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Senior School Certificate Examination

March 2018

Marking Scheme — Mathematics 65(B)

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65(B)

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|3AB| = 9|A|\,|B|$ $\frac{1}{2}$

$= -54$ $\frac{1}{2}$

2. $|x-5| = \begin{cases} x-5 & x \geq 5 \\ 5-x & x < 5 \end{cases}$ So, derivative at $x = 2$ is -1. $\frac{1}{2}+\frac{1}{2}$

3. $\int a^x e^x\, dx = \int (ae)^x\, dx = \frac{(ae)^x}{\log(ae)} + c$ 1

4. Let β be angle which $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with y axix

$\cos\beta = \frac{1}{\sqrt{(\sqrt{2})^2 + 1^2 + 1^2}} = \frac{1}{2}$ 1

SECTION B

5. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) = \frac{\pi}{2}$

$\sin^{-1}\left(\sin\left(x + \frac{\pi}{4}\right)\right) = \frac{\pi}{2}$ 1

$x = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$ 1

6. $\tan\left[\tan^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}} - \frac{\pi}{4}\right] = \tan\left[\tan^{-1}\frac{5}{12} - \tan^{-1}1\right]$ 1

$= \tan\left[\tan^{-1}\frac{\frac{5}{12}-1}{1+\frac{5}{12}\times 1}\right] = \frac{-7}{17}$ 1

7. $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 4 \\ 7 & 20 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -4 \\ 7 & 20 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} A \qquad (R_1 \to -R_1)$$ $\frac{1}{2}$

$$\Rightarrow \begin{bmatrix} 1 & -4 \\ 0 & 48 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 7 & 1 \end{bmatrix} A \qquad (R_2 \to R_2 - 7R_1)$$ $\frac{1}{2}$

$$\Rightarrow \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 7/48 & 1/48 \end{bmatrix} A \qquad (R_2 \to R_2/48)$$ $\frac{1}{2}$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -5/12 & 1/12 \\ 7/48 & 1/48 \end{bmatrix} A \qquad (R_1 \to R_1 + 4R_2)$$ $\frac{1}{2}$

$$\therefore \quad A^{-1} = \begin{bmatrix} -5/12 & 1/12 \\ 7/48 & 1/48 \end{bmatrix}$$

8. Let the curves cut at (h, k)

$y = 2e^x \Rightarrow \frac{dy}{dx} = 2e^x \Rightarrow m_1(h, k) = 2e^h$ $\frac{1}{2}$

$y = ae^{-x} \Rightarrow \frac{dy}{dx} = -ae^{-x} \Rightarrow m_2(h, k) = -ae^{-h}$ $\frac{1}{2}$

As curve cut each other orthogonally

$\therefore m_1 m_2 = -1 \Rightarrow 2e^h \times (-ae^{-h}) = -1$ $\frac{1}{2}$

$\Rightarrow a = \frac{1}{2}$ $\frac{1}{2}$

9. $y = 12x - x^3$

$\frac{dy}{dx} = 12 - 3x^2$ $\frac{1}{2}$

As $\frac{dy}{dx} = 0 \quad \therefore 12 - 3x^2 = 0 \Rightarrow x = -2, 2$ $\frac{1}{2}$

required points are $(-2, -16), (2, 16)$ 1

10. $I = \int \sqrt{x^2 - 4x + 13}\, dx = \int \sqrt{(x-2)^2 + 3^2}\, dx$ 1

$= \frac{x-2}{2}\sqrt{x^2 - 4x + 13} + \frac{9}{2}\log|x - 2 + \sqrt{x^2 - 4x + 13}| + C$ 1

11. $\left.\begin{array}{l} \overrightarrow{AB} = -2\hat{i} - 5\hat{k} \\ \overrightarrow{AC} = \hat{i} - 2\hat{j} - \hat{k} \end{array}\right\}$ any two side vectors. $\frac{1}{2} + \frac{1}{2}$

required vector $= \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ or $10\hat{i} + 7\hat{j} - 4\hat{k}$ 1

