \Rightarrow \vec{a} \perp \vec{b} \text{ and } \vec{b} \perp \vec{c} \\ \vec{a} \times \vec{c} = \vec{b} \Rightarrow \vec{a} \perp \vec{c} \text{ and } \vec{c} \perp \vec{b} \end{array} \right\} \Rightarrow \vec{a} \perp \vec{b} \perp \vec{c} \quad 1$$

$$|\vec{a} \times \vec{b}| = |\vec{c}| \text{ and } |\vec{a} \times \vec{c}| = |\vec{b}| \quad 1$$

$$\Rightarrow |\vec{a}| |\vec{b}| \sin \frac{\pi}{2} = |\vec{c}| \text{ and } |\vec{a}| |\vec{c}| \sin \frac{\pi}{2} = |\vec{b}|$$

$$\Rightarrow |\vec{a}| |\vec{b}| = |\vec{c}| \therefore |\vec{a}| |\vec{a}| |\vec{b}| = |\vec{b}| \Rightarrow |\vec{a}|^2 = 1 \Rightarrow |\vec{a}| = 1 \quad 1$$

$$\Rightarrow 1. |\vec{b}| = |\vec{c}| \Rightarrow |\vec{b}| = |\vec{c}|$$

$$14. \text{ DR's of line } (L_1) \text{ joining } (4, 3, 2) \text{ and } (1, -1, 0) \text{ are } \langle 3, 4, 2 \rangle \quad \frac{1}{2}$$

$$\text{DR's of line } (L_2) \text{ joining } (1, 2, -1) \text{ and } (2, 1, 1) \text{ are } \langle 1, -1, 2 \rangle \quad \frac{1}{2}$$

$$\text{A vector } \perp \text{ to } L_1 \text{ and } L_2 \text{ is } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 1 & -1 & 2 \end{vmatrix} = 10\hat{i} - 4\hat{j} - 7\hat{k} \quad 1\frac{1}{2}$$

$\therefore$  Equation of the line passing through  $(1, -1, 1)$  and  $\perp$  to  $L_1$  and  $L_2$  is

$$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda (10\hat{i} - 4\hat{j} - 7\hat{k}) \quad 1\frac{1}{2}$$

OR

Equation of line AB is

$$\vec{r} = (-\hat{j} + 3\hat{k}) + \lambda (5\hat{i} + 5\hat{j} + \hat{k})$$

 $\therefore$  Point Q is  $(5\lambda, -1+5\lambda, 3+\lambda)$ 

$$\overrightarrow{PQ} = (5\lambda-1)\hat{i} + (5\lambda-9)\hat{j} + (\lambda-1)\hat{k}$$

$$PQ \perp AB \Rightarrow 5(5\lambda-1) + 5(5\lambda-9) + 1(\lambda-1) = 0$$

$$51\lambda = 51 \Rightarrow \lambda = 1$$

 $\Rightarrow$  foot of perpendicular (Q) is  $(5, 4, 4)$ 

$$\text{Length of perpendicular PQ} = \sqrt{4^2 + (-4)^2 + 0^2} = 4\sqrt{2} \text{ units}$$

15.  $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$

$$\Rightarrow \sin^{-1} 6x = \left(-\frac{\pi}{2} - \sin^{-1} 6\sqrt{3}x\right)$$

$$\Rightarrow 6x = \sin \left[-\frac{\pi}{2} - \sin^{-1} 6\sqrt{3}x\right] = -\sin \left[\frac{\pi}{2} + \sin^{-1} 6\sqrt{3}x\right]$$

