Indian National Astronomy Olympiad – 2019

Question Paper

INAO – 2019

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐

Date: 2nd February 2019

Duration: **Three Hours**

Maximum Marks: 100

Please Note:

- Before starting, please ensure that you have received a copy of the question paper containing a cover page and 4 pages of questions.
- Please write your roll number in the space provided above.
- There are total 7 questions. Maximum marks are indicated in front of each question / sub-question.
- For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question paper back with you.
- Please take note of following details about Orientation-Cum-Selection Camp (OCSC) in Astronomy:
  - Tentative Dates: 22nd April to 9th May 2019.
  - This camp will be held at HBCSE, Mumbai.
  - Attending the camp for the entire duration is mandatory for all participants.

Useful Constants

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	$\approx$	5.972×10^{24} kg
Mass of the Moon	$M_☾$	$\approx$	7.347×10^{22} kg
Radius of the Earth	$R_\oplus$	$\approx$	6.371×10^{6} m
Speed of Light	c	$\approx$	$2.998 \times 10^{8}\ \mathrm{m\,s^{-1}}$
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
Radius of the Moon	R_m	$\approx$	1.737×10^{6} m
Astronomical Unit	$a_\oplus$	$\approx$	1.496×10^{11} m
Solar Constant (at Earth)	S	$\approx$	$1366\ \mathrm{W\,m^{-2}}$
Gravitational Constant	G	$\approx$	$6.674 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$

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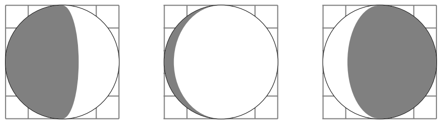
1. We can represent phases of the Moon diagramatically as follows:



It is known that there would be a new moon day on 28th February 2025. Sameer, who is writing a science fiction story, visualised various hypothetical situations for the phases of Moon during the month of March 2025. For each of the cases described below, draw the phases of Moon as it will appear on 3rd March, 8th March, 11th March, 15th March, 20th March and 27th March. High precision diagrams are not expected. Any approximate drawings (like the ones shown above) will be accepted.

(a) (4 marks) First he thought that the lead character of his novel should be a resident of the Moon and she can see the Earth in her sky. Draw phases of Earth as seen by her.

(b) (4 marks) As he did not find first case interesting enough, he brought his leading lady back to the Earth. Next he assumed that the orbit of Moon around Earth is perpendicular (inclination=90°) to the orbit of Earth around the Sun (i.e. ecliptic plane) and the normal to the orbit is always towards the Sun. Draw phases of the Moon in this case.

(c) (4 marks) Lastly, he assumed the orbit of the Moon is in the ecliptic plane (inclination=0°), but the shape of the Moon is cylindrical, with the diameter of the cylinder being equal to its height. The axis of the cylinder is perpendicular to the ecliptic plane. For the observer based on Earth, draw the phases of the Moon.

2. Below is a table of data indicating orbital radii and masses of solar system planets and some dwarf planets. Assume all orbits to be circular around the centre of mass of the solar system (C) and all objects to be in ecliptic plane.

Object	**Mass** (kg)	**Orbital Radius** (km)		**Object**	**Mass** (kg)	**Orbital Radius** (km)
Mercury	3.301×10^{23}	5.791×10^{7}		Saturn	5.683×10^{26}	1.427×10^{9}
Venus	4.867×10^{24}	1.082×10^{8}		Uranus	8.681×10^{25}	2.871×10^{9}
Earth	5.972×10^{24}	1.496×10^{8}		Neptune	1.024×10^{26}	4.498×10^{9}
Mars	6.417×10^{23}	2.279×10^{8}		Pluto	1.309×10^{22}	5.906×10^{10}
Ceres	9.470×10^{20}	4.137×10^{8}		Eris	1.660×10^{22}	1.018×10^{11}
Jupiter	1.898×10^{27}	7.783×10^{8}		–	–	–

(a) (3 marks) Either qualitatively describe or schematically draw what should be the configuration of these bodies around the Sun such that,

(i) point C is as far as possible from the centre of the Sun (S).

(ii) points C and S are as close to each other as possible.

(b) (7 marks) Assuming all planets to be collinear, estimate the smallest possible distance between points C and S.

3. Ameya has come up with an ambitious plan to set up a giant lens at an appropriate place between the Earth and the Sun to reduce the flux of light falling on all points on the Earth.

