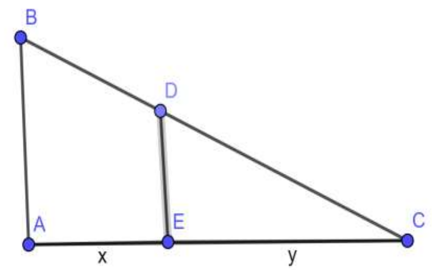
Marking Scheme
Class XII
Mathematics (Code – 041)
Section : A (Multiple Choice Questions- 1 Mark each)

Question No	Answer	Hints/Solution
1.	(c)	In a skew-symmetric matrix, the (i, j)th element is negative of the (j, i)th element. Hence, the (i, i)th element = 0
2.	(a)	$\lvert AA'\rvert = \lvert A\rvert\lvert A'\rvert = (-3)(-3) = 9$
3.	(b)	The area of the parallelogram with adjacent sides AB and AC = $\lvert\overrightarrow{AB} \times \overrightarrow{AC}\rvert$. Hence, the area of the triangle with vertices A, B, C $=\frac{1}{2}\lvert\overrightarrow{AB} \times \overrightarrow{AC}\rvert$
4.	(c)	The function f is continuous at x = 0 if $lim_{x\to0}f(x) = f(0)$ We have f(0) = k and $lim_{x\to0}f(x) = lim_{x\to0}\frac{1-cos}{8x^2} = lim_{x\to0}\frac{2sin^2 2x}{8x^2} = lim_{x\to0}\frac{sin^2 2x}{4x^2}$ $=lim_{x\to0}\left(\frac{sin2x}{2x}\right)^2 = 1$ Hence, k =1
5.	(b)	$\frac{x^2}{2} + \log\lvert x\rvert + C \left(\because f(x) = \int\left(x + \frac{1}{x}\right)dx\right)$
6.	(c)	The given differential equation is $4\left(\frac{dy}{dx}\right)^3\frac{d^2y}{dx^2} = 0.$ Here, m = 2 and n = 1 Hence, m + n= 3
7.	(b)	The strict inequality represents an open half plane and it contains the origin as (0, 0) satisfies it.
8.	(a)	Scalar Projection of $3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ *on vector* $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ $=\frac{(3\hat{\imath}-\hat{\jmath}-2\hat{k}).(\hat{\imath}+2\hat{\jmath}-3\hat{k})}{\lvert\hat{\imath}+2\hat{\jmath}-3\hat{k}\rvert} = \frac{7}{\sqrt{14}}$
9.	(c)	$\int_2^3\frac{x}{x^2+1} = \frac{1}{2}[log(x^2+1)]_2^3 = \frac{1}{2}(log10 - log5) = \frac{1}{2}log\left(\frac{10}{5}\right)$ $=\frac{1}{2}log2$
10.	(c)	$(AB^{-1})^{-1} = (B^{-1})^{-1}A^{-1} = BA^{-1}$
11.	(d)	The minimum value of the objective function occurs at two adjacent corner points (0.6, 1.6) and (3, 0) and there is no point in the half plane $4x + 6y < 12$ in common with the feasible region. So, the minimum value occurs at every point of the line-segment joining the two points.
12.	(d)	$2 - 20 = 2x^2 - 24 \Rightarrow 2x^2 = 6 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$
13.	(b)	$\lvert adjA\rvert = \lvert A\rvert^{n-1} \Rightarrow \lvert adjA\rvert = 25$
14.	(c)	$P(A' \cap B') = P(A') \times P(B')$ (As A and B are independent, A' *and* B' are also independent.) = 0.7 × 0.4 = 0.28
15.	(c)	$ydx - xdy = 0 \Rightarrow ydx - xdy = 0 \Rightarrow \frac{dy}{y} = \frac{dx}{x}$ $\Rightarrow \int\frac{dy}{y} = \int\frac{dx}{x} + logK, K > 0 \Rightarrow \log\lvert y\rvert = \log\lvert x\rvert + \log K$ $\Rightarrow \log\lvert y\rvert = \log\lvert x\rvert K \Rightarrow \lvert y\rvert = \lvert x\rvert K \Rightarrow y = \pm Kx \Rightarrow y = Cx$