12. $\left.\begin{array}{l} A\text{: Number on the card is odd} \\ B\text{: Number on card} > 2 \end{array}\right\}$ $\frac{1}{2}$

B = {3, 4, 5, 6, 7, 8} $\frac{1}{2}$

$A \cap B$: {3, 5, 7} $\frac{1}{2}$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{3/8}{6/8} = \frac{1}{2}$ $\frac{1}{2}$

SECTION C

13. $C_1 \to aC_1,\ C_2 \to bC_2,\ C_3 \to cC_3$

$\Delta = \frac{1}{abc}\begin{vmatrix} a^2 & b^2 - bc & c^2 + bc \\ a^2 + ac & b^2 & c^2 - ac \\ a^2 - ab & b^2 + ab & c^2 \end{vmatrix}$ 1

$C_1 \to C_1 + C_2 + C_3$

$\Delta = \frac{1}{abc}\begin{vmatrix} a^2 + b^2 + c^2 & b^2 - bc & c^2 + bc \\ a^2 + b^2 + c^2 & b^2 & c^2 - ac \\ a^2 + b^2 + c^2 & b^2 + ab & c^2 \end{vmatrix}$ $\frac{1}{2}$

$$= \frac{a^2+b^2+c^2}{abc}\begin{vmatrix} 1 & b^2-bc & c^2+bc \\ 1 & b^2 & c^2-ac \\ 1 & b^2+ab & c^2 \end{vmatrix}$$ $\frac{1}{2}$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$

$$= \frac{a^2+b^2+c^2}{abc}\begin{vmatrix} 1 & b^2-bc & c^2-bc \\ 0 & bc & -bc-ac \\ 0 & ab+bc & -bc \end{vmatrix}$$ 1

$$= \frac{a^2+b^2+c^2}{abc}\left[-b^2c^2 + ab^2c + a^2bc + b^2c^2 + abc^2\right]$$

$$= \frac{(a^2+b^2+c^2)}{abc}\, abc\,(a+b+c) = (a+b+c)(a^2+b^2+c^2) = \text{RHS}$$ 1

OR

Clearly A is of order 2×2 Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$

$$\begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$ 1

$$\left.\begin{matrix} a+4b=-7 & c+4d=2 \\ 2a+5b=-8 & 2c+5d=4 \\ 3a+6b=-9 & 3c+6d=6 \end{matrix}\right\}$$ $\frac{1}{2}$

Solving we get

$a = 1, \ b = -2, \ c = 2, \ d = 0$ 1

$\therefore A = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ $\frac{1}{2}$

14.

LHL at $x = 0$	RHL at $x = 0$	$f(0)$	
$\lim\limits_{h\to 0} f(0-h)$	$\lim\limits_{h\to 0} \frac{\sqrt{h}}{\sqrt{4+\sqrt{h}}-2}$	k	
$\lim\limits_{h\to 0} \frac{2-2\cos(-2h)}{(-h)^2}$	$\lim\limits_{h\to 0} \frac{\sqrt{h}(\sqrt{4+\sqrt{h}}+2)}{\cancel{4}+\sqrt{h}-\cancel{4}}$		
$\lim\limits_{h\to 0} \frac{4\sin^2 h}{h^2}$	$2 + 2$		$1\frac{1}{2}+1\frac{1}{2}$ (LHL + RHL)
4	4		

$\therefore f(x)$ is continuous at $x = 0$ if $k = 4$ 1

OR

Taking log on both sides

$y \log x = (x - y)$ 1

$y(1 + \log x) = x$

$$y = \frac{x}{1+\log x}$$ $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{(1+\log x)1 - x\frac{1}{x}}{(1+\log x)^2}$$ 2

$$= \frac{\log x}{(1+\log x)^2},$$ Hence proved $\frac{1}{2}$

15. $$\frac{dx}{dt} = a(-\sin t + t\cos t + \sin t) = at\cos t$$ 1

$$\frac{dy}{dt} = a(\cos t + t\sin t - \cos t) = at\sin t$$ 1

$$\frac{dy}{dx} = \frac{at\sin t}{at\cos t} = \tan t$$ $\frac{1}{2}$

$$\frac{d^2y}{dx^2} = \sec^2 t\frac{dt}{dx} = \sec^2 t\frac{1}{at\cos t} = \frac{\sec^3 t}{at}$$ $1+\frac{1}{2}$

