**Paper Specific Instructions**

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

## SECTION – A

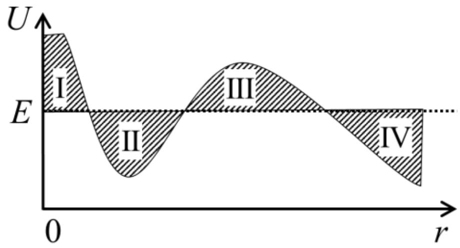
### MULTIPLE CHOICE QUESTIONS (MCQ)

**Q. 1 – Q.10 carry one mark each.**

Q.1 The function $f(x) = \dfrac{8x}{x^2+9}$ is continuous everywhere except at

(A) $x = 0$ (B) $x = \pm 9$ (C) $x = \pm 9i$ (D) $x = \pm 3i$

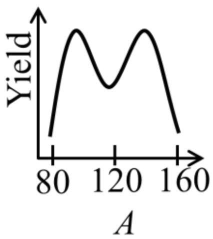
Q.2 A classical particle has total energy $E$. The plot of potential energy ($U$) as a function of distance ($r$) from the centre of force located at $r = 0$ is shown in the figure. Which of the regions are forbidden for the particle?




(A) I and II (B) II and IV (C) I and IV (D) I and III

Q.3 In the thermal neutron induced fission of $^{235}$U, the distribution of relative number of the observed fission fragments (Yield) versus mass number ($A$) is given by

(A)

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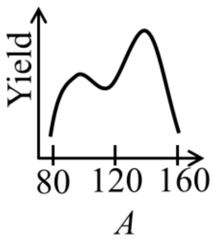


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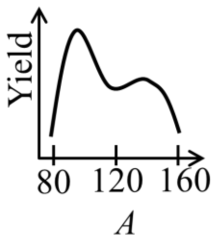


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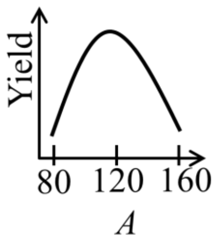
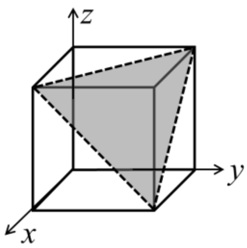



Q.4 Which one of the following crystallographic planes represent (101) Miller indices of a cubic unit cell?

(A)

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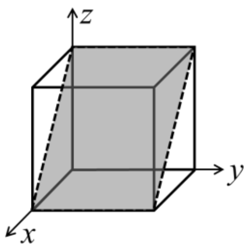


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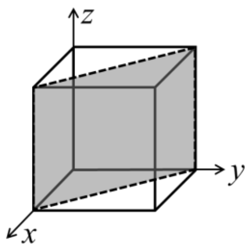


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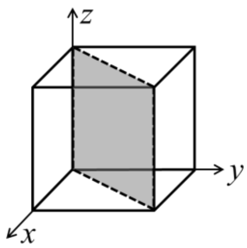


Q.5 The Fermi-Dirac distribution function $[n(\varepsilon)]$ is
($k_B$ is the Boltzmann constant, $T$ is the temperature and $\varepsilon_F$ is the Fermi energy)

(A) $n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\varepsilon_F}{k_B T}} - 1}$ (B) $n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon_F-\varepsilon}{k_B T}} - 1}$ (C) $n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\varepsilon_F}{k_B T}} + 1}$ (D) $n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon_F-\varepsilon}{k_B T}} + 1}$

Q.6 If $\phi(x,y,z)$ is a scalar function which satisfies the Laplace equation, then the gradient of $\phi$ is

(A) Solenoidal and irrotational
(B) Solenoidal but not irrotational
(C) Irrotational but not solenoidal
(D) Neither solenoidal nor irrotational

Q.7 In a heat engine based on the Carnot cycle, heat is added to the working substance at constant

