Indian National Astronomy Olympiad – 2014

Question Paper Roll Number:

INAO – 2014 Date: 1^{st} February 2014

Duration: **Three Hours** Maximum Marks: 100

Please Note:

- Please write your roll number on top of this page in the space provided.
- Before starting, please ensure that you have received a copy of the question paper containing total 3 pages (6 sides).
- There are total 8 questions. Maximum marks are indicated in front of each question.
- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Blank spaces are provided in the question paper for the rough work. No rough work should be done on the answer-sheet.
- No computational aides like calculators, log tables, slide rule etc. are allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question booklet back with you.
- Please be advised that tentative dates for the next stage are as follows:
  - Orientation Camp (Junior): 28^{th} April to 6^{th} May 2013.
  - Orientation Camp (Senior): 1^{st} May to 6^{th} May 2013.
  - Selection Camp (Jr. + Sr.): 27^{th} May to 6^{th} June 2013.
  - Participation in both parts (Orientation and Selection) is mandatory for all participants.

Useful Physical Constants

Mass of the Earth	M_E	$\approx$	6×10^{24} kg
Radius of the Earth	R_E	$\approx$	6.4×10^{6} m
Mass of Jupiter	M_J	$\approx$	2×10^{27} kg
Astronomical Unit	1 A. U.	$\approx$	1.5×10^{11} m
Gravitational acceleration	g	$\approx$	9.8 m/s^2
Avogadro constant	N_a	$\approx$	6.023×10^{23} mol^{-1}
Solar Constant	S	$\approx$	1400 W/m^2

1. (12 marks) Read the following passage and point out scientific inaccuracies. Give very brief argument for each mistake you point out.

> Arrival of a spacecraft on the surface of the Moon is called a Moon landing. This includes both manned and unmanned (robotic) missions. In order to get to the Moon, a spacecraft must first cross the point, beyond which the Earth's gravitational force is zero. The only practical way of accomplishing this currently is with a rocket. At every instant, the rocket has to produce thrust to propel itself to a velocity equal to critical velocity at that height. Jet engines can be used to propel the spacecraft to the Moon, but rockets provide greater advantage in terms of power required for given mass of the spacecraft.
>
> Unlike the Earth, the Moon does not have thick atmosphere to absorb most of the solar radiation and the magnetic field of the moon is too weak to deflect the UV rays. Thus, astronauts traveling to the Moon are exposed to harmful electromagnetic radiation. Spacesuits for astronauts are specially designed by keeping this in mind.
>
> On arrival near the Moon, the spacecraft is captured by the lunar gravity in an orbit around the Moon. As the Moon is much smaller than the Earth, typically these orbits are low altitude orbits (close to the lunar surface) as compared to the polar satellites around the Earth. The spacecraft can stay in this orbit forever, however, if one needs it to land on the Moon, it has to fire its engines to change course.
>
> Landing at the Moon can be of two types. A hard landing is equivalent to crash landing on the Moon. A soft landing is a controlled landing in which priority is to maintain all equipment and astronauts (if any) inside the spacecraft safe. Needless to say, all Apollo missions had a soft landing on the Moon. This was achieved by firing reversing rockets very close the lunar surface to slow down the spacecraft. Charring of the lunar soil, due to burning of these rockets, is one of the permanent impressions left by humankind on the Moon.

Solution:

- Earth's gravitational force goes to zero only at infinity. Close to the Moon, the force is small but not zero.
- Velocity needed is bigger than the critical velocity.
- For jet engines, the oxygen is taken from the air. It is not stored on-board. So they cannot be used to take spacecraft to the Moon.
- UV rays are electromagnetic waves and are not affected by the magnetic field.
- Low altitude orbits are not stable, as imperfections on the lunar surface result in change in gravitational force from point to point.

- The lunar soil won't get charred due to firing of rockets as there is no oxygen on the lunar surface.
- One mark for each correct indentification of the mistake. One mark each for corresponding reasoning.
- If a blatantly wrong argument is resulting in incorrect identification of an inaccuracy, one mark will be subtracted.
- No negative marking for subtle mistakes in reasoning.