(a) (2 marks) What type of lens should be used for this purpose?

(b) (2 marks) Where should we put the lens so that this reduction in intensity is permanent?

(c) (3 marks) What should be the minimum diameter of this lens?

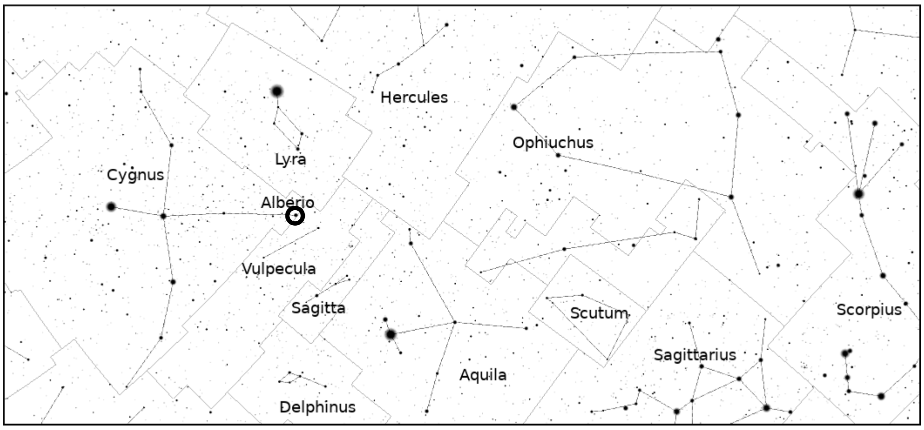
(d) (6 marks) Assuming that we want to reduce the temperature of the Earth by 2 °C, what should be the focal length of this lens?

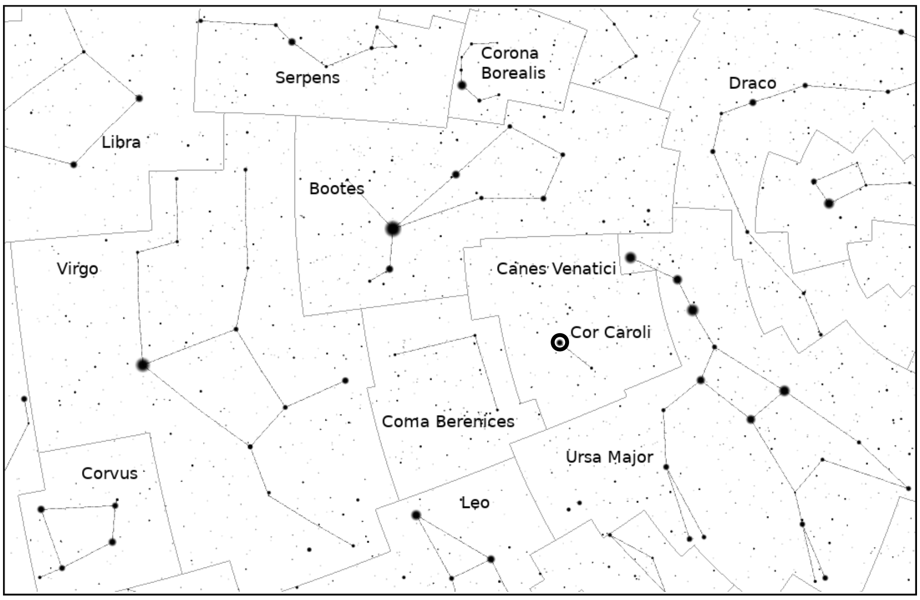
(e) (5 marks) In reality, it is impractical to make a single lens of such a diameter. So Ameya's plan B is to replace this single lens by an array (like a net with lens at every node) of lenses of diameter 1 m and focal length 10 m. How many such lenses are required to be placed at the same distance, in a circular area of same diameter as the bigger lens to produce the same temperature change?

(f) (2 marks) If we want to replace the lenses in part (e) by opaque spheres of diameter 1 m, what would be the corresponding drop in temperature?

4. (14 marks) Vinita prepared the two maps given below and gave those to students for practice on a certain date. At that time, the sky looked like the skymap in your answersheet. Use the practice maps to identify following constellations / stars in the skymap provided in your answersheet.