16.	(a)	y = $\sin^{-1}x$ $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}\,.\frac{dy}{dx} = 1$ Again , differentiating both sides w. r. to x, we get $\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}.\left(\frac{-2x}{2\sqrt{1-x^2}}\right) = 0$ Simplifying, we get $(1-x^2)y_2 = xy_1$
17.	(b)	$\left\|\vec{a}-2\vec{b}\right\|^2 = (\vec{a}-2\vec{b}).(\vec{a}-2\vec{b})$ $\left\|\vec{a}-2\vec{b}\right\|^2 = \vec{a}.\vec{a} - 4\vec{a}.\vec{b} + 4\vec{b}.\vec{b}$ $= \|\vec{a}\|^2 - 4\vec{a}.\vec{b} + 4\left\|\vec{b}\right\|^2$ $= 4 - 16 + 36 = 24$ $\left\|\vec{a}-2\vec{b}\right\|^2 = 24 \Rightarrow \left\|\vec{a}-2\vec{b}\right\| = 2\sqrt{6}$
18.	(b)	The line through the points (0, 5, -2) and (3, -1, 2) is $\frac{x}{3-0} = \frac{y-5}{-1-5} = \frac{z+2}{2+2}$ $or, \frac{x}{3} = \frac{y-5}{-6} = \frac{z+2}{4}$ Any point on the line is $(3k, -6k+5, 4k-2)$, where k is an arbitrary scalar. $3k = 6 \Rightarrow k = 2$ The z-coordinate of the point P will be $4 \times 2 - 2 = 6$
19.	(c)	$sec^{-1}x$ is defined if $x \le -1$ or $x \ge 1$. Hence, $sec^{-1}2x$ will be defined if $x \le -\frac{1}{2}$ or $x \ge \frac{1}{2}$. Hence, A is true. The range of the function $sec^{-1}x$ is $[0, \pi] - \{\frac{\pi}{2}\}$ R is false.
20.	(a)	The equation of the x-axis may be written as $\vec{r} = t\hat{\imath}$. Hence, the acute angle θ between the given line and the x-axis is given by $cos\theta = \frac{\|1\times 1 + (-1)\times 0 + 0 \times 0\|}{\sqrt{1^2+(-1)^2+0^2} \times \sqrt{1^2+0^2+0^2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

SECTION B (VSA questions of 2 marks each)

21.	$sin^{-1}[sin\left(\frac{13\pi}{7}\right)] = sin^{-1}[sin\left(2\pi - \frac{\pi}{7}\right)]$ $= sin^{-1}[sin\left(-\frac{\pi}{7}\right)] = -\frac{\pi}{7}$ **OR** Let $y \in N$(codomain). Then $\exists\ 2y \in N$(domain) such that $f(2y) = \frac{2y}{2} = y$. Hence, f is surjective. $1, 2 \in N$(domain) such that $f(1) = 1 = f(2)$ Hence, f is not injective.	.1 1 1 1
22.	Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and the length of his shadow EC be y m. Using similarity of triangles, we have $\frac{4}{1.6} = \frac{x+y}{y} \Rightarrow 3y = 2x$	½