2. (6 marks) A comet is in elliptical orbit around the Sun with period T and semi-major axis a. A second comet around the Sun has same period T, but a different orbit with same eccentricity e. The two orbits are not necessarily in the same plane. Further, we also know that when the first comet is closest to the Sun in its orbit (at perihelion), the other comet is farthest from the Sun in its orbit (at aphelion). Given this configuration, what is the minimum possible distance between the two comets, when one of the comets is at its perihelion? Justify your answer by brief arguments / sketches.

Solution: By visualising the configuration correctly, it will be realised that to achieve minimum possible separation, the two orbits should lie in the same plane with their semi-major axes along the same line, such that sun - perihelion-1 - aphelion-2 are collinear in that sequence. **(3 marks)**
Thus, the minimum separation will be

$$\begin{aligned} d &= d_{aphelion,1} - d_{perihelion,2} \\ &= a(1+e) - a(1-e) \\ &= 2ae \quad (\textbf{3 marks}) \end{aligned}$$

3. On a flight between the cities of Oslo (61°N, 8°E) and Helsinki (60.2°N, 25°E), Mayank saw the Sun just at the horizon on the west. After 20 minutes, when he checked again, he saw the Sun was now 2° above the horizon.

   (a) (2 marks) The flight was traveling from which city to which city?
   (b) (8 marks) What was the speed of the aircraft with respect to the ground (in Km/hr)?
   (c) (4 marks) How long will this aircraft take to traverse the distance between these two cities? Assume uniform speed during the entire flight.

Solution: The Sun rising from the west can be observed only if we are traveling in the direction opposite to the Earth's rotation. Thus, the flight was from Helsinki to Oslo. **(2 marks)**

In 20 minutes, the Earth would rotate by 5° eastwards. Mayank saw the Sun rising by 2° in that time. This means, the plane moved (5° + 2°) westwards in 20 minutes. **(2 marks)**

We can approximate latitudes of both places to $\phi = 60°$.

$$\begin{aligned}
\theta &= 5° + 2° = 7° \\
\omega &= \frac{\theta}{20 \times 60} \times \frac{\pi}{180} = \frac{7}{1200} \times \frac{\pi}{180} \text{ rad s}^{-1} \quad (\textbf{2 marks}) \\
v &= r\omega = R_{\oplus}\omega\cos\phi \\
&= 6.4 \times 10^6 \times \frac{7\pi}{1200 \times 180} \times \cos 60° \\
&= 64 \times \frac{7\pi}{1.2 \times 1.8} \times \frac{1}{2} \approx \frac{28}{0.27} \times \frac{22}{7} \\
&\approx 326 \text{ m/s} \approx \frac{8800}{27} \times \frac{18}{5} \\
&\approx 1170 \text{ km/hr} \quad (\textbf{4 marks})
\end{aligned}$$

Difference between longitudes of the two cities is 17°. Thus,

$$\begin{aligned}
t &= \frac{17 \times 20}{7} \\
&\approx 49\text{min}
\end{aligned}$$

The flight will take about 49 minutes. **(4 marks)**

Note: The solution above is a simplification as θ is not estimated correctly. When Maynak will see the Sun 2° above horizon, it would actually mean that at 60° latitude, the Sun has rolled back by 4° along its apparent path.
Thus, solution using $\theta = 9°$ is more correct. However, students are not expected to know this at INO level and hence solution given above is accepted for full credit.

4. The skymap below corresponds to sky above Nagpur (21°N, 79°E at 09:00 am on 1st February 2014. If you are not used to using sky maps, it is important to note that sky map is usually seen lying down on the ground (feet to the South), facing the sky with map in your hand. Thus, East is on the left of the map and West is on the right. Answer the following questions:

   (a) (1 mark) Mark Polaris with letter 'P'.

   (b) (2 marks) Circumpolar stars for a given place are the stars, which will never go below the horizon. Draw boundary of this region and mark it by the letter 'C'.

   (c) (2 marks) The celestial equator is just a projection of the Earth's equator in the sky. It will be locus of points which are equidistant from the north and the south pole. Draw the equator on the map approximately and mark it with 'Q'.