(A) Entropy (B) Pressure (C) Temperature (D) Volume

Q.8 Isothermal compressibility is given by

(A) $\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ (B) $\frac{1}{P}\left(\frac{\partial P}{\partial V}\right)_T$ (C) $-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ (D) $-\frac{1}{P}\left(\frac{\partial P}{\partial V}\right)_T$

Q.9 For using a transistor as an amplifier, choose the correct option regarding the resistances of base-emitter ($R_{BE}$) and base-collector ($R_{BC}$) junctions

(A) Both $R_{BE}$ and $R_{BC}$ are very low
(B) Very low $R_{BE}$ and very high $R_{BC}$
(C) Very high $R_{BE}$ and very low $R_{BC}$
(D) Both $R_{BE}$ and $R_{BC}$ are very high

Q.10 A unit vector perpendicular to the plane containing $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + \hat{k}$ is

(A) $\frac{1}{\sqrt{26}}(-\hat{i} + 3\hat{j} - 4\hat{k})$
(B) $\frac{1}{\sqrt{19}}(-\hat{i} + 3\hat{j} - 3\hat{k})$
(C) $\frac{1}{\sqrt{35}}(-\hat{i} + 5\hat{j} - 3\hat{k})$
(D) $\frac{1}{\sqrt{35}}(-\hat{i} - 5\hat{j} - 3\hat{k})$

**Q. 11 – Q. 30 carry two marks each.**

Q.11 A thin lens of refractive index 3/2 is kept inside a liquid of refractive index 4/3. If the focal length of the lens in air is 10 cm, then its focal length inside the liquid is

(A) 10 cm (B) 30 cm (C) 40 cm (D) 50 cm

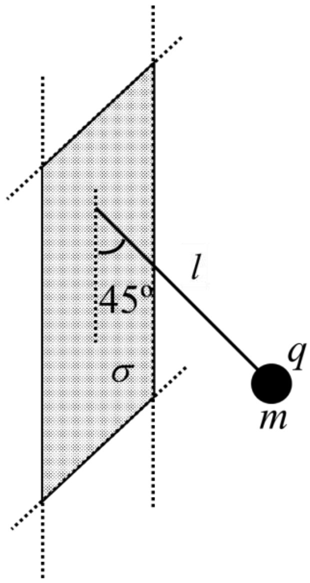
Q.12 The eigenvalues of $\begin{pmatrix} 3 & i & 0 \\ -i & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ are

(A) 2, 4 and 6 (B) $2i$, $4i$ and 6 (C) $2i$, 4 and 8 (D) 0, 4 and 8

Q.13 For a quantum particle confined inside a cubic box of side $L$, the ground state energy is given by $E_0$. The energy of the first excited state is

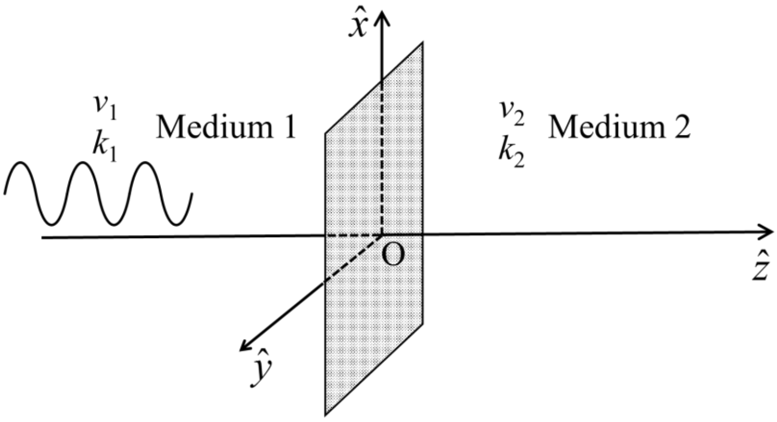
(A) $2E_0$ (B) $\sqrt{2}\,E_0$ (C) $3E_0$ (D) $6E_0$