	 Differentiating both sides w.r.to t, we get $3\frac{dy}{dt} = 2\frac{dx}{dt}$ $$\frac{dy}{dt} = \frac{2}{3} \times 0.3 \Rightarrow \frac{dy}{dt} = 0.2$$ At any time t seconds, the tip of his shadow is at a distance of $(x + y)$ m from AB. The rate at which the tip of his shadow moving $$= \left(\frac{dx}{dt} + \frac{dy}{dt}\right) m/s = 0.5\, m/s$$ The rate at which his shadow is lengthening $$= \frac{dy}{dt}\; m/s = 0.2\, m/s$$	½ ½ 1/2
23.	$\vec{a} = \hat{\imath} - \hat{\jmath} + 7\hat{k}$ and $\vec{b} = 5\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$ Hence $\vec{a} + \vec{b} = 6\,\hat{\imath} - 2\hat{\jmath} + (7 + \lambda)\hat{k}$ and $\vec{a} - \vec{b} = -4\hat{\imath} + (7 - \lambda)\hat{k}$ $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ will be orthogonal if, $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$ i.e., if, $-24 + (49 - \lambda^2) = 0 \Longrightarrow \lambda^2 = 25$ i.e., if, $\lambda = \pm 5$ **OR** The equations of the line are $6x - 12 = 3y + 9 = 2z - 2$, which, when written in standard symmetric form, will be $\frac{x-2}{\frac{1}{6}} = \frac{y-(-3)}{\frac{1}{3}} = \frac{z-1}{\frac{1}{2}}$ Since, lines are parallel, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Hence, the required direction ratios are $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right)$ or (1,2,3) and the required direction cosines are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$	½ ½ 1 ½ ½ 1
24.	$y\sqrt{1 - x^2} + x\sqrt{1 - y^2} = 1$ Let $sin^{-1}x = A$ and $sin^{-1}y = B$. Then x = sinA and y = sinB $y\sqrt{1 - x^2} + x\sqrt{1 - y^2} = 1 \Longrightarrow sinBcosA + sinAcosB = 1$ $\Longrightarrow \sin(A + B) = 1 \Longrightarrow A + B = sin^{-1}1 = \frac{\pi}{2}$ $\Longrightarrow sin^{-1}x + sin^{-1}y = \frac{\pi}{2}$ Differentiating w.r.to x, we obtain $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$	½ ½ 1
25.	Since $\vec{\boldsymbol{a}}$ is a unit vector, $\therefore |\vec{a}| = 1$	½

$(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) = 12.$

$\Rightarrow \vec{x}.\vec{x} + \vec{x}.\vec{a} - \vec{a}.\vec{x} - \vec{a}.\vec{a} = 12$ ½

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12.$ ½

$\Rightarrow |\vec{x}|^2 - 1 = 12$

$\Rightarrow |\vec{x}|^2 = 13 \Rightarrow |\vec{x}| = \sqrt{13}$ 1/2

SECTION C

(*Short Answer Questions of 3 Marks each*)

26.

$$\int \frac{dx}{\sqrt{3-2x-x^2}}$$

$= \int \frac{dx}{\sqrt{-(x^2+2x-3)}} = \int \frac{dx}{\sqrt{4-(x+1)^2}}$ 2

$= sin^{-1}\left(\frac{x+1}{2}\right) + C \ \left[\int \frac{dx}{\sqrt{a^2-x^2}} = sin^{-1}\left(\frac{x}{a}\right) + C\right]$ 1

27.

P(not obtaining an odd person in a single round) = P(All three of them throw tails or All three of them throw heads)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{4}$ 1+1/2

P(obtaining an odd person in a single round)

$= 1 -$ P(not obtaining an odd person in a single round) $= \frac{3}{4}$ ½

The required probability

= P('In first round there is no odd person' and 'In second round there is no odd person' and 'In third round there is an odd person')

$= \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{64}$ 1

OR

Let X denote the Random Variable defined by the number of defective items.

$P(X=0) = \frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$

$P(X=1) = 2 \times \left(\frac{2}{6} \times \frac{4}{5}\right) = \frac{8}{15}$

$P(X=2) = \frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$ 2

x_i	0	1	2
p_i	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$
$p_i x_i$	0	$\frac{8}{15}$	$\frac{2}{15}$

1/2

Mean $= \sum p_i x_i = \frac{10}{15} = \frac{2}{3}$ 1/2

28.

