केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली

सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)

परीक्षार्थी प्रवेश–पत्र के अनुसार भरें

विषय Subject : Mathematics

विषय कोड Subject Code : 041

परीक्षा का दिन एवं तिथि
Day & Date of the Examination : Tuesday, 17.03.2020

उत्तर देने का माध्यम
Medium of answering the paper : - English

प्रश्न पत्र के ऊपर लिखे कोड को दर्शाएं
Write code No. as written on the top of the question paper :

Code Number	Set Number
65/1/2	① ● ③ ④

अतिरिक्त उत्तर–पुस्तिका (ओं) की संख्या
No. of supplementary answer-book(s) used: —

बेंचमार्क विकलांग व्यक्ति हाँ / नहीं
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यदि दृष्टिहीन है तो उपयोग में लाए गये सॉफ्टवेयर का नाम :
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*एक खाने में एक अक्षर लिखें। नाम के प्रत्येक भाग के बीच एक खाना रिक्त छोड़ें। यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।
Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

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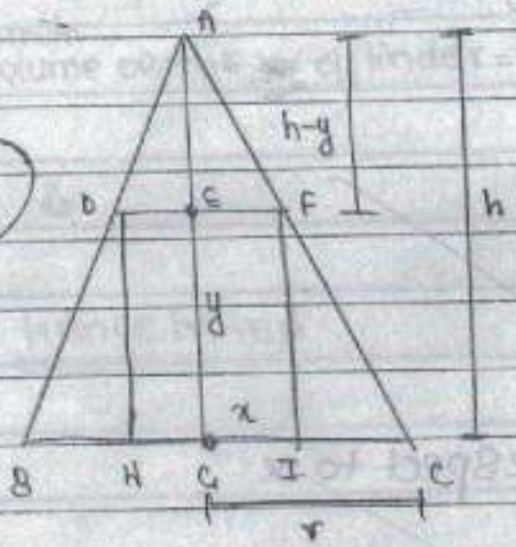
Section D

33. 

let the cone be represented by ABC and the cylinder have radius and height x and y respectively.

~~Volume~~ ∴ As ΔAEF and ΔAGC are similar

$$\frac{h-y}{h} = \frac{x}{r}$$

$$rh - ry = xh \Rightarrow \frac{rh - xh}{r} = y.$$

Volume of the cylinder $= V = \pi x^2 y$

$$\Rightarrow V = \frac{\pi x^2}{r}(rh - xh)$$

$$V = \pi x^2 h - \pi x^3 h$$

Differentiating with respect to x

$$\Rightarrow \frac{dV}{dx} = 2\pi xh - \frac{3\pi x^2 h}{r} \quad \text{--- (i)}$$

For maximum volume $\frac{dV}{dx} = 0$

$$\Rightarrow 2\pi xh = \frac{3\pi x^2 h}{r}$$

$$\Rightarrow 2r = 3x \Rightarrow x = \frac{2}{3}r$$

Differentiating equation (i) with respect to x

$$\Rightarrow \frac{d^2V}{dx^2} = 2\pi h - \frac{6\pi xh}{r}$$

$$\left(\frac{d^2V}{dx^2}\right)_{(x=\frac{2}{3}r)} = 2\pi h - \frac{6\pi 2rh}{3r} = 2\pi h - 4\pi h = -2\pi h < 0$$

For maximum volume $x = \frac{2}{3}r$

$$y = \frac{rh - xh}{r} = \frac{rh - \frac{2}{3}rh}{r} = \frac{h}{3}$$

$\therefore$ Height of the right cylinder with maximum volume is $\frac{1}{3}$rd height of cone.