Q.14 A small spherical ball having charge $q$ and mass $m$, is tied to a thin massless non-conducting string of length $l$. The other end of the string is fixed to an infinitely extended thin non-conducting sheet with uniform surface charge density $\sigma$. Under equilibrium, the string makes an angle 45° with the sheet as shown in the figure. Then $\sigma$ is given by ($g$ is the acceleration due to gravity and $\varepsilon_0$ is the permittivity of free space)




(A) $\frac{mg\varepsilon_0}{q}$ (B) $\sqrt{2}\frac{mg\varepsilon_0}{q}$ (C) $2\frac{mg\varepsilon_0}{q}$ (D) $\frac{mg\varepsilon_0}{q\sqrt{2}}$

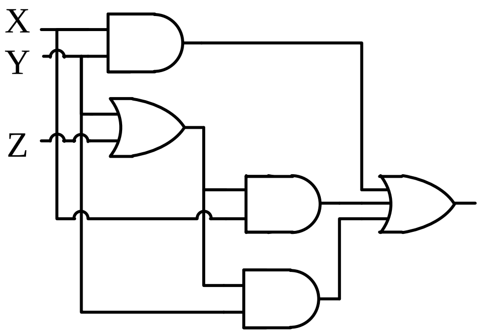
Q.15 Consider the normal incidence of a plane electromagnetic wave with electric field given by $\vec{E} = E_0 \exp[i(k_1 z - \omega t)]\hat{x}$ over an interface at $z = 0$ separating two media [wave velocities $v_1$ and $v_2$ ($v_2 > v_1$) and wave vectors $k_1$ and $k_2$, respectively] as shown in figure. The magnetic field vector of the reflected wave is
($\omega$ is the angular frequency)




(A) $\frac{E_0}{v_1}\exp[i(k_1 z - \omega t)]\hat{y}$ (B) $\frac{E_0}{v_1}\exp[i(-k_1 z - \omega t)]\hat{y}$

(C) $\frac{-E_0}{v_1}\exp[i(-k_1 z - \omega t)]\hat{y}$ (D) $\frac{-E_0}{v_1}\exp[i(k_1 z - \omega t)]\hat{y}$

Q.16 The output of following logic circuit can be simplified to




(A) X + YZ (B) Y + XZ (C) XYZ (D) X + Y + Z

Q.17 A red star having radius $r_R$ at a temperature $T_R$ and a white star having radius $r_W$ at a temperature $T_W$, radiate the same total power. If these stars radiate as perfect black bodies, then

(A) $r_R > r_W$ and $T_R > T_W$ (B) $r_R < r_W$ and $T_R > T_W$

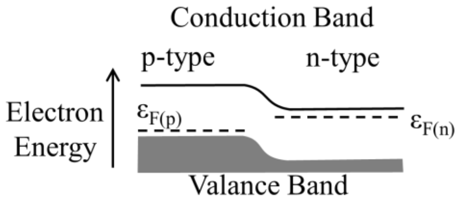
(C) $r_R > r_W$ and $T_R < T_W$ (D) $r_R < r_W$ and $T_R < T_W$

Q.18 The mass per unit length of a rod (length 2 m) varies as $\rho = 3x$ kg/m. The moment of inertia (in kg m$^2$) of the rod about a perpendicular-axis passing through the tip of the rod (at $x = 0$) is

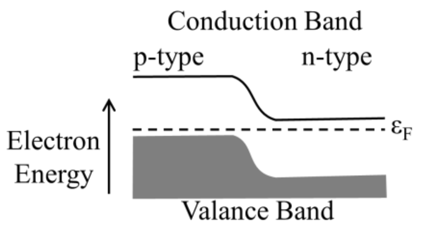
(A) 10 (B) 12 (C) 14 (D) 16

Q.19 For a forward biased p-n junction diode, which one of the following energy-band diagrams is correct?
($\varepsilon_F$ is the Fermi energy)