Let I $= \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{cosx}}{\sqrt{sinx}+\sqrt{cosx}} dx$ ..(i)

	Using $\int_a^b \boldsymbol{f(x)\ dx} = \int_a^b \boldsymbol{f(a+b-x)dx}$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{cos(\frac{\pi}{6}+\frac{\pi}{3}-x)}}{\sqrt{sin(\frac{\pi}{6}+\frac{\pi}{3}-x)}+\sqrt{cos(\frac{\pi}{6}+\frac{\pi}{3}-x)}} dx$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{sinx}}{\sqrt{cosx}+\sqrt{sinx}}$ dx ..(ii). Adding (i) and (ii), we get $2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{cosx}}{\sqrt{sinx}+\sqrt{cosx}} dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{sinx}}{\sqrt{cosx}+\sqrt{sinx}}$dx $2I = \int_{\pi/6}^{\pi/3} dx$ $= [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ Hence, $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}} = \frac{\pi}{12}$ **OR** $\int_0^4 \lvert x-1 \rvert\, dx = \int_0^1 (1-x)dx + \int_1^4 (x-1)dx$ $=\left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$ $= (1-\frac{1}{2}) + (8-4) - (\frac{1}{2} - 1)$ $= 5$	 1 1 1 1 1 1
29.	$ydx + (x - y^2)dy = 0$ Reducing the given differential equation to the form $\boldsymbol{\frac{dx}{dy} + Px = Q}$ we get, $\frac{dx}{dy} + \frac{x}{y} = y$ I.F $= e^{\int Pdy} = e^{\int \frac{1}{y}dy} = e^{logy} = y$ The general solution is given by $x.IF = \int Q.IFdy \Rightarrow xy = \int y^2 dy$ $\Rightarrow xy = \frac{y^3}{3} + C$, which is the required general solution **OR** $xdy - ydx = \sqrt{x^2+y^2}\, dx$ It is a **Homogeneous Equation as** $\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x} = \sqrt{1+(\frac{y}{x})^2} + \frac{y}{x} = f\left(\frac{y}{x}\right)$. Put $y = vx$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$	 ½ 1 1 ½ ½

$v + x\frac{dv}{dx} = \sqrt{1+v^2} + v$ ½

Separating variables, we get

$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$ ½

Integrating, we get $log\left|v + \sqrt{1+v^2}\right| = log|x| + logK, K > 0$

$log\left|y + \sqrt{x^2+y^2}\right| = logx^2K$

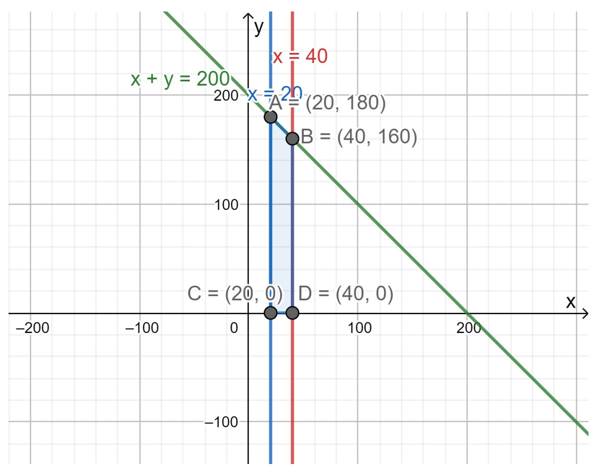
$\Rightarrow y + \sqrt{x^2+y^2} = \pm Kx^2$

$\Rightarrow y + \sqrt{x^2+y^2} = Cx^2$, which is the required general solution 1+1/2

30. We have Z= 400x +300y subject to

x + y $\leq 200, x \leq 40, x \geq 20, y \geq 0$

The corner points of the feasible region are C(20,0), D(40,0), B(40,160), A(20,180)



1

Corner Point	Z = 400x + 300y
C(20,0)	8000
D(40,0)	16000
B(40,160)	64000
A(20,180)	62000

