Sample Question Paper

CLASS: XII

Session: 2022-23

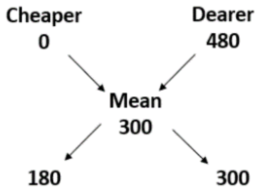
Applied Mathematics (Code-241)

Marking Scheme

Section – A

Each question carries 1-mark weightage

1.	$x \equiv 27(\text{mod } 4)$ $\Rightarrow x - 27 = 4k$, for some integer k $\Rightarrow x = 31$ as $27 < x \leq 36$ **(C) option**	
2.	**(D) option**	
3.	n = 26$\Rightarrow |t| = 3.07 > t_{25}(0.05) = 2.06$ **(B) option**	
4.	n = 34 $\Rightarrow v = 34 - 1 = 33$ **(B) option**	
5.	Speed of boat downstream = u = 10 km/h And, speed of boat upstream = v = 6 km/h $\Rightarrow$ Speed of stream = $\frac{1}{2}$(u – v) = 2 km/h **(B) option**	
6.	**(C) option**	
7.	Truck A carries water = 100 – $(\frac{20\times1500}{1000})$ = 70 l Truck B carries water = 80 – $(\frac{20\times1000}{1000})$ = 60 l **(C) option**	
8.	Let the face value of the bond = x Then, $\frac{10}{200}x = 1800 \Rightarrow x = 36000$ **(D) option**	
9.	**(C) option**	
10.	**(D) option**	
11.	D = $\frac{C-S}{n} = \frac{480000-25000}{10} = 45500$ **(B) option**	
12.	**(A) option**	
13.	$\int \frac{dy}{y\, log y} = \int \frac{dx}{x}$ $\Rightarrow \log(log y) = \log|x| + \log|C|$ $\Rightarrow \log(log y) = \log|Cx|$ $\Rightarrow y = e^{|Cx|}$	

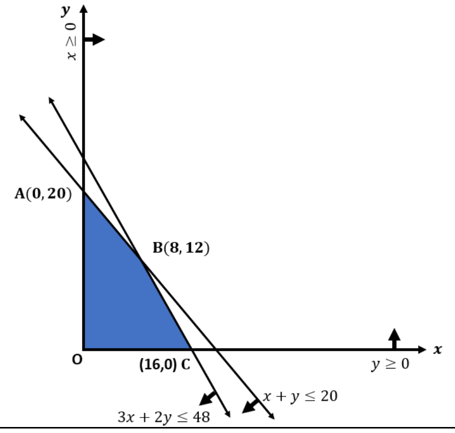
	(B) option	
14.	$\left[\left(\frac{60000}{10000}\right)^{\frac{1}{4}} - 1\right] \times 100 = [\sqrt[4]{6} - 1] \times 100$ **(C) option**	
15.	 ⇒ 180 : 300 = 3 : 5 **(C) option**	
16.	**(D) option**	
17.	**(C) option**	
18.	**(B) option**	
19.	P(Win in one game) = P(Lose in one game) = ½ ⇒ P (Beena to win in 3 out of 4 games) = $^4C_3 . \left(\frac{1}{2}\right)^4 = \frac{1}{4} = 25\%$ Assertion is correct and Reason is the correct explanation for it **(A)option**	
20.	Effective rate of interest = Nominal rate – inflation rate = 12.5 – 2 = 10.5% Assertion is correct Reason is true but not supportive of assertion **(B) option**	
	Section – B Each question carries 2-mark weightage	
21.	P = 250000, R = 7500, $i = r/400$ $\Rightarrow 250000 = \frac{7500 \times 400}{r} \Rightarrow r = 12$	1
	$\Rightarrow r = 12$	1
22.	a – 8 = 1 ⇒ $a = 9$ 3b = –2 ⇒ $b = -\frac{2}{3}$ –c + 2 = –28 ⇒ $c = 30$	1
	⇒ 2a + 3b – c = – 14	1
	OR Expanding C_1, we get $\Delta = 1(2x^2 + 4) - 2(-4x - 20) = 86$	1
	$\Rightarrow x^2 + 4x - 21 = 0$ $\therefore x = 3, -7$	1
23.	Let the number of hardcopy and paperback copies be x and y respectively ⇒ Maximum profit Z = (72x + 40y) –(9600 + 56x + 28y) = 16x + 12y– 9600	1