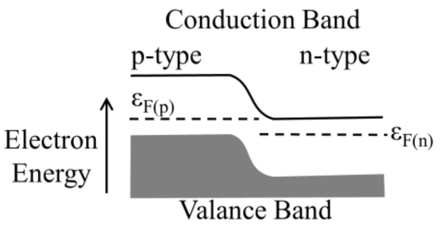
(A)



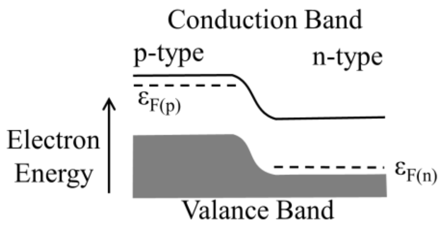

(B)




(C)



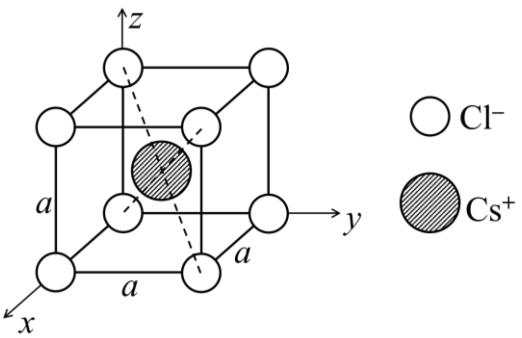

(D)




Q.20 The amount of work done to increase the speed of an electron from $c/3$ to $2c/3$ is
($c = 3\times10^8$ m/s and rest mass of electron is 0.511 MeV)

(A) 56.50 keV (B) 143.58 keV (C) 168.20 keV (D) 511.00 keV

Q.21 The location of $Cs^+$ and $Cl^-$ ions inside the unit cell of CsCl crystal is shown in the figure. The Bravais lattice of CsCl is




(A) simple cubic
(B) body centered orthorhombic
(C) face centered cubic
(D) base centered orthorhombic

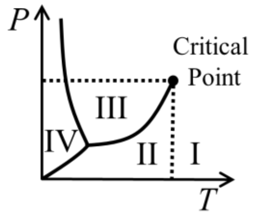
Q.22 A γ-ray photon emitted from a $^{137}Cs$ source collides with an electron at rest. If the Compton shift of the photon is $3.25 \times 10^{-13}$ m, then the scattering angle is closest to
(Planck's constant $h = 6.626 \times 10^{-34}$ J s, electron mass $m_e = 9.109 \times 10^{-31}$ kg and velocity of light in free space $c = 3\times10^8$ m/s)

(A) 45° (B) 60° (C) 30° (D) 90°

Q.23 During free expansion of an ideal gas under adiabatic condition, the internal energy of the gas

(A) Decreases
(B) Initially decreases and then increases
(C) Increases
(D) Remains constant

Q.24 In the given phase diagram for a pure substance, regions I, II, III, IV, respectively represent




(A) Vapor, Gas, Solid, Liquid
(B) Gas, Vapor, Liquid, Solid
(C) Gas, Liquid, Vapor, Solid
(D) Vapor, Gas, Liquid, Solid

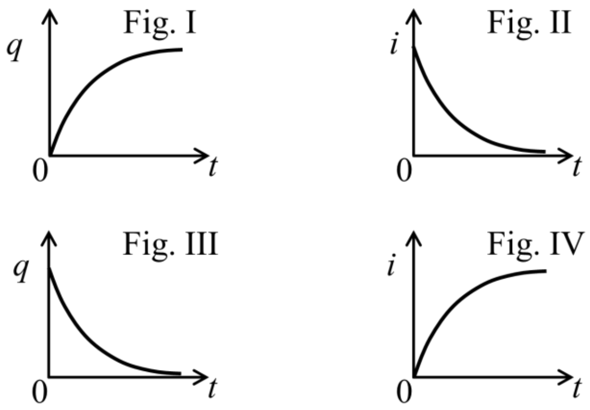
Q.25 Light of wavelength λ (in free space) propagates through a dispersive medium with refractive index $n(\lambda)=1.5 + 0.6\lambda$. The group velocity of a wave travelling inside this medium in units of $10^8$ m/s is