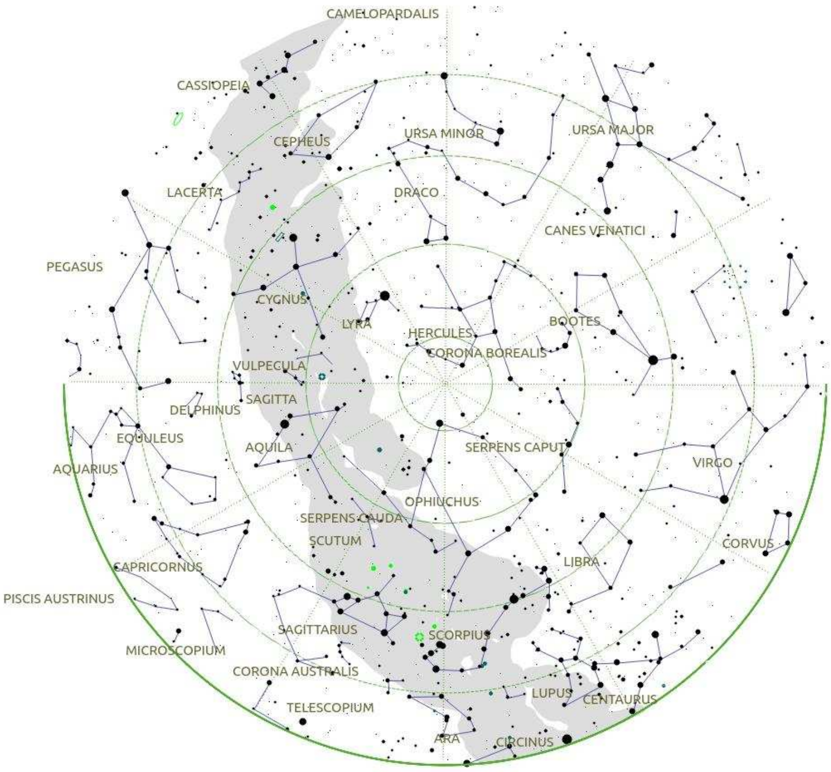
(d) (2 marks) The ecliptic is the imaginary yearly path of the Sun in the sky. Mark this approximately on the map and mark it with 'E'.

(e) (2 marks) Mark approximate position of the Sun on the map as 'S'.

(f) (2 marks) Yesterday was a new moon day. Mark the current position of the moon on the map as 'M'.

(g) (2 marks) Which star was very close to the Zenith at 06:00 am today? Mark it on the map as 'N'.

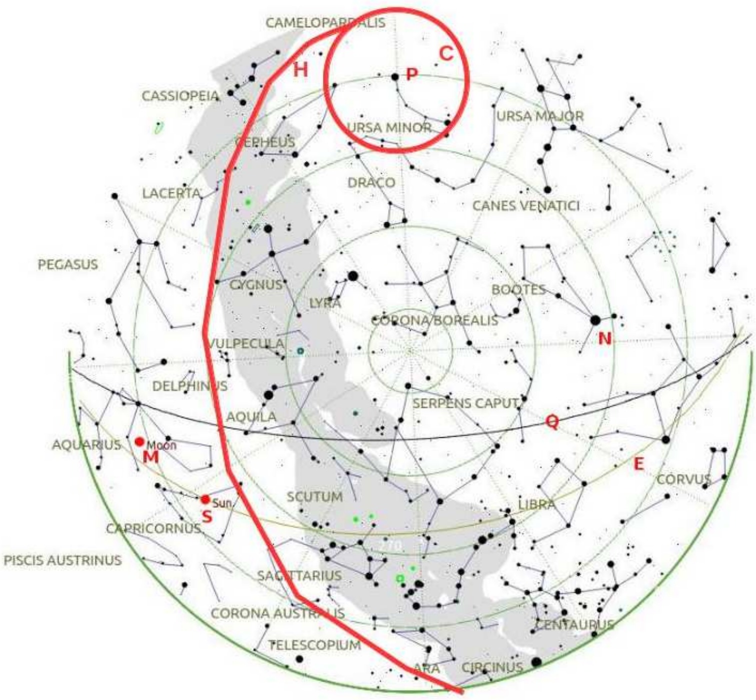
(h) (2 marks) Draw a line across sky showing horizon line as at 07:00 am today as 'H'.



Solution: Notes:

- Pole star should be exact.
- Circumpolar region should be a circle around 'P' with radius equal to Polaris-Northern horizon distance.
- Equator should be a smooth curve passing through East and West points and below zenith (centre) in the middle.
- Ecliptic should be a smooth curve roughly passing through the zodiacal signs (i.e. mostly below equator) and cutting equator in Virgo.