	Subject to constraints: $x + y \leq 960$ $5x + y \leq 2400$ $x, y \geq 0$	1
24.	Speed of boat in still waters = x km/h Speed of stream = y km/h Distance travelled = d km Time taken to travel downstream = $\frac{d}{x+y}$ Time taken to travel upstream = $\frac{d}{x-y}$	1
	Then, $\frac{2d}{x+y} = \frac{d}{x-y} \Rightarrow x : y = 3:1$	1
	OR Param runs 5 m in 3 seconds $\Rightarrow$ time taken to run 200 m $= \frac{3}{5} \times 200 = 120$ seconds	1
	Anuj 's time = 120 – 3 = 117 seconds	1
25.	$V_f = 437500, V_i = 350000$ Nominal rate = $\frac{V_f - V_i}{V_i} \times 100$	1
	$= \frac{437500 - 350000}{350000} \times 100 = 25\%$	1
Section – C Each question carries 3-mark weightage		
26.	$f'(x) = x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$ $\Rightarrow x = 1,2,3$	1
	Strictly increasing in $(1,2) \cup (3,\infty)$	1
	Strictly decreasing in $(-\infty,1) \cup (2,3)$	1
27.	Daily diet of team A = $\begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2500 & 65 \\ 1900 & 50 \\ 2000 & 54 \end{bmatrix} = \begin{bmatrix} 12700 \\ 334 \end{bmatrix}$ Team A consumes 12700 calories and 334 g vitamin	1.5
	Daily diet of team B = $\begin{bmatrix} 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2500 & 65 \\ 1900 & 50 \\ 2000 & 54 \end{bmatrix} = \begin{bmatrix} 10300 \\ 273 \end{bmatrix}$ Team B consumes 10300 calories and 273 g vitamin	1.5
28.	$\int \frac{dx}{(1+e^x)(1+e^{-x})}$ $= \int \frac{e^x dx}{(1+e^x)^2}$	3

	$= \int \frac{dt}{t^2}$, *where* $t = e^x + 1$ *and* $dt = e^x dx$ $= \frac{-1}{t} + C$ $= \frac{-1}{1+e^x} + C$ **OR** $\int \underset{II}{x} \, \underset{I}{log(1+x^2)} dx$, Integration by parts $= \log(1+x^2). \int x dx - \int [\frac{d}{dx} \log(1+x^2) . \int x \, dx] \, dx$ $= \frac{x^2}{2} \log(1+x^2) - \int [\frac{2x}{1+x^2} . \frac{x^2}{2}] \, dx$ $= \frac{x^2}{2} \log(1+x^2) - \int \frac{x^3}{1+x^2} \, dx$ $= \frac{x^2}{2} \log(1+x^2) - \int [x - \frac{x}{1+x^2}] \, dx$ $= \frac{x^2}{2} \log(1+x^2) - \frac{x^2}{2} + \frac{1}{2} \log(1+x^2) + C$ $= \frac{1}{2} [(1+x^2) \log(1+x^2) - x^2] + C$	
29.	Under pure competition, $p_d = p_s$ $\Rightarrow \frac{8}{x+1} - 2 = \frac{x+3}{2}$ $\Rightarrow x^2 + 8x - 9 = 0$ $\Rightarrow x = -9, 1$ $\therefore x = 1$	1.5
	When $x_0 = 1 \Rightarrow p_0 = 2$ $\therefore$ Produce surplus = $2 - \int_0^1 \frac{x+3}{2} dx = 2 - [\frac{x^2}{4} + \frac{3x}{2}]_0^1 = \frac{1}{4}$	1.5
	OR $p = 274 - x^2$ $\Rightarrow R = px = 274x - x^3$ $\frac{dR}{dx} = 274 - 3x^2$ Given MR = $4 + 3x$ In profit monopolist market, MR = $\frac{dR}{dx} \Rightarrow 4 + 3x = 274 - 3x^2$ $\Rightarrow x^2 + x - 90 = 0$	1.5