(A) 1.5 (B) 2.0 (C) 3.0 (D) 4.0

Q.26 The maximum number of intensity minima that can be observed in the Fraunhofer diffraction pattern of a single slit (width 10 μm) illuminated by a laser beam (wavelength 0.630 μm) will be

(A) 4 (B) 7 (C) 12 (D) 15

Q.27 During the charging of a capacitor C in a series RC circuit, the typical variations in the magnitude of the charge $q(t)$ deposited on one of the capacitor plates, and the current $i(t)$ in the circuit, respectively are best represented by




(A) Fig. I and Fig. II
(B) Fig. I and Fig. IV
(C) Fig. III and Fig. II
(D) Fig. III and Fig. IV

Q.28 Which one of the following is an impossible magnetic field $\vec{B}$?

(A) $\vec{B} = 3x^2z^2\hat{x} - 2xz^3\hat{z}$

(B) $\vec{B} = -2xy\,\hat{x} + yz^2\hat{y} + \left(2yz - \frac{z^3}{3}\right)\hat{z}$

(C) $\vec{B} = (xz + 4y)\hat{x} - yx^3\hat{y} + \left(x^3z - \frac{z^2}{2}\right)\hat{z}$

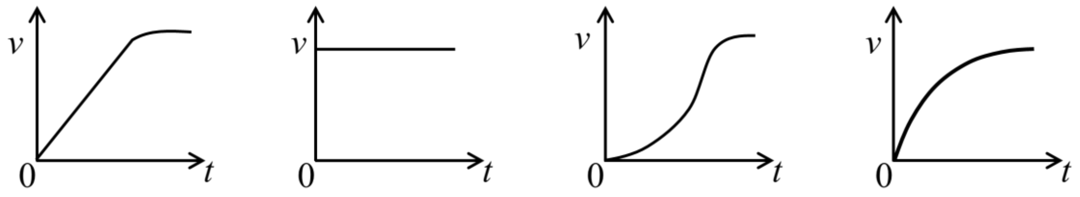
(D) $\vec{B} = -6xz\,\hat{x} + 3yz^2\hat{y}$

Q.29 If the motion of a particle is described by $x = 5\cos(8\pi t)$, $y = 5\sin(8\pi t)$ and $z = 5t$, then the trajectory of the particle is

(A) Circular (B) Elliptical (C) Helical (D) Spiral

Q.30 A ball of mass $m$ is falling freely under gravity through a viscous medium in which the drag force is proportional to the instantaneous velocity $v$ of the ball. Neglecting the buoyancy force of the medium, which one of the following figures best describes the variation of $v$ as a function of time $t$?

(A) (B) (C) (D)




## SECTION - B

## MULTIPLE SELECT QUESTIONS (MSQ)

**Q. 31 – Q. 40 carry two marks each.**

Q.31 The relation between the nuclear radius ($R$) and the mass number ($A$), given by $R = 1.2\,A^{1/3}$ fm, implies that

(A) The central density of nuclei is independent of $A$
(B) The volume energy per nucleon is a constant
(C) The attractive part of the nuclear force has a long range
(D) The nuclear force is charge dependent

Q.32 Consider an object moving with a velocity $\vec{v}$ in a frame which rotates with a constant angular velocity $\vec{\omega}$. The Coriolis force experienced by the object is

(A) along $\vec{v}$
(B) along $\vec{\omega}$
(C) perpendicular to both $\vec{v}$ and $\vec{\omega}$
(D) always directed towards the axis of rotation

Q.33 The gradient of a scalar field $S(x,y,z)$ has the following characteristic(s).