- The Sun should be in Capricornus.
- The Moon should be in Aquarius.
- Any star reasonably close to Archturus is accepted as answer for 'N'.
- Horizon at 7:00am should not cut current northern horizon, roughly pass close to the Sun and cut bottom edge on the west of south.



5. (12 marks) Gliese-876 is a star in the constellation of Aquarius which has four confirmed planets going around it. We know following facts about this system:

- the four planets have names 'b', 'c', 'd' and 'e'.
- the four planets have masses (in Earth masses) of $7M_\oplus$, $15M_\oplus$, $225M_\oplus$ and $720M_\oplus$. *(not necessarily in the same order)*
- the orbital radii of the four planets (in A.U.) are 1.93, 30, 61, 124. *(not necessarily in the same order)*
- the orbital eccentricities of the planets are 0.03, 0.06, 0.21, 0.26. *(not necessarily in the same order)*
- planet 'e' is farthest from the star.
- the heaviest planet has the lowest eccentricity.
- the lightest planet is also the closest to the star.
- the two giant planets are neither the closest nor the farthest from the star.
- the planet with a mass of $7M_\oplus$ has eccentricity of 0.21 and is closer to the star than planet 'c'.

- eccentricity of planet 'd' is more than planet 'e', but less than 'c'.
- planet 'c' is closer to the star as compared to planet 'b'.

Use the information above to correctly identify mass, orbital radius and orbital eccentricity of each of the planet. For your help, you may use the grid given in the answer sheet. The grid has six sub-grids of 4×4 size. You can tick mark in the boxes corresponding to matching pairs and cross mark the boxes where there is no match.

Note that this grid is only for your help. It is your choice if you wish to use this grid or some other method. **Final answer must be written in the second table below the grid. Only the values written in the table will be considered for giving marks.**

Solution:

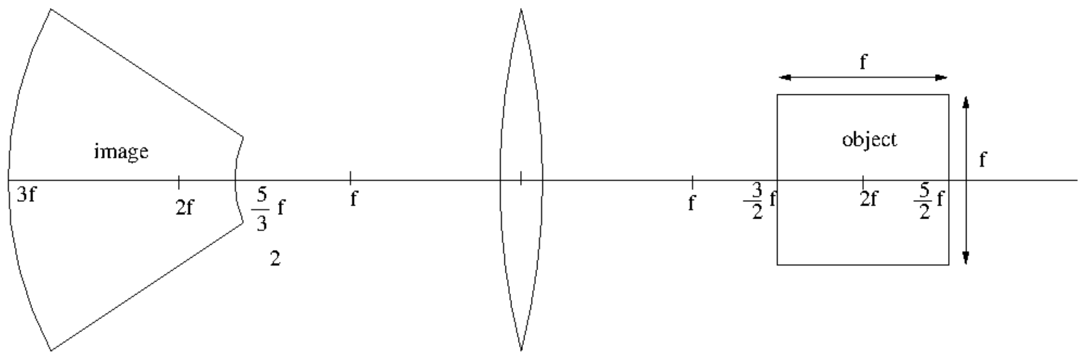
	mass	eccentricity	orbital radius
b	720	0.03	61
c	225	0.26	30
d	7	0.21	1.93
e	15	0.06	124

One mark per value.

6. (a) (9 marks) A thin convex lens, of focal length of f, is placed in y-z plane. The principal axis of the lens is along the x-axis. A luminous square sheet is placed in the x-y plane with its sides parallel to x and y axes. The centre of the sheet is exactly at a distance of $2f$ from the centre of the lens and length of each side of the sheet is f. The sheet is slightly rotated about the y-axis to make the entire sheet visible from the other side of lens. Sketch the shape of the image of the sheet and mark all relevant lengths in terms of f.

   (b) (6 marks) A pin P is placed in front of a concave mirror such that its mid-point lies on the principal axis at a distance d from the pole of the mirror. An observer on the principal axis at a distance D, where $D \gg d$, finds the pin and its real image to overlap. When the observer moves slightly to the left of the principal axis, the image is viewed to the right of the pin. From this observation, what can be concluded about value of d? Justify your answer.