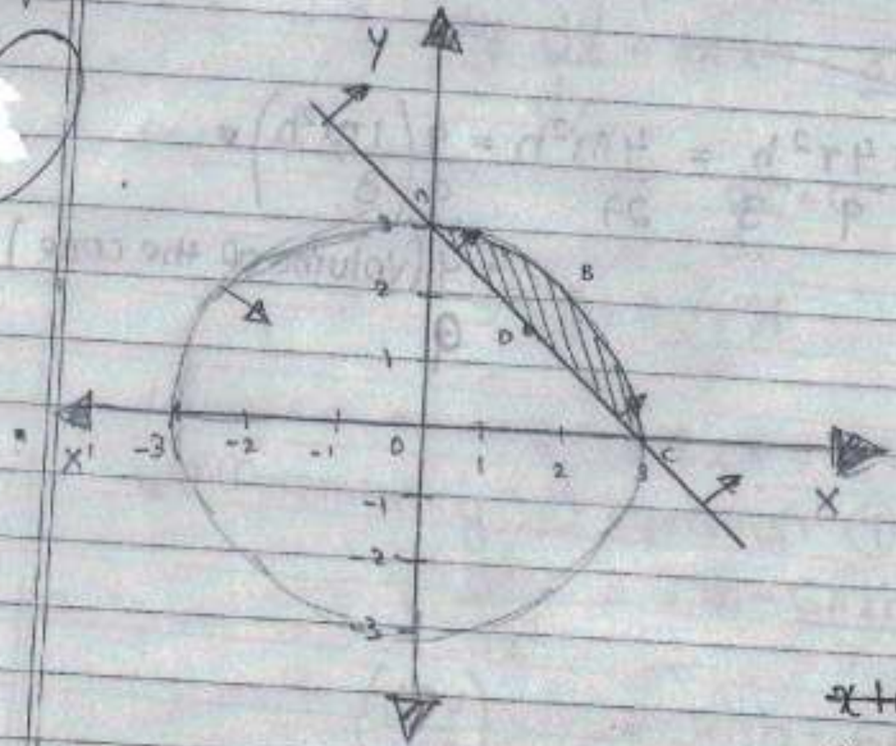
Volume of the cone $= \frac{1}{3}\pi r^2 h$

Maximum Volume of the ~~m~~ cylinder $= \pi x^2 y = \pi \frac{4r^2}{9} \frac{h}{3} = \frac{4\pi r^2 h}{27} = \frac{4}{9}\left(\frac{1}{3}\pi r^2 h\right)$

$= \frac{4}{9}$ (Volume of the cone)

Hence Proved

34.



$x^2+y^2 \leq 9$ is the equation of a circle with centre at origin and radius = 3 cm. O (0,0) satisfies the inequation $x^2+y^2 \leq 9$

∴ The area required is within the circle

~~x+y ≥ 3 is th~~

$x+y=3$ is the equation of a line passing through (0,3) and (3,0)

O(0,0) does not satisfy the inequation.

The area required is the area above the line

∴ The required area is represented by ~~BCBD~~. ADCB

Required area $= \int_0^3 y_{circle} - y_{line}\, dx$

$= \int_0^3 \sqrt{9-x^2} - (3-x)\, dx$

$= \int_0^3 \sqrt{9-x^2} + x - 3\, dx$

$= \left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{x^2}{2} - 3x\right]_0^3$

$= \left[\frac{9}{2}\cdot\frac{\pi}{2} + \frac{9-9}{2}\right]$

$= \cancel{\frac{9\pi}{4}}$ ~~square units~~ $\left(\frac{9\pi}{4} - \frac{9}{2}\right)$ square units

$= \frac{9}{2}\left(\frac{\pi}{2} - 1\right)$ square units

35. $P(-2,-4,7)$ The equation of the line in cartesian form is $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-6}{2} = \lambda$

Any point on the line can be represented as

$x_1 = 2\lambda+3$, $y_1 = -\lambda-2$, $z_1 = 2\lambda+6$

equation of the plane in cartesian form is

$x-y+z=6$

(x_1, y_1, z_1) satisfies the equation of the plane where the line intersects it.