(A) Line integral of a gradient is path-independent
(B) Closed line integral of a gradient is zero
(C) Gradient of $S$ is a measure of the maximum rate of change in the field $S$
(D) Gradient of $S$ is a scalar quantity

Q.34 A thermodynamic system is described by the $P$, $V$, $T$ coordinates. Choose the valid expression(s) for the system.

(A) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial P}{\partial T}\right)_V$

(B) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial P}{\partial T}\right)_V$

(C) $\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -\left(\frac{\partial V}{\partial P}\right)_T$

(D) $\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = \left(\frac{\partial V}{\partial P}\right)_T$

Q.35 Which of the following statement(s) is/are true?

(A) Newton's laws of motion and Maxwell's equations are both invariant under Lorentz transformations.
(B) Newton's laws of motion and Maxwell's equations are both invariant under Galilean transformations.
(C) Newton's laws of motion are invariant under Galilean transformations and Maxwell's equations are invariant under Lorentz transformations.
(D) Newton's laws of motion are invariant under Lorentz transformations and Maxwell's equations are invariant under Galilean transformations.

Q.36 For an underdamped harmonic oscillator with velocity $v(t)$,

(A) Rate of energy dissipation varies linearly with $v(t)$
(B) Rate of energy dissipation varies as square of $v(t)$
(C) The reduction in the oscillator frequency, compared to the undamped case, is independent of $v(t)$
(D) For weak damping, the amplitude decays exponentially to zero

Q.37 Out of the following statements, choose the correct option(s) about a perfect conductor.

(A) The conductor has an equipotential surface
(B) Net charge, if any, resides only on the surface of conductor
(C) Electric field cannot exist inside the conductor
(D) Just outside the conductor, the electric field is always perpendicular to its surface

Q.38 In the X-ray diffraction pattern recorded for a simple cubic solid (lattice parameter $a$ = 1 Å) using X rays of wavelength 1 Å, the first order diffraction peak(s) would appear for the

(A) (100) planes
(B) (112) planes
(C) (210) planes
(D) (220) planes

Q.39 Consider a classical particle subjected to an attractive inverse-square force field. The total energy of the particle is E and the eccentricity is $\varepsilon$. The particle will follow a parabolic orbit if

(A) $E > 0$ and $\varepsilon = 1$
(B) $E < 0$ and $\varepsilon < 1$
(C) $E = 0$ and $\varepsilon = 1$
(D) $E < 0$ and $\varepsilon = 1$

Q.40 An atomic nucleus X with half-life $T_X$ decays to a nucleus Y, which has half-life $T_Y$. The condition(s) for secular equilibrium is(are)

(A) $T_X \simeq T_Y$
(B) $T_X < T_Y$
(C) $T_X \ll T_Y$
(D) $T_X \gg T_Y$

## SECTION – C

## NUMERICAL ANSWER TYPE (NAT)

**Q. 41 – Q. 50 carry one mark each.**

Q.41 In a typical human body, the amount of radioactive $^{40}K$ is $3.24 \times 10^{-5}$ percent of its mass. The activity due to $^{40}K$ in a human body of mass 70 kg is ________ kBq.
(Round off to 2 decimal places)
(Half-life of $^{40}K = 3.942 \times 10^{16}$ s, Avogadro's number $N_A = 6.022 \times 10^{23}$ mol$^{-1}$)

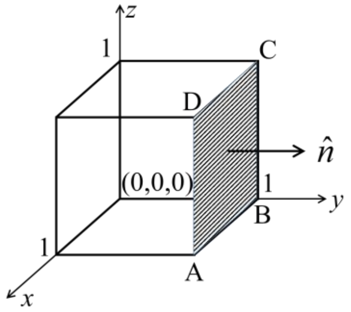
Q.42 Sodium (Na) exhibits body-centered-cubic (BCC) crystal structure with atomic radius 0.186 nm. The lattice parameter of Na unit cell is ________ nm.
(Round off to 2 decimal places)