Solution: (a) The image will look as follows:



- Correct drawing of object: 1 mark
- Correct extremities of image on the axis: 3 marks
- Top and bottom edges NOT curved: 2 marks
- left and right edges curved on 'correct' side: 3 marks

(b) As you move you observing position laterally, the closer objects appear to move faster than the farther objects.
Thus, when you move your eye to the left, the one closer out of the object and the image will appear to shift to the right of other. **(3 marks)**
It means that image distance is more than object distance. **(1 mark)**
Hence, the object is between f and $2f$. **(2 marks)**

7. (8 marks) Chandrayaan-1, the lunar mission launched by India in 2008, had all gold wiring. A particular instrument on board the mission, was operating at the same temperature as the space surrounding the Chandrayaan. Instrument specifications demanded that no wire in the instrument should offer a resistance more than 7 $m\Omega$. All the instruments on Chandrayaan were fabricated in ISRO labs at room temperature. If the radius of all wires in the instrument was 0.1mm (as measured in ISRO lab), what should be the maximum allowable length (as measured in ISRO lab) of any wire? Physical properties of gold:

   - Resistivity (at room temperature), $\rho = 2.214 \times 10^{-8}\ \Omega\,\mathrm{m}$
   - Temperature coefficient of resistivity, $\beta = 0.0032\ \mathrm{K}^{-1}$
   - Linear expansion coefficient, $\alpha = 1.5 \times 10^{-5}\ \mathrm{K}^{-1}$
   - Density, $\varrho = 2 \times 10^4\ \mathrm{kg/m}^3$
   - Molar heat capacity, $C_p = 25\ \mathrm{J\,mol}^{-1}\,\mathrm{K}^{-1}$
   - Molecular Weight, $W \approx 197$ g

Solution: Mass of the wire is,

$$M = \pi r_1^2 l_1 \varrho = \pi r_2^2 l_2 \varrho$$
$$\therefore r_1^2 l_1 = r_2^2 l_2$$

Resistance of wire is given by,

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$$
$$\rho_2 = \rho_1(1 - \beta\Delta T)$$
$$l_2 = l_1(1 - \alpha\Delta T)$$
$$R_2 = \frac{\rho_2 l_2}{\pi r_2^2} \quad (\textbf{2 marks})$$
$$= \frac{\rho_1(1-\beta\Delta T)l_2}{\pi \frac{l_1 r_1^2}{l_2}}$$
$$= \frac{\rho_1(1-\beta\Delta T)l_2^2}{\pi l_1 r_1^2}$$
$$= \frac{\rho_1(1-\beta\Delta T)(l_1(1-\alpha\Delta T))^2}{\pi l_1 r_1^2}$$
$$= \frac{\rho_1\left(1-\beta\Delta T - 2\alpha\Delta T + \alpha^2\Delta T^2\right) l_1}{\pi r_1^2} \quad (\textbf{2 marks})$$

Typical room temperature can be assumed to be 30° Celcius. Temperature in space is typically 3 Kelvin. Thus, temperature difference will be about 300 Kelvin. **(1 mark)**

Now, as α^2 term is too small. Further, as α is 100 times smaller than *beta*, it can be ignored too. **(1 mark)**

$$R_2 \approx \frac{\rho_1(1-\beta\Delta T)l_1}{\pi r_1^2}$$
$$\therefore l_1 \approx \frac{\pi r_1^2 R_2}{\rho_1(1-\beta\Delta T)}$$
$$\approx \frac{(10^{-4})^2 \times 7 \times 10^{-3}}{2.214 \times 10^{-8} \times (1 - 0.0032 \times 300)} \times \frac{22}{7}$$
$$\approx \frac{10^{-2}}{(1-0.96)} = \frac{1}{4}$$
$$= 25 \text{ cm} \quad (\textbf{2 marks})$$

Thus, the wire can be at max 25 cm long.

8. (18 marks) We are in the year 2020 and the spacecraft Voyager-1 is at a distance of around 200 A. U. from the Sun. It is still continuously sending information about its speed with respect to inertial frame of the surrounding interstellar medium every 10 days. Following are the readings reported by the Voyager over course of 100 days.

Time (days)	0	10	20	30	40	50
Velocity (m/s)	1000	1061.5	1124.4	1189.0	1255.4	1323.8
Time (days)	60	70	80	90	100	
Velocity (m/s)	1394.4	1467.4	1543.1	1621.8	1703.8	

Swapnil claimed that this indicates existence of an unknown body directly ahead in the path of Voyager. Those of you who agree with him, find mass and distance of this unknown body from Voyager. Those who disagree with him should give alternate explanation for the data observed and justify their arguments.