$\Rightarrow 2\lambda+3+2+\lambda+2\lambda+6 = 6$

$5\lambda = -5$

$\lambda = -1$

$\therefore$ Point $Q(1,-1,4)$ OR In vector ~~vector~~ form $\vec{q} = \hat{i} - \hat{j} + 4\hat{k}$

Distance $PQ = \sqrt{(1+2)^2+(4-1)^2+(7-4)^2}$

$= \sqrt{3^2+3^2+3^2}$

$= \sqrt{27}$

$= 3\sqrt{3}$ units.

Vector equation of the line PQ =

$$\vec{r} = \hat{i} - \hat{j} + 4\hat{k} + \lambda(3\hat{i} + 3\hat{j} - 3\hat{k})$$

$$= \hat{i} - \hat{j} + 4\hat{k} + \mu(\hat{i} + \hat{j} - \hat{k}), \quad \mu = 3\lambda = \text{scalar}$$

36. OR

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} b^2+c^2+2bc & a^2 & bc \\ a^2+c^2+2ac & b^2 & ca \\ a^2+b^2+2ab & c^2 & ab \end{vmatrix}$$

$C_1' \to C_1 + C_2 - 2C_3$

$$\Rightarrow \Delta = \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from C_1

$$\Rightarrow \Delta = (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$R_2' \to R_2 - R_1$ and $R_3' \to R_3 - R_1$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a^2 & bc \\ 0 & (b-a)(b+a) & c(a-b) \\ 0 & (c-a)(c+a) & b(a-c) \end{vmatrix} (a^2+b^2+c^2)$$

Taking $(a-b)$ common from R_2 and $(c-a)$ common from R_3

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a^2 & bc \\ 0 & -(b+a) & c \\ 0 & c+a & -b \end{vmatrix} (a^2+b^2+c^2)(a-b)(c-a)$$

expanding along R_1

$$\Rightarrow \Delta = [(b+a)(b) - (c)(c+a)](a^2+b^2+c^2)(a-b)(c-a)$$

$$\Rightarrow \Delta = [b^2+ab-c^2-ac](a^2+b^2+c^2)(a-b)(c-a)$$

$$\Rightarrow \Delta = [(b+c)(b-c)+a(b-c)](a^2+b^2+c^2)(a-b)(c-a)$$

$$\Rightarrow \Delta = (a+b+c)(b-c)(a-b)(c-a)(a^2+b^2+c^2)$$

Hence Proved

Section C

27. let the trader buy x number of chairs
and y number of tables

He has to maximise profit $Z = 150x + 250y$

He has only ₹ 50000 to invest.

$\therefore \quad 1000x + 2000y \leq 50000$

$\Rightarrow \quad x + 2y \leq 50$

Also he has a storage space of ~~ent~~ at most 35 items.

$\therefore \; x + y \leq 35$.

$\therefore$ Required LPP is

Maximize $Z = 150x + 250y$

subject to the constraints

$x + 2y \leq 50$

$x + y \leq 35$

$x \geq 0$ $y \geq 0$ (Minimum constraints)

Converting $x+2y \leq 50$ into an equation

$x+2y=50$

x	0	50
y	25	0

(0,0) satisfy the inequality.

Converting $x+y \leq 35$ into an equation

$x+y=35$

x	0	35
y	35	0

(0,0) satisfies the inequality

solving both equations simultaneously we get (20,15)

Feasible area is represented by BODE.

Corner Points	$Z = 150x + 250y$	
$O(0,0)$	$0+0=0$	$= 0$
$D(35,0)$	35×150	$= 5250$
$B(0,25)$	25×250	$= 6250$
$E(20,15)$	$20 \times 150 + 15 \times 250$	$= 6750 \Rightarrow$ Maximum

for maximum profit he will trade in 20 items of chairs and 15 items of tables.