Q.43 Light of wavelength 680 nm is incident normally on a diffraction grating having 4000 lines/cm. The diffraction angle (in degrees) corresponding to the third-order maximum is ________.
(Round off to 2 decimal places)

Q.44 Two gases having molecular diameters $D_1$ and $D_2$, and mean free paths $\lambda_1$ and $\lambda_2$, respectively, are trapped separately in identical containers.
If $D_2 = 2D_1$, then $\lambda_1/\lambda_2 =$ ________.
(Assume there is no change in other thermodynamic parameters)

Q.45 An object of 2 cm height is placed at a distance of 30 cm in front of a concave mirror with radius of curvature 40 cm. The height of the image is ________ cm.

Q.46 The flux of the function $\vec{F} = (y^2)\hat{x} + (3xy - z^2)\hat{y} + (4yz)\hat{z}$ passing through the surface ABCD along $\hat{n}$ is ________.
(Round off to 2 decimal places)

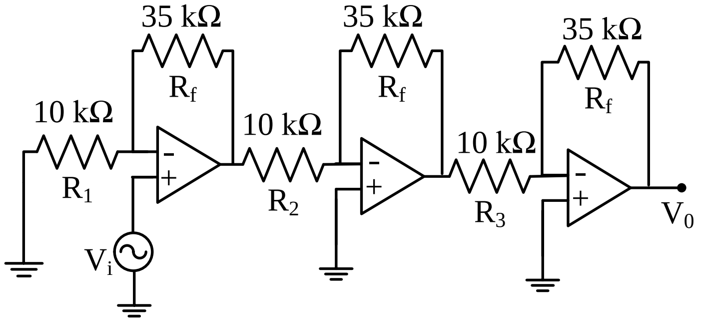



Q.47 The electrostatic energy (in units of $\frac{1}{4\pi\varepsilon_0}$ J) of a uniformly charged spherical shell of total charge 5 C and radius 4 m is ________.
(Round off to 3 decimal places)

Q.48 An infinitely long very thin straight wire carries uniform line charge density $8\pi \times 10^{-2}$ C/m. The magnitude of electric displacement vector at a point located 20 mm away from the axis of the wire is ______ C/m$^2$.

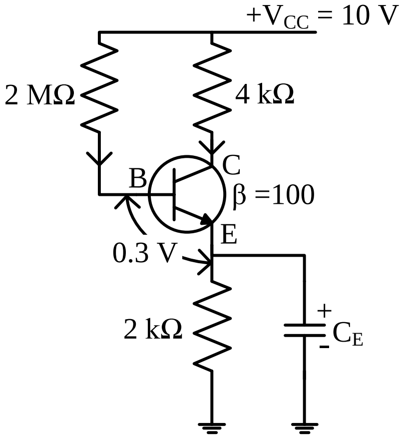
Q.49 The 7$^{th}$ bright fringe in the Young's double slit experiment using a light of wavelength 550 nm shifts to the central maxima after covering the two slits with two sheets of different refractive indices $n_1$ and $n_2$ but having same thickness 6 μm. The value of $|n_1 - n_2|$ is ________.
(Round off to 2 decimal places)

Q.50 For the input voltage $V_i$ = (200 mV) sin (400$t$), the amplitude of the output voltage ($V_0$) of the given OPAMP circuit is ________ V.
(Round off to 2 decimal places)



**Q. 51 – Q. 60 carry two marks each.**

Q.51 The value of emitter current in the given circuit is ________ μA.
(Round off to 1 decimal place)



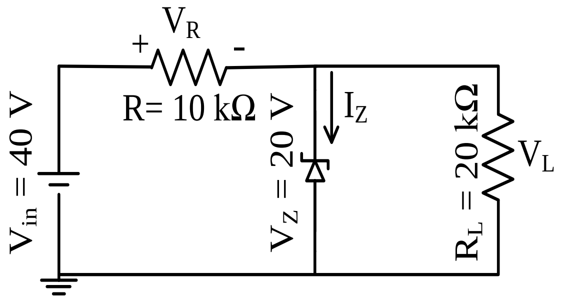

Q.52 The value of $\left| \int_0^{3+i} (\bar{z})^2 dz \right|^2$, along the line $3y = x$, where $z = x + iy$ is ________.
(Round off to 1 decimal place)

