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Indian National Astronomy Olympiad – 2020

Question Paper INAO – 2020

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐ Date: 1st February 2020

Duration: **Three Hours** Maximum Marks: 100

Please Note:

- Please write your roll number in the space provided above.
- There are a total of 7 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question paper back with you.
- Please take note of following details about Orientation-Cum-Selection Camp (OCSC) in Astronomy:
  - Tentative Dates: 21st April to 8th May 2020.
  - This camp will be held at HBCSE, Mumbai.
  - Attending the camp for the entire duration is mandatory for all participants.

Useful Constants

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	$\approx$	5.972×10^{24} kg
Mass of the Moon	M_m	$\approx$	7.347×10^{22} kg
Radius of the Earth	$R_\oplus$	$\approx$	6.371×10^{6} m
Speed of Light	c	$\approx$	2.998×10^{8} m s^{-1}
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
Radius of the Moon	R_m	$\approx$	1.737×10^{6} m
Astronomical Unit (au)	$a_\oplus$	$\approx$	1.496×10^{11} m
Solar Luminosity	$L_\odot$	$\approx$	3.826×10^{26} W
Gravitational Constant	G	$\approx$	6.674×10^{-11} N m^2 kg^{-2}
Gravitational acceleration	g	$\approx$	9.8 m/s^2
1 parsec (pc)	1 pc	$=$	3.086×10^{16} m
Stefan's Constant	σ	$=$	5.670×10^{-8} Wm^{-2}K^{-4}

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1. (10 marks) On the evening of an autumnal equinox day Siddhant noticed that Mars was exactly along the north-south meridian in his sky at the exact moment when the sun was setting. In other words, the Sun and Mars subtended an angle of exactly 90° as measured from the Earth. If the orbital radius of Mars is 1.52 au, What will be the approximate rise time of the mars on the next autumnal equinox day?

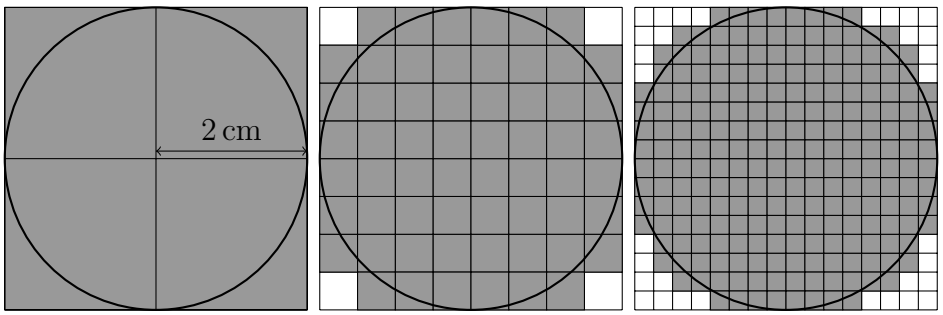
2. Manoj was determined to claim some world record. He got an idea from the fairy tale of Rapunzel that he will never cut his hair and he can claim world record for growing longest hair.

   (a) (3 marks) Estimate maximum length of the hair that he can grow in his whole life, if he hasn't cut his hair from his birth.

   (b) (7 marks) What will be mass of these hair? (density of typical hair strand is 1.3 g/cm^3)

If you make any simplifying assumptions, discuss qualitatively how answer would have been affected if those assumptions were not made.

3. (10 marks) A curious middle school student wants to actually measure perimeter of a circle of radius 2 cm. He has a scale and plenty of time at his disposal, but does not have a thread to measure lengths along arcs. Hence, he comes up with the following strategy:



   1. He makes a circumscribing square of the circle.
   2. He divides the square into 4 equal smaller squares (left panel).
   3. He measures the outer boundary (perimeter) of a shape formed by all the squares that overlap (fully or partially) with the circle (shape is highlighted in gray in the figure). He calls this estimate as P_1.
   4. He then makes a finer grid by dividing each smaller square into 4x4 grid. He again repeats step 3 to estimate the perimeter and calls it P_2 (middle panel).
   5. He keeps refining the grid again and again n times and finds final estimate P_n.