35
15
175
350
525

350
175
525

25
15
125
25×
375

35
15
175
350
525

350
175
525

25
15
125
25×
375

28. $x = a\sec^3\theta$

Differentiating with respect to x

$\Rightarrow \frac{dx}{d\theta} = 3a\sec^2\theta\sec\theta\tan\theta = 3a\sec^3\theta\tan\theta$.

$y = a\tan^3\theta$

Differentiating with respect to y

$\Rightarrow \frac{dy}{d\theta} = 3a\tan^2\theta\sec^2\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\tan^2\theta\sec^2\theta}{3a\sec^3\theta\tan\theta} = \frac{\tan\theta}{\sec\theta} = \sin\theta$. —(i)

Differenting equation (i) with respect to x

$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\sin\theta \frac{d\theta}{dx}$

$\frac{d^2y}{dx^2} = \frac{\cos\theta}{3a\sec^3\theta\tan\theta} = \frac{\cos\theta\cos^3\theta\cos\theta}{3a\sin\theta} = \frac{\cos^5\theta}{3a\sin\theta}$

29. $\int \frac{2x+1 \, dx}{\sqrt{3+2x-x^2}}$.

Now $2x+1 = A(2-2x)+B$

$2x+1 = x(-2A) + B+2A$.

$A=-1 \quad B=3$

$I = -1\int \frac{2-2x}{\sqrt{3+2x-x^2}} dx + 3\int \frac{dx}{\sqrt{3+2x-x^2}}$

Now let $3+2x-x^2 = z^2$

$(2-2x)dx = 2z\,dz$

and

$3+2x-x^2$

$= -(x^2-2x-3)$

$= -(x^2-2.x+1-4)$

$= 2^2-(x-1)^2$

$\therefore I = -\int \frac{2z\,dz}{z} + 3\int \frac{dx}{\sqrt{2^2-(x-1)^2}}$

$= -2z + 3\left[\sin^{-1}\left(\frac{x-1}{2}\right)\right] + C$

$= -2\sqrt{3+2x-x^2} + 3\sin^{-1}\left(\frac{x-1}{\cdot}\right) + C$, 'c' is integration

30.

I	II
3R	4R
5B	3B

4

let B_1 be the event that a black ball is transferred to Bag 2.
let R_1 be the event that a red ball is transferred to Bag 2.
let B_2 be the event that a black ball is picked from Bag 2.

$P(B_1) = \frac{5}{8}$ $\quad P(R_1) = \frac{3}{8}$ $\quad P(B_2/B_1) = \frac{4}{8}$. $P(B_2/R_1) = \frac{3}{8}$.

According to Bayes theorem

$$P(B_1/B_2) = \frac{P(B_1)\,P(B_2/B_1)}{P(B_1)P(B_2/B_1) + P(R_1)P(B_2/R_1)}$$

$$= \frac{\frac{5}{8}\times\frac{4}{8}}{\frac{5}{8}\times\frac{4}{8}+\frac{3}{8}\times\frac{3}{8}} = \frac{20}{20+9} = \frac{20}{29}.$$

31. OR.

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right) = f(y/x) \quad -(i)$$

∴ It is a homogeneous function.

let $\frac{y}{x} = V \Rightarrow y = Vx$

Differentiating with respect to x

$$\Rightarrow \frac{dy}{dx} = V + x\frac{dV}{dx}.$$

∴ equation (i) can be written as

$$V + x\frac{dV}{dx} = V - \tan V$$

$$\Rightarrow x\frac{dV}{dx} = -\tan V$$

$$\Rightarrow \int -\cot V \, dV = \int \frac{dx}{x}$$

...nstant.

