Indian National Astronomy Olympiad – 2018

Question Paper INAO – 2018

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐ Date: 27th January 2018

Duration: **Three Hours** Maximum Marks: 100

Please Note:

- Please write your roll number on top of this page in the space provided.
- Before starting, please ensure that you have received a copy of the question paper containing total 4 pages (2 sheets).
- There are total 8 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question booklet back with you.
- Please be advised that tentative dates for the next stage are as follows:
  - Orientation Cum Selection Camp (Senior): 24th April to 11th May 2018. This will be held at HBCSE, Mumbai.
  - Dates for IAO selection camp (Junior) will be announced by NCSM later.
  - Attending the camp for the entire duration is mandatory for all participants.

Useful Constants

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	$\approx$	5.972×10^{24} kg
Mass of the Moon	$M_☾$	$\approx$	7.347×10^{22} kg
Radius of the Earth	$R_\oplus$	$\approx$	6.371×10^{6} m
Speed of Light	c	$\approx$	2.998×10^{8} m s^{-1}
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
Radius of the Moon	R_m	$\approx$	1.737×10^{6} m
Astronomical Unit	1 A. U.	$\approx$	1.496×10^{11} m
Solar Constant (at Earth)	S	$\approx$	1366 W m^{-2}
Gravitational Constant	G	$\approx$	6.674×10^{-11} N m^2 kg^{-2}

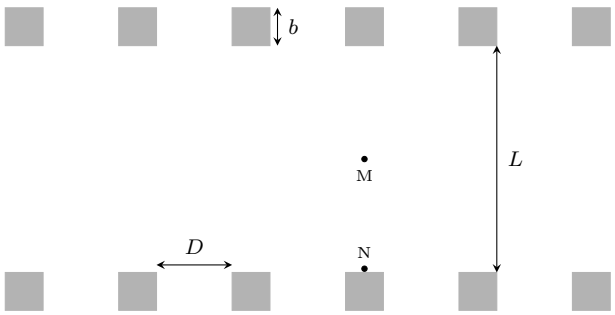
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

1. (5 marks) In Newtonian mechanics force between two objects is felt instantaneously irrespective of distance. However, according to relativity, no interaction is truly instantaneous. For an object to feel the force, the information of the force is carried by field particles. But nothing can travel faster than the speed of light.
   Two friends, 'A' (a follower of Newton) and 'B' (a follower of Einstein), were once debating about what will happen to the Earth if the Sun suddenly vanishes. They both calculated the direction in which the Earth will go, when this calamity happens. What will be the angular difference in the directions predicted by them?

2. Most buses in India run on diesel, whose calorific value is $44.8 \times 10^6\,\mathrm{J\,kg^{-1}}$ and density is $0.832\,\mathrm{kg\,l^{-1}}$. A typical bus can go 3 km, on average, for every litre of diesel consumed.
   (a) (5 marks) Estimate power consumption of a typical bus.
   (b) (3 marks) Calculate the maximum amount of solar power incident on roof of a typical bus.
   (c) (2 marks) Best commercial solar cells in the market have an efficiency of 20%. If the bus is to be run purely on solar power (during the day), estimate the area of solar cells required. What is the ratio of this area to the area of the rooftop of the bus.

3. The star Kepler-13A has a Jupiter-like planet (Kepler-13Ab) revolving around it. The period of revolution and rotation for this planet are equal. Analysis of the planet's atmosphere has revealed that it contains vapourized Titanium Oxide (TiO), a key component in sunscreen lotions. Based on the boiling point of TiO, astronomers have estimated the temperature of Kepler-13Ab's upper atmosphere to be about 3000 K.
   (a) (7 marks) What is the orbital distance of Kepler-13Ab?
   Given: Surface temperature of the star Kepler-13A = 7560 K, Radius of the star Kepler-13A = 1.71 $R_\odot$
   (b) (2 marks) Further analysis reveals condensed TiO as part of the lower atmosphere in Kepler-13Ab. Can you think of an explanation for this phenomenon?

4. Aditya is fascinated by the famous 'infinity corridor' of IITB. The corridor is a very long, straight, roofed pathway connecting different departments. The roof of the corridor is supported by square pillars of width b on either side. Let D be the distance between consecutive pillars on the same side and L be the width of the pathway (see figure).



   (a) (8 marks) One day Aditya was standing at point M (see figure), in the middle of the corridor, looking ahead along the path. He found that he cannot see any object outside the corridor beyond the n^{th} pillar on his either side. Derive an expression for n.