   Estimate P_n.

   Hint: Eliminate white squares one by one, starting from corners, to arrive at the perimeter of gray shape.

4. Let us consider three stars A, B and C. It was observed that

   - As seen from star B, star A is barely visible to the naked eye,
   - As seen from star C, star B is barely visible to the naked eye,

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- As seen from star A, star C is barely visible to the naked eye,

Let us denote the distance between star A and star B as d_1, and distance between star B and C as d_2 and star C and A as d_3.
Absolute magnitude of star A, i.e. $M_A = 2.00\,\text{mag}$ and that of star B is $M_B = 3.00\,\text{mag}$.
(For explanation of magnitude system, see the box below.)

(a) (4 marks) Find the distances d_1 and d_2.

(b) (5 marks) Find the interval (in magnitudes) in which the absolute magnitude M_C has to belong so that the above described configuration is allowed.

(c) (3 marks) If $M_C = 4.00\,\text{mag}$, find the largest angle γ in this stellar triangle.

(d) (6 marks) Show that if we change the values of the three absolute magnitudes so that their differences remain the same, the angles in the triangle will not change.

A note about magnitude system:

The brightness of a star as seen by some observer, is dependent on the observer's distance from the star as well the intrinsic brightness of the star (ignoring any absorption in the intervening space).
In astronomy, brightness of a star, as seen by some observer, is measured in terms of its 'apparent magnitude' (m). For two stars (1 and 2) with fluxes f_1 and f_2 respectively, their apparent magnitudes m_1 and m_2 are related by

$$m_1 - m_2 = -2.5 \times log_{10}\left(\frac{f_1}{f_2}\right)$$

Absolute magnitude (M) of any star is its apparent magnitude, if the star was exactly 10 pc away from the observer. Thus, M only depends on the intrinsic brightness of the star.
The relation between m and M is given as,

$$m - M = -5 + 5\ log_{10}(d)$$

where d is measured in parsec (pc).
By convention, the faintest stars visible to naked eye, in ideal viewing conditions, have been assigned an apparent magnitude of $m = +6.0\,\text{mag}$.

5. An extremely powerful cannon is placed horizontally at the north pole (at ground level) and its barrel is aligned with the Prime Meridian (Greenwich Meridian). The cannon is fired at $t = 0$, and the cannonball has sufficient velocity to travel in a circular orbit around the Earth. Assume the Earth to be a perfect sphere and that no objects obstruct the path of the cannonball.

(a) (3 marks) What is the period of the orbit of this cannonball?

(b) (10 marks) Plot the trajectory of the cannonball (latitude (ϕ) vs. longitude (λ)) on a regular graph paper given in your answersheet. The plot should correspond to one full orbit.

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(c) (7 marks) The second graph given in the answersheet is a part of a large polar projection plot having concentric arcs around the centre as latitudes and radial lines as longitudes. Define appropriate scale and show the path of cannonball on this graph from the moment it is fired till the moment it crosses the equator.

6. Pulsars are a type of fast rotating, high density stars, which are known for their regular pulses of radiation. They are also sometimes called as the 'light houses of the universe'. In this problem, we assume the pulsars to be spheres of uniform density, which are gravitationally bound.

   (a) (4 marks) Rotation period of a pulsar was measured to be $P = 1.500\,\text{ms}$. What limit does this put on its density?

   (b) (2 marks) If mass of this pulsar is $1.5\text{M}_\odot$, what limit can we place on its radius?

   A pulsar is formed from a massive progenitor star which typically has a magnetic field of 0.1 T and average density of $10^{-4}\,\text{g}\,\text{cm}^{-3}$. Such a star loses about 90% of its mass towards the end of its life. The remnant mass forms a pulsar of the kind described above.

   (c) (2 marks) Along the lines of part (a), what limit does this put on the rotation period of the progenitor star?

   (d) (4 marks) Assuming that the magnetic flux is conserved during the formation of a pulsar, find the typical magnetic field at the surface of the pulsar?