$\Rightarrow \log \sin V + \log x = \log c$

$\Rightarrow x \sin V = c$

$\Rightarrow x \sin\left(\frac{y}{x}\right) = c$

At $x = 1, y = \pi/4$

$\sin\left(\frac{\pi}{4}\right) = c = \frac{1}{\sqrt{2}}$

$\Rightarrow x \sin\left(\frac{y}{x}\right) = \frac{1}{\sqrt{2}}$ Answer

32. $R : N \times N$

$(a,b) R (c,d)$ iff $ad = bc$

$a,b,c,d \in N$.

for reflexive

let ~~(a,b) ∈~~ $a, b \in N$

$(a,b) R (a,b)$

as $ab = ba$

∴ for all $a, b \in N$ ~~(a,b) ∈ R~~ $(a,b) R (a,b)$

∴ The relation is reflexive.

For symmetric

let $a, b, c, d \in N$

such that $(a,b) R (c,d)$

$\Rightarrow ad = bc$

$\Rightarrow cb = da$

$\Rightarrow (c,d) R (a,b)$

$\therefore$ The relation is symmetric

For transitive

let $a, b, c, d, e, f \in N$

such that $(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow ad = bc$ — (i) $\Rightarrow cf = de$

$\Rightarrow c = \frac{de}{f}$ — (ii)

from equations (i) and (ii)