$$= -\cos [\sin^{-1} 6\sqrt{3}x] = -\sqrt{1-108x^2}$$

$$\Rightarrow 36x^2 = 1-108x^2 \Rightarrow 144x^2 = 1$$

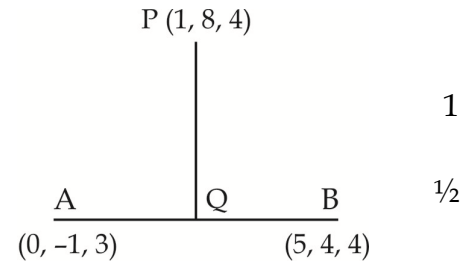
$$\Rightarrow x = \pm \frac{1}{12}$$

since  $x = \frac{1}{12}$  does not satisfy the given equation

$$\therefore x = -\frac{1}{12}$$

OR

$$\text{LHS} = 2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31}$$



$$= 2 \tan^{-1} \frac{3}{4} - \tan^{-1} \frac{17}{31} \quad 1$$

$$= \tan^{-1} \left( \frac{2 \cdot \frac{3}{4}}{1 - \frac{9}{16}} \right) - \tan^{-1} \frac{17}{31} \quad 1$$

$$= \tan^{-1} \left( \frac{24}{7} \right) - \tan^{-1} \frac{17}{31} \quad 1$$

$$= \tan^{-1} \left( \frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \cdot \frac{17}{31}} \right) = \tan^{-1} (1) = \pi/4 \quad 1$$

16.  $x = \sin t$  and  $y = \sin kt$

$$\frac{dx}{dt} = \cos t \text{ and } \frac{dy}{dt} = k \cos kt$$

$$\Rightarrow \frac{dy}{dx} = k \frac{\cos kt}{\cos t} \quad 1$$

$$\text{or } \cos t \cdot \frac{dy}{dx} = k \cdot \cos kt$$

$$\cos^2 t \left( \frac{dy}{dx} \right)^2 = k^2 \cos^2 kt$$

$$\cos^2 t \left( \frac{dy}{dx} \right)^2 = k^2 \cos^2 kt \quad 1/2$$

$$(1-x^2) \left( \frac{dy}{dx} \right)^2 = k^2 (1-y^2) \quad 1$$

Differentiating w.r.t.x

$$(1-x^2) 2 \frac{dy}{dx} \frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^2 (-2x) = -2k^2y \frac{dy}{dx} \quad 1$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + k^2y = 0 \quad 1/2$$

17. let  $u = y^x$ ,  $v = x^y$ ,  $w = x^x$

$$(i) \log u = x \log y \Rightarrow \frac{du}{dx} = y^x \left[ \log y + \frac{x}{y} \frac{dy}{dx} \right] \quad 1$$

$$(ii) \log v = y \log x \Rightarrow \frac{dv}{dx} = x^y \left[ \frac{y}{x} + \log x \frac{dy}{dx} \right] \quad 1/2$$

$$(iii) \log w = x \log x \Rightarrow \frac{dw}{dx} = x^x, (1+\log x) \quad 1/2$$

$$\Rightarrow y^x \left[ \log y + \frac{x}{y} \frac{dy}{dx} \right] + x^y \left[ \frac{y}{x} + \log x \frac{dy}{dx} \right] + x^x (1+\log x) = 0 \quad 1$$

$$\Rightarrow \frac{dy}{dx} = - \frac{x^x(1+\log x) + y x^{y-1} + y^x \log y}{x \cdot y^{x-1} + \log x} \quad 1$$

18.  $f(x) = x^3 + bx^2 + ax + 5$  on  $[1, 3]$

$$f'(x) = 3x^2 + 2bx + a$$

$$f'(c) = 0 \Rightarrow 3 \left( 2 + \frac{1}{\sqrt{3}} \right)^2 + 2b \left( 2 + \frac{1}{\sqrt{3}} \right) + a = 0 \text{ ----- (i)} \quad 1$$

$$f(1) = f(3) \Rightarrow b + a + 6 = 32 + 9b + 3a$$

$$\text{or } a + 4b = -13 \text{ ----- (ii)} \quad 1$$

$$\text{Solving (i) and (ii) to get } a=11, b=-6 \quad 1$$

19. Let  $3x + 1 = A(-2x - 2) + B \Rightarrow A = -3/2, B = -2 \quad 1$

$$I = \int \frac{-\frac{3}{2}(-2x-2)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{(\sqrt{6})^2 - (x+1)^2}} dx \quad 1+1$$

$$= -3\sqrt{5-2x-x^2} - 2 \cdot \sin^{-1} \left( \frac{x+1}{\sqrt{6}} \right) + C \quad 1$$