Note:

- For purpose of any calculations, you may ignore the influence of the Sun on Voyager. At the end of calculations, you should give reason why this is a valid assumption. Two points are reserved for this justification.
- As an approximation, assume the acceleration of the Voyager to be constant during each 10 day period between the readings.
- You may note that $8.5^2 \approx 72$.
- For calculations, you may assume one day approximately contains 90000 seconds.

Solution: We note that Voyager's velocity is changing. As we are ignoring influence of the Sun, this can only be because of presence of some unknown object close by. By noting difference between first two readings and last two readings, we realise that the change is not the same, i.e. acceleration is changing. We try to estimate average acceleration and velocities for each period by,

$$a = \frac{v-u}{t}$$

$$v_{av} = \frac{s}{t} = u + \frac{1}{2}at$$

In the table below, time is in days, velocities in m/sec and acceleration in μ m/s^2. Average velocity of the Voyager is given by v_{av} in m/s and last column is cumulative sum of v_{av}. **(8 marks)**

t	v	$v-u$	a	v_{av}	$\sum v_{av}$
0	1000	—	—	—	—
10	1061.5	61.5	68.3	1031	1031
20	1124.4	62.9	69.9	1093	2124
30	1189.0	64.6	71.8	1157	3280
40	1255.4	66.4	73.8	1222	4503
50	1323.8	68.4	76.0	1290	5792
60	1394.4	70.6	78.4	1359	7151
70	1467.4	73.0	81.1	1431	8582
80	1543.1	75.7	84.1	1505	10087
90	1621.8	78.7	87.4	1582	11670
100	1703.8	82.0	91.2	1663	13333

Now note that,

$$
\begin{aligned}
s &= v_{av} t \\
\therefore \sum s &= \sum v_{av} \times 10 \times 90000 \\
a &= \frac{GM}{r^2} \\
&= \frac{GM}{(r_0 - \sum s)^2} \\
\therefore a_1 (r_0 - (\sum s)_1)^2 &= a_2 (r_0 - (\sum s)_2)^2 \\
\sqrt{a_1}(r_0 - (\sum s)_1) &= \sqrt{a_2}(r_0 - (\sum s)_2) \\
\sqrt{71.8}(r_0 - 3280 \times 900000) &= \sqrt{81.1}(r_0 - 8582 \times 900000) \\
8.5(\frac{r_0}{9 \times 10^8} - 3.3) &\approx 9(\frac{r_0}{9 \times 10^8} - 8.6) \\
\therefore r_0 &\approx 9 \times 10^8 \times (8.6 \times 18 - 3.3 \times 17) \\
&\approx 9 \times 10^8 \times 98.7 \\
&\approx 8.9 \times 10^{10} \text{ m} \\
&\approx 0.6 \text{ A U} \quad (\textbf{5 marks})
\end{aligned}
$$

Now we use this distance to find mass of the object.

$$
\begin{aligned}
\therefore M &= \frac{a(r_0 - \sum s)^2}{G} \\
&\approx \frac{68.3 \times 10^{-6} \times (8.9 \times 10^{10} - 1031 \times 9 \times 10^5)^2 \times 3}{20 \times 10^{-11}} \\
&\approx \frac{68.3 \times (8.8)^2 \times 10^{24} \times 3}{2} \\
&\approx 68.3 \times 77.4 \times 1.5 \times 10^{24} \\
&\approx 5.3 \times 10^3 \times 1.5 \times 10^{24} \\
&\approx 8 \times 10^{27} \text{ kg} \\
&\approx 4 M_J \quad (\textbf{3 marks})
\end{aligned}
$$

Now the object is about 300 times lighter than the Sun and about 400 times closer to the Voyager as compared to the Sun.

$$
\frac{F_{obj}}{F_{Sun}} = \frac{M_{obj}}{r_{obj}^2} \times \frac{r_{Sun}^2}{M_{Sun}} = \frac{400^2}{300}
$$

Thus, gravitational force by the Sun on the Voyager will be about 500 times smaller and hence can be safely ignored. **(2 marks)**