$$ad = \frac{bde}{f}$$

$$\Rightarrow af = be$$

$$\cancel{\Rightarrow af = eb}$$

$$\therefore (a,b) R (e,f)$$

$\therefore$ For all $(a,b) R (c,d)$ and $(c,d) R (e,f)$

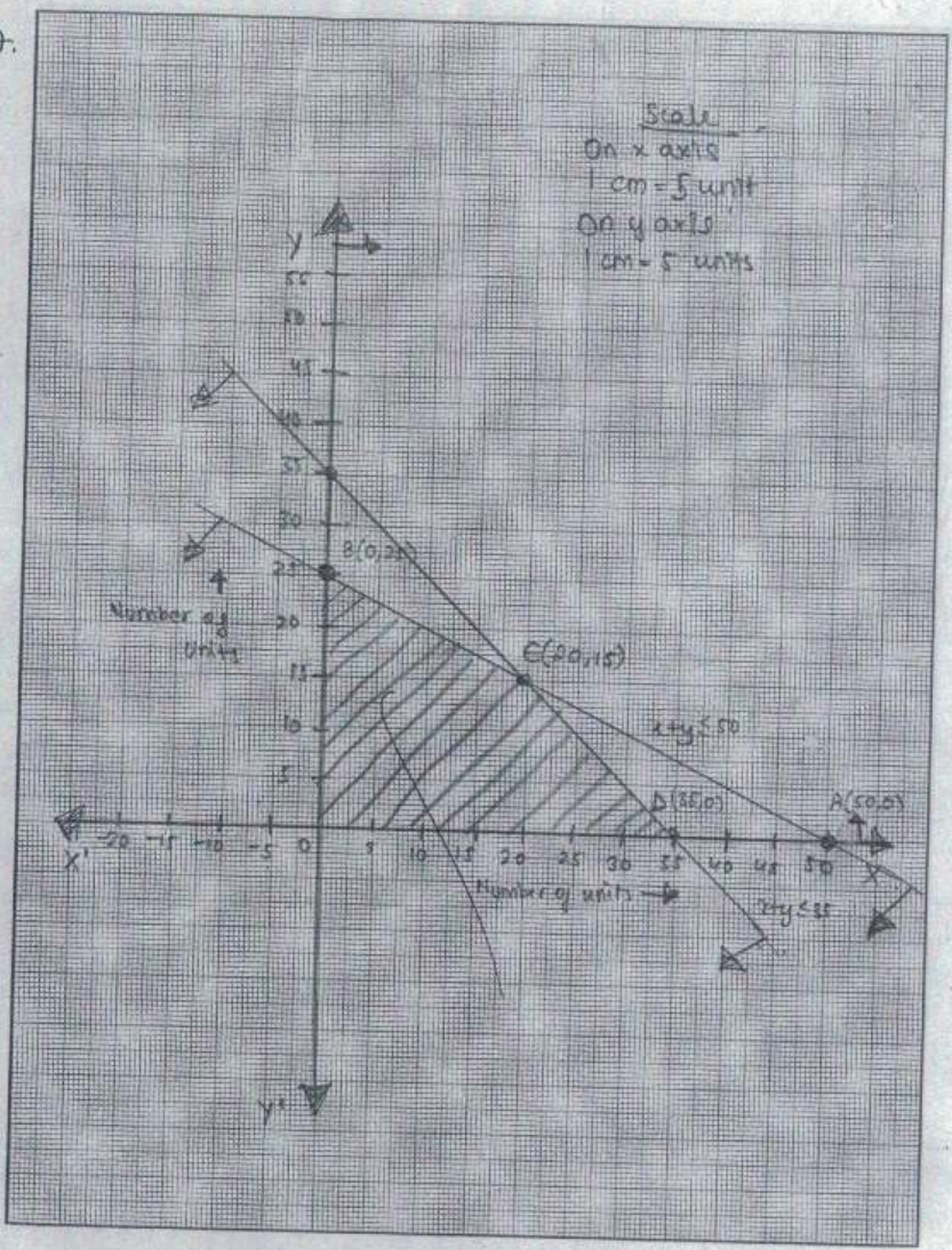
$\Rightarrow (a,b) R (e,f)$

$\forall a,b,c,d,e,f \in N$

$\therefore$ The relation is transitive

As the relation is reflexive, symmetric and

17.


Scale
On x axis
1 cm = 5 unit
On y axis
1 cm = 5 units
Y
X'
X
Y'
B(0,25)
E(20,15)
D(35,0)
A(50,0)
x+2y ≤ 50
x+y ≤ 35
Number of units
Number of units

transitive, it is an equivalence relation.

Hence Proved

Section B

21. $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$

$\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

2

A ~~unit~~ vector perpendicular to both $\vec{a}$ and $\vec{b}$ can be expressed as $\vec{c} = \lambda(\vec{a} \times \vec{b})$, λ is scalar.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix} = 24\hat{i} - 24\hat{j} - 12\hat{k} = \cancel{24\hat{i} - 2\hat{j}}$$

$$\therefore \quad \vec{c} = \pm \lambda(24\hat{i} - 24\hat{j} - 12\hat{k})$$

$$= \pm \beta(2\hat{i} - 2\hat{j} - \hat{k}) \quad , \quad \beta = \frac{1}{12}\lambda$$

$\therefore \hat{c} = \pm\left(\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}\right)$

22. $f(x) = \sin 2x$ in $[0, \pi]$

i. For continuity

~~sin~~ $f(x)$ is a sine function which is always continuous

$\therefore f(x)$ is continuous in $[0, \pi]$

ii. For differentiability

$f'(x) = 2\cos 2x$

which is a cosine function which is always ~~differentiable~~ continuous

$\therefore f(x)$ is differentiable in $(0, \pi)$

iii. $f(0) = 0$

~~f(π)~~ $f(\pi) = 0$

$\therefore f(0) = f(\pi)$.

∴ According to Rolle's theorem there exists at least one 'c' in $(0,\pi)$ where ~~f(c)~~ $f'(c)=0$

$f'(c) = 2\cos 2c = 0$

$\cos 2c = 0$

$c = \frac{(2n+1)\pi}{4},\ n \in N$.