16. $I = \int \frac{x^2}{(x^2+1)(x^2+4)} dx$

let $x^2 = t$ $\qquad \frac{t}{(t+1)(t+4)} = \frac{A}{t+1} + \frac{B}{t+4}$ 1

$A + B = 1$ & $4A + B = 0$

Solving, we get $A = -\frac{1}{3}$, $B = \frac{4}{3}$ 1

$\therefore I = -\frac{1}{3}\int \frac{dx}{x^2+1} + \frac{4}{3}\int \frac{dx}{x^2+4}$

$= -\frac{1}{3}\tan^{-1} x + \frac{2}{3}\tan^{-1}\frac{x}{2} + C$ 1+1

OR

$I = \int \frac{x-5}{(x-3)^3} e^x \, dx$

$= \int \left[\frac{x-3}{(x-3)^3} - \frac{2}{(x-3)^3}\right] e^x \, dx$ 1

$= \int \left[\frac{1}{(x-3)^2} - \frac{2}{(x-3)^3}\right] e^x \, dx$ 1

$= \frac{e^x}{(x-3)^2} + C$ Using $\int e^x (f(x) + f'(x)\, dx = e^x f(x) + C$ 1+1

17. $I = \int_0^{\pi} \frac{x \sin x}{4 + \cos^2 x} dx$ ...(1)

replacing x by $\pi - x$

$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{4 + \cos^2 x} dx$ ...(2) 1

(1) + (2) $\Rightarrow$

$2I = \int_0^{\pi} \frac{\cancel{x \sin x} + \pi \sin x - \cancel{x \sin x}}{4 + \cos^2 x} dx$

$$I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{4+\cos^2 x}\,dx \qquad \text{let } \cos x = t,\ -\sin x\, dx = dt$$ 1

$$= -\frac{\pi}{2}\int_1^{-1} \frac{1}{4+t^2}dt$$ $\frac{1}{2}$

$$= \frac{\pi}{2}\int_{-1}^{1} \frac{1}{4+t^2}\,dt = \frac{\pi}{2}\,\frac{1}{2}\left[\tan^{-1}\frac{t}{2}\Big|_{-1}^{1}\right]$$ 1

$$= \frac{\pi}{4}\left[\tan^{-1}\frac{1}{2} - \tan^{-1}\frac{-1}{2}\right]$$

$$= \frac{\pi}{4}\cdot 2\tan^{-1}\frac{1}{2} = \frac{\pi}{2}\tan^{-1}\frac{1}{2} \text{ or } \frac{\pi}{4}\tan^{-1}\frac{4}{3}$$ $\frac{1}{2}$

18. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y}{2x - x\log\frac{y}{x}}$$ $\frac{1}{2}$

Clearly it is homogeneous let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$ $\frac{1}{2}$

$$v + x\frac{dv}{dx} = \frac{v}{2-\log v}$$

$$\Rightarrow x\frac{dv}{dx} \quad \frac{v \quad v\log v}{2 \quad \log v}$$

$$\Rightarrow \frac{\log v - 2}{v(\log v - 1)}\,dv = -\frac{dx}{x}$$ 1

$$\left(\frac{1}{v} - \frac{1}{v(\log v - 1)}\right)dv = -\frac{dx}{x}$$

$\Rightarrow \log v - \log|\log v - 1| = -\log|x| + \log C$ 1

$$\Rightarrow \frac{vx}{\log v - 1} = C$$

$$\Rightarrow y = C\left(\log\frac{y}{x} - 1\right)$$ 1

19. Given differential equation can be written as

$$\frac{y}{y+2}\,dy = \frac{x+2}{x}\,dx \qquad 1$$

$$\int\left(1-\frac{2}{y+2}\right)dy = \int\left(1+\frac{2}{x}\right)dx$$

$y - 2\log|y+2| = x + 2\log|x| + C$ 2

It passes through (1, –1)