### SECTION-C

20. (i) for all  $a, b \in A$ ,  $(a, b) R (a, b)$ , as  $a + b = b + a$

$$\therefore R \text{ is reflexive} \quad 1$$

(ii) for  $a, b, c, d \in A$ , let  $(a, b) R (c, d)$

$$\therefore a + d = b + c \Rightarrow c + b = d + a \Rightarrow (c, d) R (a, b)$$

$$\therefore R \text{ is symmetric} \quad 1$$

(iii) for  $a, b, c, d, e, f \in A$ ,  $(a, b) R (c, d)$  and  $(c, d) R (e, f)$

$$\therefore a + d = b + c \text{ and } c + f = d + e$$

$$\Rightarrow a + d + c + f = b + c + d + e \text{ or } a + f = b + e$$

$$\Rightarrow (a, b) R (e, f) \therefore R \text{ is Transitive}$$

2

Hence R is an equivalence relation and equivalence class [(2, 5)] is

1/2

$$\{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$$

1 1/2

OR

Let  $y \in S$ , then  $y = 4x^2 + 12x + 15$ , for some  $x \in \mathbb{N}$

$$\Rightarrow y = (2x + 3)^2 + 6 \Rightarrow x = \frac{(\sqrt{y-6})-3}{2}, \text{ as } y > 6$$

1

$$\text{Let } g : S \rightarrow \mathbb{N} \text{ is defined by } g(y) = \frac{(\sqrt{y-6})-3}{2}$$

1

$$\therefore \text{gof}(x) = g(4x^2 + 12x + 15) = g((2x+3)^2 + 6) = \frac{\sqrt{(2x+3)^2 - 3} - 3}{2} = x$$

1

$$\text{and } \text{fog}(y) = f\left(\frac{(\sqrt{y-6})-3}{2}\right) = \left[\frac{2\{(\sqrt{y-6})-3\}}{2} + 3\right]^2 + 6 = y$$

1

Hence  $\text{fog}(y) = I_S$  and  $\text{gof}(x) = I_N$

$$\Rightarrow f \text{ is invertible and } f^{-1} = g$$

1

21. Let the lines be, AB:  $x + 2y = 2$ , BC:  $2x + y = 7$ , AC =  $y - x = 1$

1

$\therefore$  Points of intersection are

A(0,1), B(4,-1) and C(2, 3)

$$A = \frac{1}{2} \int_{-1}^3 (7 - y) dy - \int_{-1}^1 (2 - 2y) dy - \int_1^3 (y - 1) dy$$

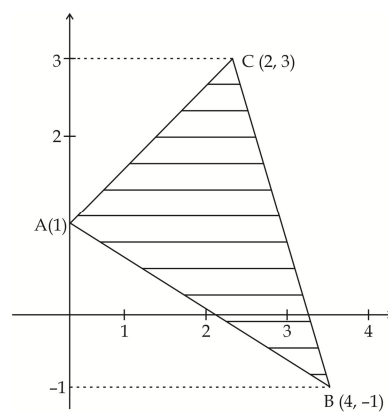
1 1/2

1 1/2

$$= \frac{1}{2} \left(7y - \frac{y^2}{2}\right)_{-1}^3 - (2y - y^2)_{-1}^1 - \left(\frac{y^2}{2} - y\right)_1^3$$