1

Maximum profit occurs at x= 40, y=160

and the maximum profit =₹ 64,000 1

31. $\int \frac{(x^3+x+1)}{(x^2-1)} dx = \int \left(x + \frac{2x+1}{(x-1)(x+1)}\right) dx$ 1

Now resolving $\frac{2x+1}{(x-1)(x+1)}$ into partial fractions as

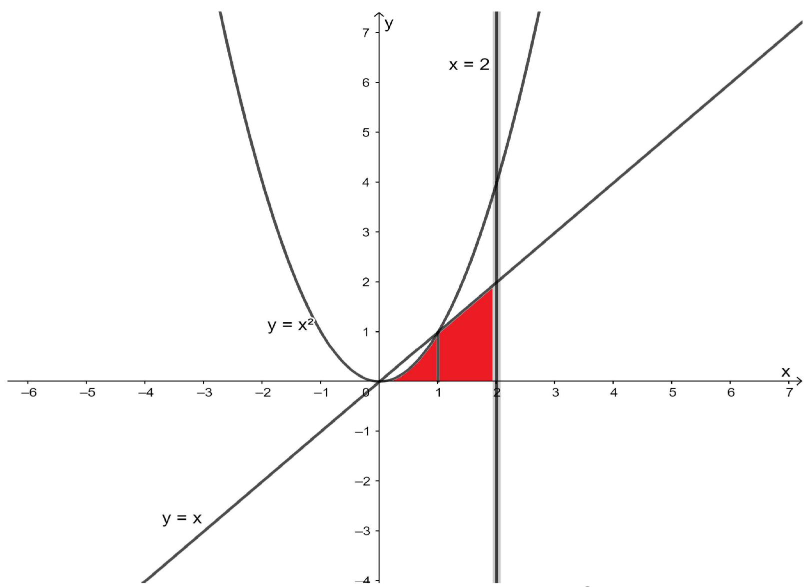
$$\frac{2x+1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

We get $\frac{2x+1}{(x-1)(x+1)} = \frac{3}{2(x-1)} + \frac{1}{2(x+1)}$ 1

	Hence, $\int \frac{(x^3+x+1)}{(x^2-1)}dx = \int \left(x + \frac{2x+1}{(x-1)(x+1)}\right)dx$ $= \int \left(x + \frac{3}{2(x-1)} + \frac{1}{2(x+1)}\right)dx$ $= \frac{x^2}{2} + \frac{3}{2}\log\lvert x-1\rvert + \frac{1}{2}\log\lvert x+1\rvert + C$ $= \frac{x^2}{2} + \frac{1}{2}(\log\lvert(x-1)^3(x+1)\rvert + C$	1

SECTION D

(Long answer type questions (LA) of 5 marks each)

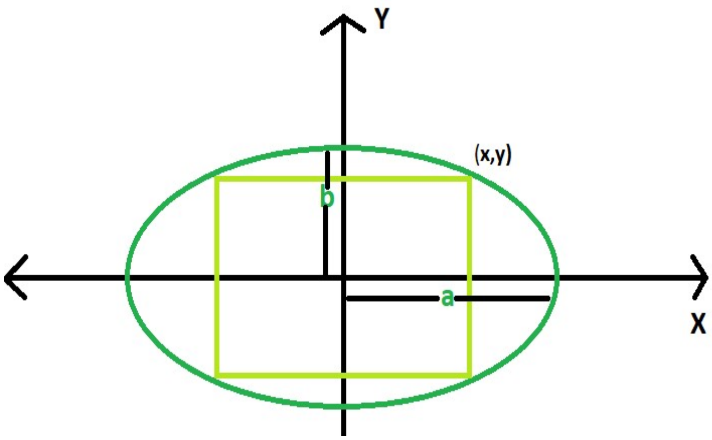
32.		(Correct Fig: 1 Mark)
	The points of intersection of the parabola $y = x^2$ and the line $y = x$ are (0, 0) and (1, 1).	½
	Required Area = $\int_0^1 y_{parabola}\,dx + \int_1^2 y_{line}\,dx$ Required Area = $\int_0^1 x^2\,dx + \int_1^2 x\,dx$	2
	$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$	1+1/2
33.	Let $(a, b) \in N \times N$. Then we have ab = ba (by commutative property of multiplication of natural numbers) $\Rightarrow (a, b)R(a, b)$ Hence, R is reflexive.	1
	Let $(a, b), (c, d) \in N \times N$ such that (a, b) R (c, d). Then ad = bc $\Rightarrow cb = da$ (by commutative property of multiplication of natural numbers $\Rightarrow (c, d)R(a, b)$ Hence, R is symmetric.	1+1/2
	Let $(a, b), (c, d), (e, f) \in N \times N$ such that	