$-1 - 0 = 1 + C \Rightarrow C = -2$ $\frac{1}{2}$

$\therefore$ required solution curve is $y - 2\log|y+2| = x + 2\log|x| - 2$ $\frac{1}{2}$

20. Sum of given vectors is $(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$ $\frac{1}{2}$

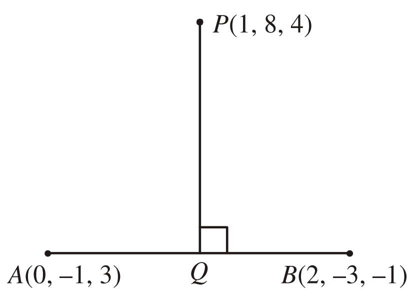
Unit vector is $\dfrac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}}$ $\frac{1}{2}$

Acc. to Question $(\hat{i} + \hat{j} + \hat{k})\cdot\dfrac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}} = 1$ 1

$\Rightarrow 2 + \lambda + 6 - 2 = \sqrt{(2+\lambda)^2 + 40}$ 1

$\Rightarrow 8\lambda = 8 \Rightarrow \lambda = 1$ 1

21.



Equation of line AB $\dfrac{x}{2} = \dfrac{y+1}{-2} = \dfrac{z-3}{-4} = \lambda$ $\frac{1}{2}$

Coordinates of $Q(2\lambda, -2\lambda - 1, -4\lambda + 3)$ for some λ $\frac{1}{2}$

Dr's of $PQ(2\lambda - 1, -2\lambda - 9, -4\lambda - 1)$ $\frac{1}{2}$

$PQ \perp AB \Rightarrow 2(2\lambda - 1) - 2(-2\lambda - 9) - 4(-4\lambda - 1) = 0$

gives $\lambda = \dfrac{-5}{6}$ 1

So coordinates of Q are $\left(\dfrac{-5}{6}, \dfrac{+2}{3}, \dfrac{19}{3}\right)$ $\frac{1}{2}$

Length of perpendicular $PQ = \sqrt{\left(\dfrac{-8}{3}\right)^2 + \left(\dfrac{22}{3}\right)^2 + \left(\dfrac{7}{3}\right)^2} = \dfrac{\sqrt{597}}{3}$ 1

22. Let X denotes the number of doublets $p = \frac{1}{6},\ q = \frac{5}{6}$ $\frac{1}{2}$

X	0	1	2
$P(X)$	$\frac{25}{36}$	$\frac{5}{36}$	$\frac{1}{36}$
$XP(X)$	0	$\frac{10}{36}$	$\frac{2}{36}$

1

$1\frac{1}{2}$

$\frac{1}{2}$

Mean $= \Sigma X\ P(X) = \frac{12}{36} = \frac{1}{3}$ $\frac{1}{2}$

23. $P(W) = \frac{4}{36} = \frac{1}{9}$ $P(L) = \frac{8}{9}$ where W: Getting sum 5

L: Not getting sum 5 1

$P(A$ winning$) = P(W) + P(LLW) + P(LLLLW) + \ldots.$

$= \frac{1}{9} + \frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9} + \frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9} + \ldots$ 2

$= \frac{9}{17}$ $\frac{1}{2}$

$P(B$ winning$) = 1 - \frac{9}{17} = \frac{8}{17}$ $\frac{1}{2}$

SECTION D

24. Reflexive

Clearly $(x, x) \in R$ $\because$ x is either odd or even. So R is reflexive 1

Symmetric

Let $(x, y) \in R$

$\therefore$ x, y are both either odd or even

$\Rightarrow y, x$ are both either odd or even

$\Rightarrow (y, x) \in R$, So R is symmetric 2

Transitive

Let $(x, y) \in R$ and $(y, z) \in R$

Case (i) x and y are both odd so y and z are both odd

$\therefore$ x and z are both odd

$\therefore$ $(x, z) \in R$

Case (ii) x and y are both even, so y and z are both even

$\therefore$ x and z are both even

$\therefore$ $(x, z) \in R$

Thus (x, y) and $(y, z) \in R \;\Rightarrow\; (x, z) \in R$ 2

R is transitive

As R is reflexive, symmetric and transitive so R is an equivalence relation.