1 1/2

$$= 12 - 4 - 2 = 6 \text{ sq. Unit.}$$



1/2

22. Given differential equation is homogenous.

$$\therefore \text{Putting } y = vx \text{ to get } \frac{dy}{dx} = v + x \frac{dv}{dx} \quad 1/2$$

$$\frac{dy}{dx} = \frac{y \sin\left(\frac{y}{x}\right) - x e^{y/x}}{x \sin\left(\frac{y}{x}\right)} \Rightarrow v + x \frac{dv}{dx} = \frac{v \sin v - e^v}{\sin v} \quad 1$$

$$\therefore v + x \frac{dv}{dx} = v - \frac{e^v}{\sin v} \text{ or } x \frac{dv}{dx} = -\frac{e^v}{\sin v}$$

$$\therefore \int \sin v e^{-v} dv = -\int \frac{dx}{x} \text{ or } I_1 = -\log x + c_1 \text{ ----- (i)} \quad 1$$

$$I_1 = \sin v \cdot e^{-v} + \int \cos v e^{-v} dv$$

$$= -\sin v \cdot e^{-v} - \cos v e^{-v} - \int \sin v \cdot e^{-v} dv$$

$$I_1 = -\frac{1}{2} (\sin v + \cos v) e^{-v} \quad 1$$

$$\text{Putting (i), } \frac{1}{2} (\sin v + \cos v) e^{-v} = \log x + C_2$$

$$\Rightarrow \left[ \sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right) \right] e^{-\frac{y}{x}} = \log x^2 + C \quad 1$$

$$x = 1, y = 0 \Rightarrow c = 1 \quad 1$$

$$\text{Hence, Solution is } \left[ \sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right) \right] e^{-\frac{y}{x}} = \log x^2 + 1 \quad 1/2$$

OR

$$(x-a)^2 + (y-b)^2 = r^2 \quad \text{.....(i)}$$

$$\Rightarrow 2(x-a) + 2(y-b) \frac{dy}{dx} = 0 \quad \text{.....(ii)} \quad 1/2$$

$$\Rightarrow 1 + (y-b) \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \quad \text{.....(iii)} \quad 1/2$$

$$\therefore (y-b) = -\frac{(1+y_1^2)}{y^2} \quad 1 1/2$$

$$\text{From (ii), } (x-a) = \frac{y_1(1+y_1^2)}{y_2} \quad 1 1/2$$

Putting these values in (i)

$$\frac{y_1^2(1+y_1^2)^2}{y_2^2} + \frac{(1+y_1^2)^2}{y_2^2} = r^2 \quad 1$$

$$\text{or } \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = r^2 \left(\frac{d^2y}{dx^2}\right)^2 \quad 1$$

23. Here  $\vec{a}_1 = -3\hat{i} + \hat{j} + 5\hat{k}$ ,  $\vec{b}_1 = 3\hat{i} + \hat{j} + 5\hat{k}$

$$\vec{a}_2 = -\hat{i} + 2\hat{j} + 5\hat{k}, \vec{b}_2 = -\hat{i} + 2\hat{j} + 5\hat{k} \quad \frac{1}{2}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 2(-5) - 1(-15 + 5) \quad 1\frac{1}{2}$$

$$= -10 + 10 = 0$$

$\therefore$  lines are co-planer. 1/2

Perpendicular vector ( $\vec{n}$ ) to the plane =  $\vec{b}_1 \times \vec{b}_2$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = -5\hat{i} + 10\hat{j} - 5\hat{k} \quad 2$$

$$\text{or } \hat{i} - 2\hat{j} + \hat{k} \quad 2$$

$$\therefore \text{Eqn. of plane is } \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = (\hat{i} - 2\hat{j} + \hat{k}) \cdot (-3\hat{i} + \hat{j} + 5\hat{k}) = 0 \quad 1\frac{1}{2}$$

$$\text{or } x - 2y + z = 0$$

24. Let  $E_1$ : Student resides in the hostel

$E_2$ : Student resides outside the hostel

$$P(E_1) = \frac{40}{100} = \frac{2}{5}, P(E_2) = \frac{3}{5} \quad \frac{1}{2} + \frac{1}{2}$$