$c = \pi/4, 3\pi/4 \in (0,\pi)$

∴ ~~The ta~~ Hence verified

The tangent is parallel to x axis when $f'(x)=0$

$x = \frac{(2n+1)\pi}{4},\ n \in N$

In the range $x = \pi/4, 3\pi/4$.

when $x = \pi/4$ $\quad x = 3\pi/4$

$y = 1$ $\quad$ ~~y=~~ $y = -1$

Points are $(\pi/4, 1), (3\pi/4, -1)$

Q.5. $f(x) = 2 + 3x - x^3$

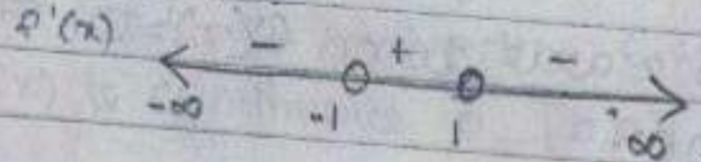
$f'(x) = 3 - 3x^2$

$= 3(1-x^2)$

$= 3(1-x)(1+x)$

For decreasing

$f'(x) < 0$



$f'(x) < 0$ when $x \in (-\infty,-1] \cup [1,\infty)$

∴ ~~The function is decreasing when $x \in (-\infty,-1] \cup [1,\infty)$~~.

Also $f(x)$ is continuous at $x = -1, 1$

∴ The function is decreasing in $(-\infty,-1] \cup [1,\infty)$

$$\begin{array}{r} 21 \\ \times 3 \\ \hline 63 \\ \times 2 \\ \hline 126 \end{array}$$

24. let A be the event of finding a green signal on 2 consecutive days.

let B_i, $i=1,2,3$ be the probability of finding a green signal on i^{th} day.

$P(B_1) = P(B_2) = P(B_3) = \frac{30}{100}$.

$P(A) = P(B_1)P(B_2)P(\overline{B_3}) + P(\overline{B_1})P(B_2)P(B_3)$

$= \frac{30}{100} \times \frac{30}{100} \times \frac{70}{100} + \frac{70}{100} \times \frac{30}{100} \times \frac{30}{100}$

$= \frac{126\cancel{000}}{1000\cancel{000}} = 12.6\%$ or $\frac{126}{1000}$

25. let $y = \sin^{-1}(2x\sqrt{1-x^2})$

let $x = \cos\theta$

$\Rightarrow \theta = \cos^{-1} x$ $\quad 0 \le \theta \le \pi/4$

$\Rightarrow y = \sin^{-1}(2\cos\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}\sin 2\theta$

$= 2\theta$ $\quad [\because 0 \le \theta \le \pi/4 \Rightarrow 0 \le 2\theta \le \pi/2]$

$= 2\cos^{-1} x$

Hence Proved

26. $\frac{x}{k} = \frac{y}{-k} = \frac{z}{1}$ - line (i)

$\frac{x-2}{1} = \frac{y+1/2}{1/2} = \frac{z-1}{-1}$ - line (ii)

dR of line (i) $\langle k, -k, 1\rangle$

dR of line (ii) $\langle 1, 1/2, -1\rangle$

As they are perpendicular

$k - \frac{k}{2} - 1 = 0$

$\frac{k}{2} = 1 \Rightarrow k = 2$

Section A

1. $|A.\text{Adj}A| = |A||\text{Adj}A|$
$= |A||A|^{n-1}$
$= |A|^n$
$= (-2)^3$
$= -8$.

C. -8.

2. A. 0 As it is a particular solution.

3. $\cos^{-1}\cos\left(\frac{13\pi}{6}\right) = \cos^{-1}\cos\left(2\pi + \frac{\pi}{6}\right) = \cos^{-1}\cos\left(\frac{\pi}{6}\right) = \pi/6$.
D. $\pi/6$

4. $2a + 4b = 4a$
$\Rightarrow 2a = 4b$
$a = 2b$

A. a

5. $P(B'/A) = \dfrac{P(B' \cap A)}{P(A)} = \dfrac{\frac{3}{4} \times \frac{1}{3}}{\frac{1}{3}} = \dfrac{3}{4}$.

C. 3/4

6. $(I-A)^3 + A$

$= I^3 - A^3 \cancel{3A} + 3A^2 I - 3A + A$.