$\left.\begin{array}{l}[1] = \{1, 3, 5, 7, 9\} \\ [2] = \{2, 4, 6, 8\}\end{array}\right\}$ are required equivalence classes $\frac{1}{2}$

OR

$a * b = a + b + ab$

$b * a = b + a + ba$

$\therefore\; a * b = b * a \quad \forall\; a, b \in A$

So * is commutative 2

$a * (b * c) = a * (b + c + bc) = a + b + c + bc + ab + ac + abc$

$(a * b) * c = (a + b + ab) * c = a + b + ab + c + ac + bc + abc$

$a * (b * c) = (a * b) * c \quad \forall\; a, b, c \in A$

$\therefore$ * is associative 3

$a * 0 = a + 0 + a \,.\, 0 = a$ 1

$0 * a = 0 + a + 0 \,.\, a = a$

and $0 \in A$ $\quad\therefore$ 0 is the identity element

25. Let A = $\begin{bmatrix} 5 & -1 & 1 \\ 3 & 2 & -5 \\ 1 & 3 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} y \\ 2 \\ 5 \end{bmatrix}$

given system is $AX = B \Rightarrow X = A^{-1}B$ provided $|A| \neq 0$ $\frac{1}{2}$

$A_{11} = 11$ $A_{12} = 1$ $A_{13} = 7$

$A_{21} = +1$ $A_{22} = -11$ $A_{23} = -16$ 2

$A_{31} = 3$ $A_{32} = 28$ $A_{33} = 13$

$|A| = 61 \neq 0$ 1

So $A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{61}\begin{bmatrix} 11 & 1 & 3 \\ 1 & -11 & 28 \\ 7 & -16 & 13 \end{bmatrix}$ $\frac{1}{2}$

$X = \frac{1}{61}\begin{bmatrix} 11 & 1 & 3 \\ 1 & -11 & 28 \\ 7 & -16 & 13 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix} = \frac{1}{61}\begin{bmatrix} 61 \\ 122 \\ 61 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $1\frac{1}{2}$

$x = 1, y = 2, z = 1$ $\frac{1}{2}$

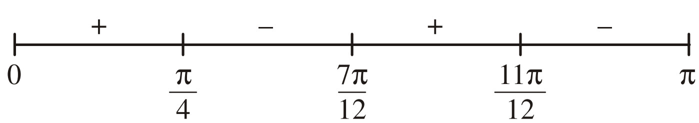
26. $f'(x) = 3 \cos 3x + 3 \sin 3x$ 1

$f'(x) = 0 \quad \Rightarrow \tan 3x = -1 = \tan \frac{3\pi}{4}$ 1

$\Rightarrow 3x = n\pi + \frac{3\pi}{4} \quad \Rightarrow \quad x = \frac{n\pi}{3} + \frac{\pi}{4} \quad n \in Z$ 2

$x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$

Sign of $f'(x)$



$f(x)$ is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$ 1

strictly decreasing in $\left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$ 1

OR

Surface Area, $S = 2\pi rh + 2\pi r^2$ and Volume $V = \pi r^2 h$ 1

$\Rightarrow h = \dfrac{S - 2\pi r^2}{2\pi r}$ thus, Volume, $V = \dfrac{Sr - 2\pi r^3}{2}$ 1