	$\Rightarrow x = -10, 9$ $\therefore x = 9$	
	When $x_0 = 9 \Rightarrow p_0 = 193$ ∴ Consumer surplus = $\int_0^9 (274 - x^2)dx - 193 \times 9$ $= \left[274x - \frac{x^3}{3}\right]_0^9$ = 486	1.5
30.	Purchase = ₹ 40,00,000 Down payment = x Balance = 40,00,000 – x $i = \frac{9}{1200} = 0.0075$, n = 25 x 12 = 300 E = ₹ 30,000	1
	$\Rightarrow 30000 = \frac{(4000000 - x) \times 0.0075}{1 - (1.0075)^{-300}}$ $\Rightarrow 30000 = \frac{(4000000 - x) \times 0.0075}{1 - 0.1062}$ $\Rightarrow x = 424800$ Down payment = ₹ 4,24,800	2
31.	n = 10 x 2 = 20, S = 10,21,760, $i = \frac{5}{200}$ = 0.025, R = ? S =R $\left[\frac{(1+i)^n - 1}{i}\right]$	1.5
	⇒1021760 = R $\left[\frac{(1+0.025)^{20} - 1}{0.025}\right]$ ⇒1021760 = R $\left[\frac{1.6386 - 1}{0.025}\right]$ ⇒R = $\left[\frac{1021760 \times 0.025}{0.6386}\right]$ $\Rightarrow$ R = ₹ 40,000 Mr Mehra set aside an amount of ₹ 40,000 at the end of every six months	1.5
	Section – D Each question carries 5-mark weightage	
32.	Probability of defective bucket = 0.03 n = 100 m = np = 100 x 0.03 = 3 Let X = number of defective buckets in a sample of 100 P (X = r) = $\frac{m^r e^{-m}}{r!}, r = 0,1,2,3, ...$	1
	(i) P (no defective bucket) = P(r = 0) = $\frac{3^0 e^{-3}}{0!} = 0.049$	2
	(ii) P (at most one defective bucket) = P(r = 0, 1) = $\frac{3^0 e^{-3}}{0!} + \frac{3^1 e^{-3}}{1!}$	2

	$= 0.049 + 0.147$ $= 0.196$	
	OR X = scores of students, $\mu = 45, \sigma = 5$ $\therefore Z = \frac{X-\mu}{\sigma} = \frac{X-45}{5}$	1
	(i) When X = 45, $Z = 0$ P (X > 45) = P (Z > 0) = 0.5 $\Rightarrow$ 50% students scored more than the mean score	2
	(ii) When X = 30, $Z = -3$ and when X = 50, $Z = 1$ P (30< X< 50) = P (-3< Z < 1) = P (-3< Z $\leq$ 1) = P $(-3 < Z \leq 0) + P$ $(0 \leq Z < 1)$ = P $(0 \leq Z < 3) + P$ $(0 \leq Z < 1)$ = 0.4987 + 0.3413 = 0.84 $\Rightarrow$ 84% students scored between 30 and 50 marks	2
33.	Let x be the number of guests for the booking Clearly, $x > 100$ to avail discount $\therefore$ Profit, P = $[4800 - \frac{200}{10}(x - 100)]\, x = 6800x - 20x^2$	2
	$\Rightarrow \frac{dP}{dx} = 6800 - 40x \Rightarrow x = 170$	1
	As $\frac{d^2P}{dx^2} = -40 < 0, \forall x$	1
	A booking for 170 guests will maximise the profit of the company And, Profit = ₹ 5,78,000	1
	OR P(x) = R(x) – C(x) = 5x – (100 + 0.025x^2)	2
	$\Rightarrow$P'(x) = 5 – 0.05 x $\Rightarrow x = 100$	1
	As P''(x) = $-$0.05 < 0, $\forall x$	1
	$\therefore$ Manufacturing 100 dolls will maximise the profit of the company And, Profit = ₹ 1,50,000	1
34.	Let the number of tables and chairs be x and y respectively (Max profit) Z = $22x + 18y$ Subject to constraints: $x + y \leq 20$ $3x + 2y \leq 48$ $x, y \geq 0$	1.5



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The feasible region OABCA is closed (bounded)

Corner points	Z = 22 x + 18 y
O (0,0)	0
A (0,20)	360
B (8,12)	392
C (16,0)	352

Buying 8 tables and 12 chairs will maximise the profit

1.5

35.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 3 & 2 \end{bmatrix}$$

⇒ |A| = 9 ⇒ A^{-1} exists

And $A^{-1} = \frac{1}{9}\begin{bmatrix} -2 & 5 & -2 \\ -2 & -4 & 7 \\ 5 & 1 & -4 \end{bmatrix}$

2

AX = B ⇒ $X = A^{-1}B$

$$\Rightarrow X = \frac{1}{9}\begin{bmatrix} -2 & 5 & -2 \\ -2 & -4 & 7 \\ 5 & 1 & -4 \end{bmatrix}\begin{bmatrix} 85 \\ 105 \\ 110 \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \\ 10 \end{bmatrix}$$