	(a, b) R (c, d) and (c, d) R (e, f). Then ad = bc, cf = de $\Longrightarrow adcf = bcde$ $\Longrightarrow af = be$ $\Longrightarrow (a,b)R(e,f)$ Hence, R is transitive.	2
	Since, R is reflexive, symmetric and transitive, R is an equivalence relation on $N \times N$.	½
	OR	
	Let $A \in P(X)$. Then $A \subset A$ $\Longrightarrow (A,A) \in R$ Hence, R is reflexive.	1
	Let $A, B, C \in P(X)$ such that $(A,B),(B,C) \in R$ $\Longrightarrow A \subset B, B \subset C$ $\Longrightarrow A \subset C$ $\Longrightarrow (A,C) \in R$ Hence, R is transitive.	2
	$\emptyset, X \in P(X)$ such that $\emptyset \subset X$. Hence, $(\emptyset, X) \in R$. But, $X \not\subset \emptyset$, which implies that $(X, \emptyset) \notin R$. Thus, R is not symmetric.	2
34.	The given lines are non-parallel lines. There is a unique line-segment PQ (P lying on one and Q on the other, which is at right angles to both the lines. PQ is the shortest distance between the lines. Hence, the shortest possible distance between the insects = PQ The position vector of P lying on the line $\vec{r} = 6\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ is $(6+\lambda)\hat{\imath} + (2-2\lambda)\hat{\jmath} + (2+2\lambda)\hat{k}$ for some λ	½
	The position vector of Q lying on the line $\vec{r} = -4\hat{\imath} - \hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$ is $(-4+3\mu)\hat{\imath} + (-2\mu)\hat{\jmath} + (-1-2\mu)\hat{k}$ for some μ	½
	$\overrightarrow{PQ} = (-10+3\mu-\lambda)\hat{\imath} + (-2\mu-2+2\lambda)\hat{\jmath} + (-3-2\mu-2\lambda)\hat{k}$	½
	Since, PQ is perpendicular to both the lines $(-10+3\mu-\lambda) + (-2\mu-2+2\lambda)(-2) + (-3-2\mu-2\lambda)2 = 0,$	½
	$i.e., \mu - 3\lambda = 4$ …(i) and $(-10+3\mu-\lambda)3 + (-2\mu-2+2\lambda)(-2) + (-3-2\mu-2\lambda)(-2) = 0,$	½
	$i.e., 17\mu - 3\lambda = 20$ …(ii) solving (i) and (ii) for λ and μ, we get $\mu = 1, \lambda = -1$.	1
	The position vector of the points, at which they should be so that the distance between them is the shortest, are $5\hat{\imath} + 4\hat{\jmath}$ and $-\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$	½
	$\overrightarrow{PQ} = -6\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ The shortest distance = $\lvert\overrightarrow{PQ}\rvert = \sqrt{6^2+6^2+3^2} = 9$	1
	OR	