$= I - A^2 A + 3A^2 - 3A + A$

$= I - A^2 + 3A - 3A + A$

$= I - A + 3A - 3A + A$

$= I$

A. I

7. Here $f(-x) = -f(x)$.

$\therefore$ It is an odd function. $\therefore$ Its value is 0.

B. 0

8. A (−2, −1, 5)

9. ~~If projection is zero~~

$$\text{Projection} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = 0$$

$$\Rightarrow \vec{a}\cdot\vec{b} = 0$$

$$2 + 3\lambda = 0$$

$$\Rightarrow \lambda = \frac{-2}{3}$$

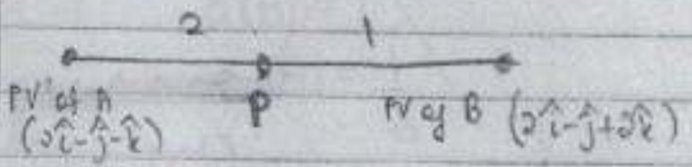
C. $-2/3$.

10. ~~A. $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$~~

C. $\hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$

b.

11.



$$\text{PV of } P = \frac{2(2\hat{i}-\hat{j}+2\hat{k}) + (2\hat{i}-\hat{j}-\hat{k})}{2+1}$$

$$= \frac{4\hat{i}-2\hat{j}+4\hat{k}+2\hat{i}-\hat{j}-\hat{k}}{3}$$

12. $y^2 = 8x$

$2y\frac{dy}{dx} = 8$

$\frac{dy}{dx} = \frac{4}{y}$.

slope of normal $= -\frac{\cancel{4}}{\cancel{y}} \; -\frac{y}{4}$.

slope of normal (at 0,0) $= -0 = 0$

$\therefore$ equation of normal $\Rightarrow$ $y-0 = 0(x-0)$

$\Rightarrow \underline{y = 0}$

(OR.

$\frac{dr}{dt} = 3$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\left(\frac{dA}{dt}\right)_{r=2} = 2\pi(2)(3) = 12\pi \; cm^2/sec$.

13. OR

A square matrix A is said to be symmetric if $A^T = A$

i.e. $a_{ij} = a_{ji}$.

14. symmetric

15. $x = 1$

16. $A^2 = 2A$

Premultiplying with A^{-1}

$A^{-1}AA = 2A^{-1}A$

$\cancel{A = 2A^{-1}}$ $\quad$ $A = 2I$

$\cancel{|A| = 8|A^{-1}|}$ $\quad$ $|A| = 8|I|$

$\cancel{|A| = 8/|A|^2}$ $\quad$ $|A| = 8$.

$\cancel{8|A| = 1}$

$\cancel{|A| = \frac{1}{8}}$

17. let A be the event one card is red and other black.

$$P(A) = \frac{26}{52} \times \frac{26}{51} \times 2 = \frac{1}{2} \times \frac{26}{51} \times 2 = \frac{26}{51}$$

18. $\int_1^3 |2x-1|\, dx$

$\cancel{f(x) = |2x-1|}$

$\cancel{= 1-2x, \quad 0 \le x \le 1/2}$

$\cancel{= 2x-1, \quad 1/2 \le}$

$= \int_1^3 2x-1 \, dx$

$= [x^2 - x]_1^3$

$= 9-3-1+1$

$= 6$

19. $\int \frac{dx}{\sqrt{9-4x^2}}$

$= \frac{1}{2}\int \frac{dx}{\sqrt{(3/2)^2 - x^2}}$

$= \frac{1}{2} \sin^{-1}\left(\frac{2x}{3}\right) + C$

20. $\int \frac{2x\,dx}{\sqrt[3]{x^2+1}}$

let $x^2+1 = z$

$2x\,dx = dz$

$= \int \frac{dz}{z^{1/3}}$

$= \int z^{-1/3}\,dz$

$= \frac{3z^{2/3}}{2} + C$

$= \frac{3(x^2+1)^{2/3}}{2} + C$