$\Rightarrow p_1 = 15, p_2 = 20, p_3 = 10$

3

Section – E

Each Case study carries 4-mark weightage

36. CASE STUDY - I

a) Pipe C empties 1 tank in 20 h ⇒ 2/5 th tank in $\frac{2}{5} \times 20 = 8$ hours

1

b) Part of tank filled in 1 hour = $\frac{1}{15} + \frac{1}{12} - \frac{1}{20} = \frac{1}{10}$ th

⇒ time taken to fill tank completely = 10 hours

1

c) At 5 am,

2

Let the tank be completely filled in 't' hours
⇒pipe A is opened for 't' hours
pipe B is opened for 't−3' hours
And, pipe C is opened for 't−4' hours

⇒ In one hour,
part of tank filled by pipe A = $\frac{t}{15}$ th
part of tank filled by pipe B = $\frac{t-3}{15}$ th
and, part of tank emptied by pipe C = $\frac{t-4}{15}$ th
Therefore $\frac{t}{15} + \frac{t-3}{12} - \frac{t-4}{20} = 1$
$\Rightarrow t = 10.5$
Total time to fill the tank = 10 hours 30 minutes

OR
6 am, pipe C is opened to empty ½ filled tank
Time to empty = 10 hours
Time for cleaning = 1 hour
Part of tank filled by pipes A and B in 1 hour= $\frac{1}{15} + \frac{1}{12} = \frac{3}{20}$th tank
⇒ time taken to fill the tank completely = $\frac{20}{3}$ hours
Total time taken in the process = 10 + 1 + $\frac{20}{3}$ = 17 hour 40 minutes

37. CASE STUDY - II

a)

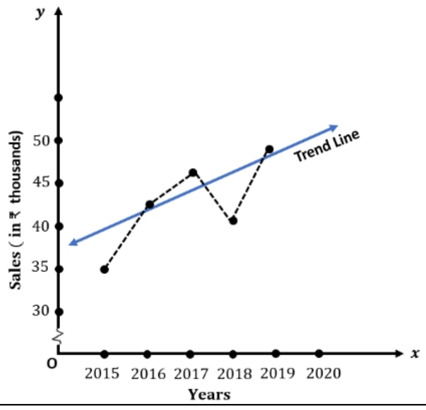
Year	Y	X	X^2	XY
2015	35	-2	4	-70
2016	42	-1	1	-42
2017	46	0	0	0
2018	41	1	1	41
2019	48	2	4	96
	212		10	25

a = $\frac{\sum Y}{n} = \frac{212}{5}$ = 42.4 and b = $\frac{\sum XY}{\sum X^2} = \frac{25}{10} = 2.5$ 2

$$Y_C = 42.4 + 2.5X$$

OR

Year	Y	3-year moving average
2015	35	-
2016	42	41
2017	46	43
2018	41	45
2019	48	-



b) For year 2022,

$$Y_{2022} = 42.4 + 2.5(2022 - 2017) = 54.9$$

⇒ the estimated sales for year 2022 = ₹ 54,900 — 1

c)
$$Y_C = 42.4 + 2.5X$$
$$\Rightarrow 67.4 = 42.4 + 2.5X$$
$$\Rightarrow X = 10$$

Sales will be ₹ 67,400 in year (2017+ 10) = year 2027 — 1

38. CASE STUDY - III

a) $\frac{k}{6} + \frac{2k}{6} + \frac{3(1-k)}{6} + \frac{4k}{2} = 1 \Rightarrow k = \frac{1}{4}$ — 1

b) P (getting admission on applying at least 2 weeks ahead of application deadline)

= P (X = 2,3,4)

$= \frac{1}{12} + \frac{3}{8} + \frac{1}{2} = \frac{23}{24}$

[alternate method: 1 – P (X = 1) = 1 - $\frac{1}{24} = \frac{23}{24}$] — 1

c) X = week applied ahead of application deadline

X	1	2	3	4
P(X)	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$
XP(X)	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{9}{8}$	2

∴ E(X) = $\frac{80}{24} = 3\frac{1}{3}$ weeks — 2

OR

X = Scholarship money awarded for the week applied in, before the deadline

Week applied in	1	2	3	4
X	9600	12000	20000	50000

P(X)	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$
XP(X)	$\frac{9600}{24}$	$\frac{12000}{12}$	$\frac{60000}{8}$	$\frac{50000}{2}$

∴ E(X) = ₹ 33,900