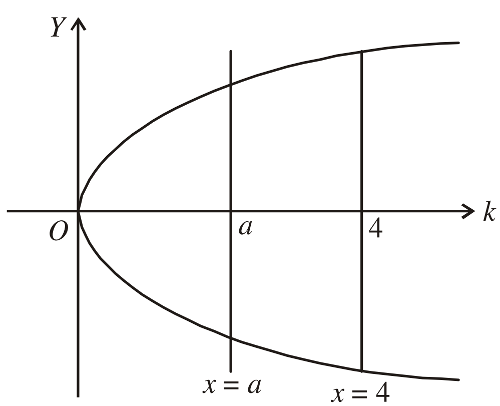
$\dfrac{dV}{dr} = \dfrac{S - 6\pi r^2}{2}$, $\dfrac{dV}{dr} = 0 \Rightarrow S = 6\pi r^2$ 1+1

$\dfrac{d^2V}{dr^2} = -6\pi r < 0 \quad \therefore V$ is maximum when $S = 6\pi r^2$ 1

$\Rightarrow h = \dfrac{4\pi r^2}{2\pi r} = 2r$ 1

Thus, height = diameter.

27.



According to question

$\not{2}\int_0^a \sqrt{x}\,dx = \not{2}\int_a^4 \sqrt{x}\,dx$ 2

$\dfrac{\not{2}}{\not{3}} x^{3/2}\Big|_0^a = \dfrac{\not{2}}{\not{3}} x^{3/2}\Big|_a^4$ 2

$a^{3/2} = 8 - a^{3/2}$

$2a^{3/2} = 8$

$a^{3/2} = 4$ 1

$\Rightarrow a = 4^{2/3}$ or $\sqrt[3]{16}$ 1

OR

$f(x) = 3x^2 + 2x + e^x \qquad a = 1,\ b = 3,\ nh = 2$ $\frac{1}{2}$

$$\int_1^3 (3x^2 + 2x + e^x)\, dx = \lim_{h \to 0} h[f(1) + f(1+h) + \ldots + f(1 + \overline{n-1}h)]$$ 1

$$= \lim_{h \to 0} h[5 + e^1 + (3h^2 + 8h + 5 + e^{1+h}) + \ldots + 5 + 3(n-1)^2 h^2 + 8(n-1)h + e^{1+(n-1)h}]$$

$$= \lim_{h \to 0} h[5n + 3h^2\, \Sigma(n-1)^2 + 8h\,\Sigma(n-1) + \Sigma\, e^{1+(n-1)h}]$$ $1\frac{1}{2}$

$$= \lim_{h \to 0} 5nh + 3h^3 \frac{(n-1)n(2n-1)}{6} + \frac{8h^2(n-1)n}{2} + he\frac{e^{nh}-1}{e^h-1}$$ 1

$$= \lim_{h \to 0} 5nh + \frac{(nh-h)\,nh(2nh-h)}{2} + 4(nh-h)nh + e\frac{e^{nh}-1}{\frac{e^n-1}{h}}$$ 1

$= 10 + 8 + 16 + e(e^2 - 1)$ 1

$= 34 + e^3 - e$ Ans.

28. Lines are $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$ 1

Consider $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0 \quad \because R_1 = R_3$ 2

$\therefore$ Lines are coplanar

Let Dr's of normal be A, B, C

$\left.\begin{aligned} 2A + 4B - C &= 0 \\ \text{and} \quad 3A - 2B + C &= 0 \end{aligned}\right\}$ gives $\frac{A}{2} = \frac{B}{-5} = \frac{C}{-16}$ 2

Also plane passes through point (1, 3, 0)

equation of required plane is $2(x - 1) - 5(y - 3) - 16(z) = 0$

i.e., $2x - 5y - 16z = -13$ 1

29. Let distance travelled at 40 km/h = x km

& distance travelled at 70 km/h = y km 1

Our LPP is

(i) Maximize $z = x + y$ 1

subject to constraints

$$\frac{x}{40} + \frac{y}{70} \leq 1$$

$2x + 7y \leq 100$ 2

$x \geq 0,\ y \geq 0$

(ii) Driving vehicle at slow speeds helps in (a) saving fuel (b) preventing accidents 1

(or any relevant answer)

(iii) No, child below 18 years should not be allowed to drive motorcycle as

(a) It is against law (b) It is unsafe for children and others travelling on the road. 1

(or any relevant answer))