7. The picture below was taken on 24$^{\text{th}}$ December 2019, from some place in India, showing the crescent of the Moon near the horizon (the horizontal dashed line marks the horizon of the place). Field of View (FOV) of the image is 60°.

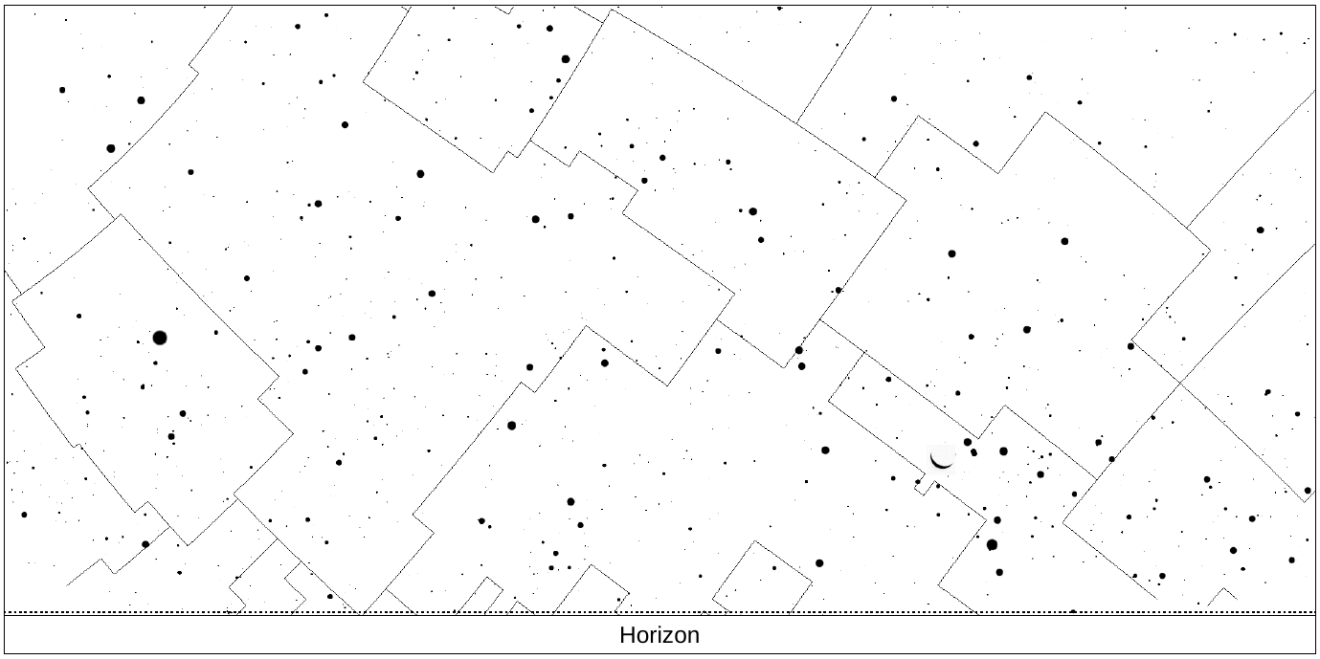

Figure 1: Negative image of certain patch of the sky on 24$^{\text{th}}$ December 2019

Note: The images printed are colour-inverted, i.e. the bright parts of the image appear black and dark parts appear white. Thus, the black dots are stars and planet and the dark cresent is actually bright crescent of the moon.

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(a) (5 marks) At which of the following times this picture may have been taken? Give justification for your answer.
18:00 hrs, 22:00 hrs, 01:00 hrs, 05:00 hrs

(b) (5 marks) Write the names of the constellations present in the image of the sky.

(c) (1 mark) The map also includes a planet. Mark the planet on the map with a circle and label as 'P'.

(d) (5 marks) A zoomed-in image of the lunar crescent is given in the answersheet. On this image mark the approximate directions to the cardinal points. [East-West-North-South] Note: You may assume the box in the answersheet has a linear angular scale along the horizon.

(e) (4 marks) Find approximate latitude of the place.

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