(b) (3 marks) Assume that the space between the successive pillars is $D = 3\,\text{m}$, width of each pillar is $b = 0.3\,\text{m}$ and width of the corridor is $L = 5\,\text{m}$. Find numerical value of n.

(c) (4 marks) How will the answer in part (b) change, if Aditya was at position N instead of M (see figure) and looking through pillars along the top row of the figure?

5. Astronauts have found that the angular diameter of the Earth as seen from the surface of the Moon is 1.9°. We assume that:

- the observer is standing on the equator of the Earth.
- the Moon is in the equatorial plane.
- the Moon's orbit is circular.

(a) (10 marks) Find the time it takes for the Moon to completely rise above the Earth's horizon.

(b) (3 marks) Briefly state why the three assumptions above are relevant to the solution.

(c) (2 marks) How much time would it take for the Earth to completely rise above Moon's horizon?

6. (a) (3 marks) Let us say we are observing sky from a dark location and all planets are visible in the sky. Arrange the planets of the solar system in the descending order of their apparent brightness as observed from the Earth.

(b) (14 marks) Shinjini was observing the sky from a location on the equator on the night of 20-21 March and she made following observations in her diary.

- Today is 11 days prior to the full Moon.
- Saturn is seen in constellation of Sagittarius.
- Jupiter is seen rising at the time of the Moon set.
- Mars' position was coinciding with the centre of Milky Way.
- Mercury set about 2 hours before the Moon.
- Venus was seen in the evening sky for about 2 hours after sunset.

In the answersheet, you will find a circle which is passing through East, Zenith (point exactly above the head of the observer), West and Nadir (point exactly below the observer). Use the information given above to mark positions of the Sun, the Moon and the 5 planets on this circle at 11 am on the Vernal Equinox day (21$^{\text{st}}$ March).
For each object, write a 1-2 line explanation stating why you think it is the correct position of the object.

7. Yash discovered a binary star, consisting of a very light star A in a circular orbit around a massive star B ($m_A \ll m_B$). He could measure the component of velocity of star A along the line of sight via Doppler shift. His measurements are tabulated below. All the data were obtained on 17$^{\text{th}}$ July 2009, and the times given are in UT.

(a) (10 marks) Plot the data, and measure the orbital period of star A from the plot.

(b) (5 marks) Use this information to calculate the mass of star B, in terms of solar mass.

(c) (3 marks) Is the plot symmetric with resepect to time axis? If yes, what does it signify? If no, what is the reason?

Time (UT)	Velocity (km/s)	Time (UT)	Velocity (km/s)
01:47:44	-292	02:37:21	98
01:41:31	-180	02:43:34	186
01:35:18	-60	02:49:51	290
01:53:57	-421	02:56:04	336
01:59:45	-438	03:02:22	344
02:06:13	-417	03:08:35	350
02:12:26	-361	03:14:48	294
02:18:41	-266	03:21:01	186
02:24:54	-163	03:27:15	130
02:31:07	-32	03:33:28	-40

8. In this question, we will investigate construction of large mirrors of modern telescopes.

   (a) (2 marks) Typical large modern telescopes do not have a single (monolithic) mirror. Instead, the mirror is divided in many small hexagonal concave mirror segments. Suppose one such telescope has segments with individual focal length of 30 m and length of their hexagonal side is 87.5 cm. What is the distance between two parallel edges of such a mirror segment?

   (b) (6 marks) To avoid image distortion (due to spherical aberration), the telescope mirrors are made in the shape of paraboloid of revolution (a parabola rotated around its axis). Let us take one such mirror facing upwards and observing a star at zenith (exactly overhead). As you may be aware, such a parabola follows equation of $x^2 = 4ay$, where a is the distance of the focus of the parabola from its vertex. You may also recall that slope of a tangent to the parabola at any point (x_0, y_0) is given by $x_0/2a$.
   Show that all rays from the star will converge at the focus of the parabola.

   (c) (3 marks) In part (b) above, show that all these rays will arrive at the focus in same phase.

Note for IAOSP candidates

- **Blackbody -**
  - emits as much energy as it absorbs,
  - emits in all directions equally.

  The thermal energy radiated by a blackbody per unit time per unit area is proportional to the fourth power of its absolute temperature and is given by

  $$\frac{L}{A} = \sigma T^4 \quad \mathrm{Jm^{-2}s^{-1}}$$

  where L is luminosity, A is area, T is absolute temperature and σ is Stefan's constant.