<table>
<tr><td></td><td>Eliminating t between the equations, we obtain the equation of the path $\frac{x}{2} = \frac{y}{-4} = \frac{z}{4}$, which are the equations of the line passing through the origin having direction ratios <2, -4, 4>. This line is the path of the rocket.
When t = 10 seconds, the rocket will be at the point (20, -40, 40).
Hence, the required distance from the origin at 10 seconds =
$$\sqrt{20^2 + 40^2 + 40^2}km = 20 \times 3\ km = 60\ km$$
The distance of the point (20, -40, 40) from the given line
$= \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|} = \frac{|-30\hat{j} \times (10\hat{i} - 20\hat{j} + 10\hat{k})|}{|10\hat{i} - 20\hat{} + 10\hat{k}|}\ km = \frac{|-300\hat{i} + 300\hat{k}|}{|10\hat{i} - 20\hat{j} + 10\hat{k}|}\ km$
$= \frac{300\sqrt{2}}{10\sqrt{6}}\ km = 10\sqrt{3}\ km$</td><td>1
½
1
2
1/2</td></tr>
<tr><td>35.</td><td>$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$
$|A| = 2(0) + 3(-2) + 5(1) = -1$
$A^{-1} = \frac{adjA}{|A|}$
$adjA = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}, A^{-1} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$
$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$
$= \frac{1}{(-1)}\begin{bmatrix} 0 + 5 - 6 \\ 22 + 45 - 69 \\ 11 + 25 - 39 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)}\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \Rightarrow x = 1, y = 2, z = 3.$</td><td>½
3
1+1/2</td></tr>
</table>

SECTION E(Case Studies/Passage based questions of 4 Marks each)

<table>
<tr><td>36.</td><td>(i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12)
(ii)$f'(x) = -\ 0.2x + m$
Since, 6 is the critical point,
$f'(6) = 0 \Rightarrow m = 1.2$
(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$
$f'(x) = -\ 0.2x + 1.2 = -0.2(x - 6)$
<table><tr><th>In the Interval</th><th>$f'(x)$</th><th>Conclusion</th></tr><tr><td>(0, 6)</td><td>+ve</td><td>f is strictly increasing in [0, 6]</td></tr><tr><td>(6, 12)</td><td>-ve</td><td>f is strictly decreasing in [6, 12]</td></tr></table></td><td>1
1
1+1</td></tr>
</table>

	OR (iii) $f(x) = -0.1x^2 + 1.2x + 98.6$, $f'(x) = -0.2x + 1.2, f'(6) = 0$, $f''(x) = -0.2$ $f''(6) = -0.2 < 0$ Hence, by second derivative test 6 is a point of local maximum. The local maximum value = $f(6) = -0.1 \times 6^2 + 1.2 \times 6 + 98.6 = 102.2$ We have $f(0) = 98.6, f(6) = 102.2, f(12) = 98.6$ 6 is the point of absolute maximum and the absolute maximum value of the function = 102.2. 0 and 12 both are the points of absolute minimum and the absolute minimum value of the function = 98.6.	1 1/2 1/2
37.	(i)  Let $(x, y) = \left(x, \frac{b}{a}\sqrt{a^2 - x^2}\right)$ be the upper right vertex of the rectangle. The area function $A = 2x \times 2\frac{b}{a}\sqrt{a^2 - x^2}$ $= \frac{4b}{a}x\sqrt{a^2 - x^2}, x \in (0, a)$. (ii)$\frac{dA}{dx} = \frac{4b}{a}\left[x \times \frac{-x}{\sqrt{a^2-x^2}} + \sqrt{a^2 - x^2}\right]$ $= \frac{4b}{a} \times \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}} = -\frac{4b}{a} \times \frac{2\left(x + \frac{a}{\sqrt{2}}\right)(x - \frac{a}{\sqrt{2}})}{\sqrt{a^2 - x^2}}$ $\frac{dA}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}$. $x = \frac{a}{\sqrt{2}}$ is the critical point. (iii)For the values of x less than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}, \frac{dA}{dx} > 0$ and for the values of x greater than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}, \frac{dA}{dx} < 0$. Hence, by the first derivative test, there is a local maximum at the critical point $x = \frac{a}{\sqrt{2}}$. Since there is only one critical point, therefore, the area of the soccer field is maximum at this critical point $x = \frac{a}{\sqrt{2}}$ Thus, for maximum area of the soccer field, its length should be $a\sqrt{2}$ and its width should be $b\sqrt{2}$. OR	1 ½ 1/2 1 1/2 ½

(iii) $A = 2x \times 2\frac{b}{a}\sqrt{a^2 - x^2}, x \in (0, a)$.