Mark stars with circles around them and for constellations draw stick figures as shown in the practice maps below.
Sagitta, Cor Caroli, Alberio, Lyra, Corvus, Corona Borealis





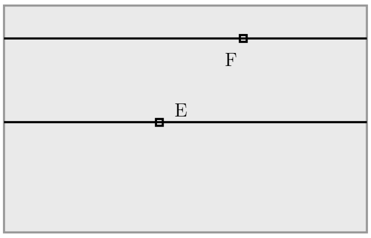
5. The maps of the Earth which you typically find in textbooks are printed with following criteria:

   - All the latitudes and longitudes are straight lines.
   - If any two points on the Earth along any longitude or along the equator have equal angular separation then on the map they will have equal linear distance.

   Prathyush has a globe of Earth of radius R. He also has a map of Earth with a projection described above. He noticed that the length of equator in this map as well as on the globe was exactly the same.

   (a) (3 marks) Find the ratio of the total surface area of the map A_{map} to the area of the globe A_{sph}.

   (b) (5 marks) Two small squares E and F of identical size are drawn on this map (see the figure below). Square E lies very near to the equator, while square F lies at a latitude of 60°. Find the ratios $\frac{A_{E|map}}{A_{E|sph}}$, $\frac{A_{F|map}}{A_{F|sph}}$ and $\frac{A_{F|sph}}{A_{E|sph}}$
   where, $A_{E|sph}$ corresponds to area of E as measured on the globe
   $A_{E|map}$ corresponds to area of E as measured on the map.
   $A_{F|sph}$ corresponds to area of F as measured on the globe
   $A_{F|map}$ corresponds to area of F as measured on the map.



   (c) (2 marks) A large square is drawn on the map such that its sides are u units ($u \sim R/4$). Lower side of this square is located on the equator. Draw approximate shape of the projection of this square as it appears on the globe.

6. (10 marks) Consider a cylindrical metallic wire of cross-sectional area A, length l, and resistance R_0 at ambient temperature T_0, which is also the temperature of the wire at the start of experiment. The resistance of the wire varies as a function of temperature $R = R_0\, e^{\alpha(T-T_0)}$. Assume it acts like a perfect blackbody with an indefinitely high melting point. The wire undergoes cooling only via radiation and its dimensions are unaffected by any change in temperature. Sketch the approximate temperature *vs.* time graph if the ends of the wire are maintained at a constant potential difference (V_0).

7. Separated from his friends, Yash landed up on an island on 23rd September. On the same day, with nothing better to do, he meticulously measured height of tide at every hour. This data is tabulated in Table 1 below.

   To model the tide height as a function of time, he made following simplifying assumptions;

   - Tides are caused **ONLY** by the gravitational pull of the Moon (i.e. ignore gravitational pull due to Sun).
   - Ignore the effect of Earth's rotation on the tides.

Table 1: Data for height of tide at every hour (local time) on a 23rd September.

Time (hrs)	Height (m)		Time (hrs)	Height (m)		Time (hrs)	Height (m)
00:00	0.538		08:00	0.45		16:00	0.756
01:00	0.616		09:00	0.341		17:00	0.812
02:00	0.708		10:00	0.307		18:00	0.786
03:00	0.753		11:00	0.33		19:00	0.733
04:00	0.786		12:00	0.372		20:00	0.666
05:00	0.741		13:00	0.482		21:00	0.622
06:00	0.639		14:00	0.572		22:00	0.583
07:00	0.554		15:00	0.676		23:00	0.545

- Ignore friction between the surface of the Earth and the water.

(a) (10 marks) Plot the given data.

(b) (4 marks) Using the plot, find the Sun-Earth-Moon angle.

(c) (2 marks) Find the date of the nearest New Moon.

(d) (6 marks) Later he explored interiors of the island, including a large cave system, and had erratic eating and sleeping schedule. As a result, when he returned to the beach, he had lost track of time. He again started measuring tide height to get a sense of date. However, this time, he was also busy in other activities such as food gathering and raft making. So his readings, as given in Table 2, are taken only sporadically. Find the height of the tide at 16:00 hours on that day, by overplotting this data on the same graph paper.

(e) (2 marks) Which is the date corresponding to Table 2?

Table 2: Height versus time data

Time (hrs)	Height (m)		Time (hrs)	Height (m)
00:00	1.014		12:00	0.748
01:00	1.079		13:00	0.855
02:00	1.033		14:00	0.844
03:00	0.905		18:00	0.279
07:00	0.255		19:00	0.213
08:00	0.235		20:00	0.203
09:00	0.322		21:00	0.307
10:00	0.445		23:00	0.682