Q.53 If the wavelength of $K\alpha_2$ X-ray line of an element is 1.544Å, then the atomic number ($Z$) of the element is ________.
(Rydberg constant R = $1.097\times10^7 \text{m}^{-1}$ and velocity of light $c = 3\times10^8$ m/s)

Q.54 A proton is confined within a nucleus of size $10^{-13}$ cm. The uncertainty in its velocity is ________ $\times 10^8$ m/s.
(Round off to 2 decimal places)
(Planck's constant $h = 6.626 \times 10^{-34}$ J s and proton mass $m_p = 1.672 \times 10^{-27}$ kg)

Q.55 Given the wave function of a particle $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L} x\right)$ for $0 < x < L$ and 0 elsewhere, the probability of finding the particle between $x = 0$ and $x = L/2$ is _______.
(Round off to 1 decimal place)

Q.56 The Zener current $I_Z$ for the given circuit is ________ mA.

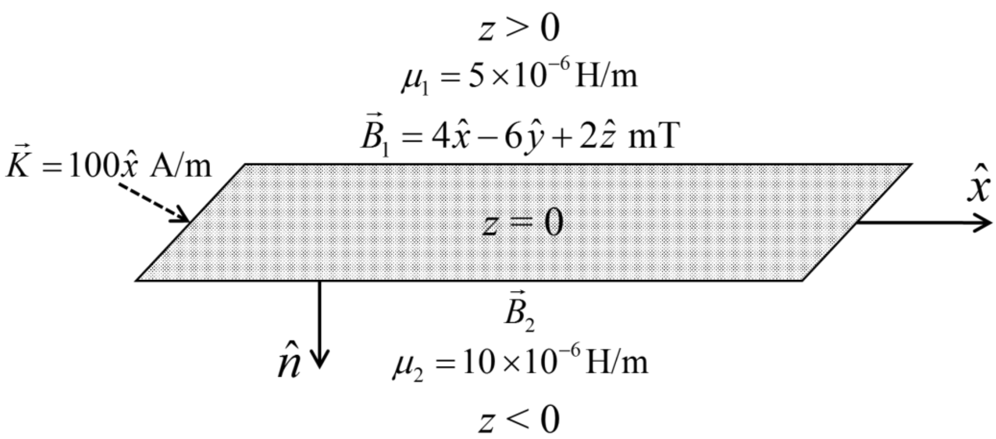



Q.57 If the diameter of the Earth is increased by 4% without changing the mass, then the length of the day is ________hours.
(Take the length of the day before the increment as 24 hours. Assume the Earth to be a sphere with uniform density.)
(Round off to 2 decimal places)

Q.58 A di-atomic gas undergoes adiabatic expansion against the piston of a cylinder. As a result, the temperature of the gas drops from 1150 K to 400 K. The number of moles of the gas required to obtain 2300 J of work from the expansion is ________.
(The gas constant $R = 8.314$ J mol$^{-1}$K$^{-1}$.)
(Round off to 2 decimal places)

Q.59 The decimal equivalent of the binary number 110.101 is ________.

Q.60 A surface current $\vec{K} = 100\,\hat{x}$ A/m flows on the surface $z = 0$, which separates two media with magnetic permeabilities $\mu_1$ and $\mu_2$ as shown in the figure. If the magnetic field in the region 1 is $\vec{B}_1 = 4\hat{x} - 6\hat{y} + 2\hat{z}$ mT, then the magnitude of the normal component of $\vec{B}_2$ will be ________ mT.




**END OF THE QUESTION PAPER**