Squaring both sides, we get

$Z = A^2 = \frac{16b^2}{a^2}x^2(a^2 - x^2) = \frac{16b^2}{a^2}(x^2a^2 - x^4), x \in (0, a)$.

A is maximum when Z is maximum.

$$\frac{dZ}{dx} = \frac{16b^2}{a^2}(2xa^2 - 4x^3) = \frac{32b^2}{a^2}x(a + \sqrt{2}x)(a - \sqrt{2}x)$$

$\frac{dZ}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}$.

$$\frac{d^2Z}{dx^2} = \frac{32b^2}{a^2}(a^2 - 6x^2)$$

$$\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{a}{\sqrt{2}}} = \frac{32b^2}{a^2}(a^2 - 3a^2) = -64b^2 < 0$$ 1

Hence, by the second derivative test, there is a local maximum value of Z at the critical point $x = \frac{a}{\sqrt{2}}$. Since there is only one critical point, therefore, Z is maximum at $x = \frac{a}{\sqrt{2}}$, hence, A is maximum at $x = \frac{a}{\sqrt{2}}$. 1/2

Thus, for maximum area of the soccer field, its length should be $a\sqrt{2}$ and its width should be $b\sqrt{2}$. 1/2

38. (i)Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

$E_1 = PQ, E_2 = \bar{P}\bar{Q}, E_3 = \bar{P}Q, E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06, P(E_2) = 0.7 \times 0.8 = 0.56, P(E_3) = 0.7 \times 0.2 = 0.14, P(E_4) = 0.3 \times 0.8 = 0.24$

$$P\left(\frac{E}{E_1}\right) = 0, P\left(\frac{E}{E_2}\right) = 0, P\left(\frac{E}{E_3}\right) = 1, P\left(\frac{E}{E_4}\right) = 1$$ 1

$$P(E) = P(E_1).P\left(\frac{E}{E_1}\right) + P(E_2).P\left(\frac{E}{E_2}\right) + P(E_3).P\left(\frac{E}{E_3}\right) + P(E_4).P\left(\frac{E}{E_4}\right)$$ 1

= 0.14 + 0.24 = 0.38

(ii)By Bayes' Theorem, $P\left(\frac{E_3}{E}\right) = \frac{P(E_3).P\left(\frac{E}{E_3}\right)}{P(E_1).P\left(\frac{E}{E_1}\right)+P(E_2).P\left(\frac{E}{E_2}\right)+P(E_3).P\left(\frac{E}{E_3}\right)+P(E_4).P\left(\frac{E}{E_4}\right)}$

$$= \frac{0.14}{0.38} = \frac{7}{19}$$ 2

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as $P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0$

Alternative way of writing the solution:

(i)P(Shell fired from exactly one of them hits the plane)

= P[(Shell from A hits the plane and Shell from B does not hit the plane) or (Shell from A does not hit the plane and Shell from B hits the plane)] 1

$= 0.3 \times 0.8 + 0.7 \times 0.2 = 0.38$ 1

(ii)P(Shell fired from B hit the plane/Exactly one of them hit the plane)

$$= \frac{\text{P(Shell fired from B hit the plane} \cap \text{Exactly one of them hit the plane)}}{\text{P(Exactly one of them hit the plane)}}$$

	$=\frac{P(Shell\ from\ only\ B\ hit\ the\ plane)}{P(Exactly\ one\ of\ them\ hit\ the\ plane)}$	1
	$=\frac{0.14}{0.38}=\frac{7}{19}$	1