A: Getting A grade in the examination

$$P\left(\frac{A}{E_1}\right) = \frac{50}{100} = \frac{1}{2} \quad P\left(\frac{A}{E_2}\right) = \frac{30}{100} = \frac{3}{10} \quad 1+1$$



$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)} \quad 1$$

$$= \frac{\frac{2}{5} \cdot \frac{1}{2}}{\frac{2}{5} \cdot \frac{1}{2} + \frac{3}{5} \cdot \frac{3}{10}} = \frac{10}{19} \quad 1+1$$

25. Let the distance travelled @ 50 km/h be  $x$  km.

and that @ 80 km/h be  $y$  km.

∴ LPP is

$$\text{Maximize } D = x + y$$

$$\text{St. } 2x + 3y \leq 120$$

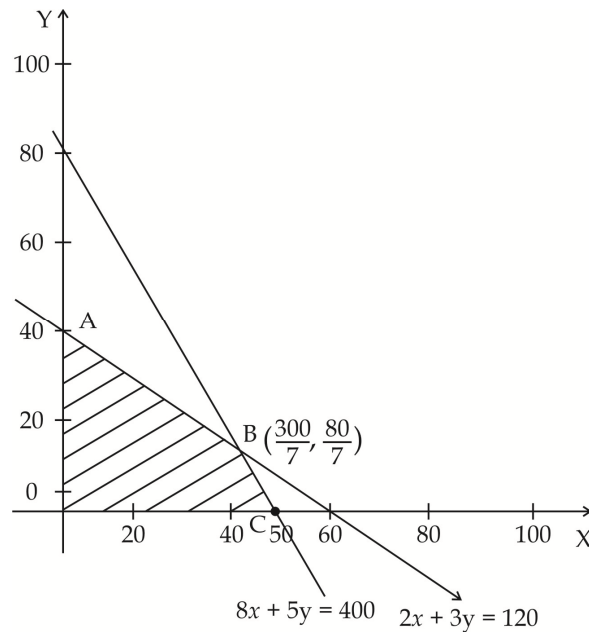
$$\frac{x}{50} + \frac{y}{80} \leq 1 \text{ or } 8x + 5y \leq 400$$

$$x \geq 0, y \geq 0$$

} 1/2

2

2



Vertices are.

$$(0, 40), \left(\frac{300}{7}, \frac{80}{7}\right), (50, 0)$$

$$\text{Max. D is at } \left(\frac{300}{7}, \frac{80}{7}\right)$$

$$\text{Max. D} = \frac{380}{7} = 54\frac{2}{7} \text{ km.} \quad 1\frac{1}{2}$$

26. Let P(x, y) be the position of the jet and the soldier is placed at A(3, 2)

$$\Rightarrow AP = \sqrt{(x-3)^2 + (y-2)^2} \quad \text{.....(i)} \quad \frac{1}{2}$$

$$\text{As } y = x^2 + 2 \Rightarrow y - 2 = x^2 \quad \text{.....(ii)} \Rightarrow AP^2 = (x-3)^2 + x^4 = z \text{ (say)} \quad \frac{1}{2}$$

$$\frac{dz}{dx} = 2(x-3) + 4x^3 \text{ and } \frac{d^2z}{dx^2} = 12x^2 + 2 \quad 2$$

$$\frac{dz}{dx} = 0 \Rightarrow x = 1 \text{ and } \frac{d^2z}{dx^2} \text{ (at } x = 1) > 0 \quad 1+1$$

$\therefore z$  is minimum when  $x = 1$ , when  $x = 1$ ,  $y = 1+2 = 3$

$$\therefore \text{minimum distance} = \sqrt{(3-1)^2 + 1^2} = \sqrt{5} \quad 1$$