Indian National Physics Olympiad – 2020

Date: 02 February 2020
Time : **09:00-12:00 (3 hours)**

Roll Number: 2 0 ☐☐-☐☐☐☐-☐☐☐☐
Maximum Marks: **80**

Extra sheets attached : ☐

INO Centre (e.g. Ranchi)

(Do not write below this line)

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Instructions

1. This booklet consists of 23 pages (excluding this page) and total of 5 questions.
2. This booklet is divided in two parts: **Questions with Summary Answer Sheet** and **Detailed Answer Sheet**. Write roll number at the top wherever asked.
3. **The final answer to each sub-question should be neatly written in the box provided below each sub-question in the Questions & Summary Answer Sheet.**
4. You are also required to show your **detailed work** for each question in a reasonably neat and coherent way in the **Detailed Answer Sheet**. You must write the relevant Question Number(s) on each of these pages.
5. Marks will be awarded on the basis of what you write on both the Summary Answer Sheet and the Detailed Answer Sheet. Simple short answers and plots may be directly entered in the Summary Answer Sheet. Marks may be deducted for absence of detailed work in questions involving longer calculations. Strike out any rough work that you do not want to be considered for evaluation.
6. Adequate space has been provided in the answersheet for you to write/calculate your answers. In case you need extra space to write, you may request additional blank sheets (maximum two) from the invigilator. Write your roll number on the extra sheets and get them attached to your answersheet and indicate number of extra sheets attached at the top of this page.
7. Non-programmable scientific calculators are allowed. Mobile phones **cannot** be used as calculators.
8. Use blue or black pen to write answers. Pencil may be used for diagrams/graphs/sketches.
9. **This entire booklet must be returned at the end of the examination.**

Table of Constants

Speed of light in vacuum	c	3.00×10^{8} m·s^{-1}
Planck's constant	h	6.63×10^{-34} J·s
	$\hbar$	$h/2\pi$
Universal constant of Gravitation	G	6.67×10^{-11} N·m^2·kg^{-2}
Magnitude of electron charge	e	1.60×10^{-19} C
Rest mass of electron	m_e	9.11×10^{-31} kg
Value of $1/4\pi\epsilon_0$		9.00×10^{9} N·m^2·C^{-2}
Avogadro's number	N_A	6.022×10^{23} mol^{-1}
Acceleration due to gravity	g	9.81 m·s^{-2}
Universal Gas Constant	R	8.31 J· K^{-1}·mol^{-1}
	R	0.0821 l·atm·mol^{-1}·K^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ H·m^{-1}

Question	Marks	Score
1	13	
2	12	
3	15	
4	20	
5	20	
Total	80	

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1. A certain gas obeys the equation of state $U(S,V,N) = aS^7/V^4N^2$, where a is a dimensioned constant. Here U represents the internal energy of the gas, S the entropy, V the volume and N the fixed number of particles of the system.

(a) Let such a gas be filled in a box of volume V and the internal energy of the system be U. A partition is placed to divide the box into two equal parts, each having volume $V/2$. For each part, the internal energy is now αU and the dimensioned constant be βa. Obtain α and β. **[3]**

$\alpha =$	$\beta =$

(b) The temperature T can be expressed in terms of the derivative of internal energy as **[2]**

$$T = \left(\frac{dU}{dS}\right)_{V,N}$$

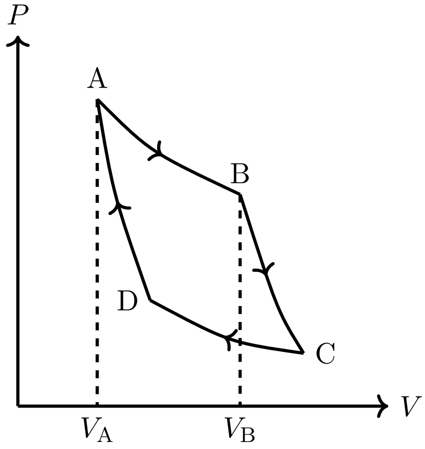
where the subscripts indicate that the differentiation has been carried out keeping V and N constant. In a similar way, express pressure P in terms of a derivative of the internal energy.

$P =$

(c) Find the equation of state of the given system relating P, T , and V. **[1]**

$P =$

(d) One mole of this gas executes a Carnot cycle ABCDA between reservoirs at temperatures T_1 and T_2 ($T_1 > T_2$). Obtain the heat change in the process AB (Q_{AB}) and work done by the system in the processes AB and BC ($W_{\text{AB}}, W_{\text{BC}}$) of the cycle. Express your answers only in terms of temperatures T_1, T_2, volumes V_A, V_B, and the other constants. **[7]**



$Q_{\text{AB}} =$

$W_{\mathrm{AB}} =$

$W_{\mathrm{BC}} =$

Detailed answers can be found on page numbers:

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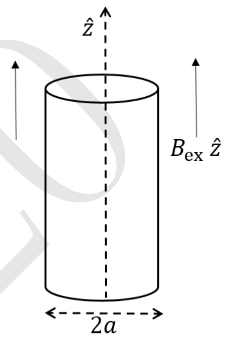
2. An insulating uniformly charged cylindrical shell of radius a lies with its axis along the z axis. The shell's moment of inertia per unit length about the z axis and the surface charge density are I and σ respectively. The cylinder is placed in an external uniform magnetic field $B_{\mathrm{ex}}\hat{z}$, and is initially at rest. Starting at $t = 0$ the external magnetic field is slowly reduced to zero. What is the final angular velocity ω of the cylinder? **[12]**



$\omega =$

Detailed answers can be found on page numbers:

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3. Consider the Bohr model of the hydrogen atom. Let m_e and e be the mass and magnitude of the charge of the electron respectively. Let a_0 be the ground state radius (Bohr radius).

   (a) Obtain an expression for the ionisation energy I_{H} of the ground state of the hydrogen atom in terms of a_0 and constants. **[1]**

   $I_{\mathrm{H}} =$

   (b) Consider a singly ionised helium atom He^+. Obtain the ground state ionisation energy I_{He^+} of He^+ in terms of I_{H}. **[2]**

$I_{\mathrm{He}^+} =$

(c) Now consider a two electron system with arbitrary atomic number Z. Use the Bohr model to obtain the ground state radius ($r(Z)$) in terms of a_0 and Z. Assume the two electrons are in the same circular orbit and as far apart as possible. **[1]**

$r(Z) =$

(d) Derive an expression for the first ionisation energy I_Z^{th} for two electron system with arbitrary Z in terms of Z and I_{H}. **[3]**

$I_Z =$

(e) The table below contains the experimental data of I_Z^{expt} (in units of Rydberg where 1 Ryd = 13.6 eV) versus Z for various two-electron systems. **[8]**

	Z	I_Z^{expt}
H^-	1	0.055
He	2	1.81
Li^+	3	5.56
Be^{++}	4	11.32
B^{3+}	5	19.07
C^{4+}	6	28.83
N^{5+}	7	40.60
O^{6+}	8	54.37
F^{7+}	9	70.15

Experimental values were not found to be equal to the theoretical predictions. This difference arises mainly from non-inclusion of Pauli's principle in the theoretical derivation of part (d). It was suggested that if the value of Z was reduced by some fixed amount α ($Z^* = Z - \alpha$) in the final expression of I_Z^{th} obtained in part (d), then $I_{Z^*}^{\mathrm{th}} \approx I_Z^{\mathrm{expt}}$. Draw a suitable linear plot and from the graph find α. Two graph papers are provided with this booklet in case you make a mistake.

$\alpha =$

Detailed answers can be found on page numbers:

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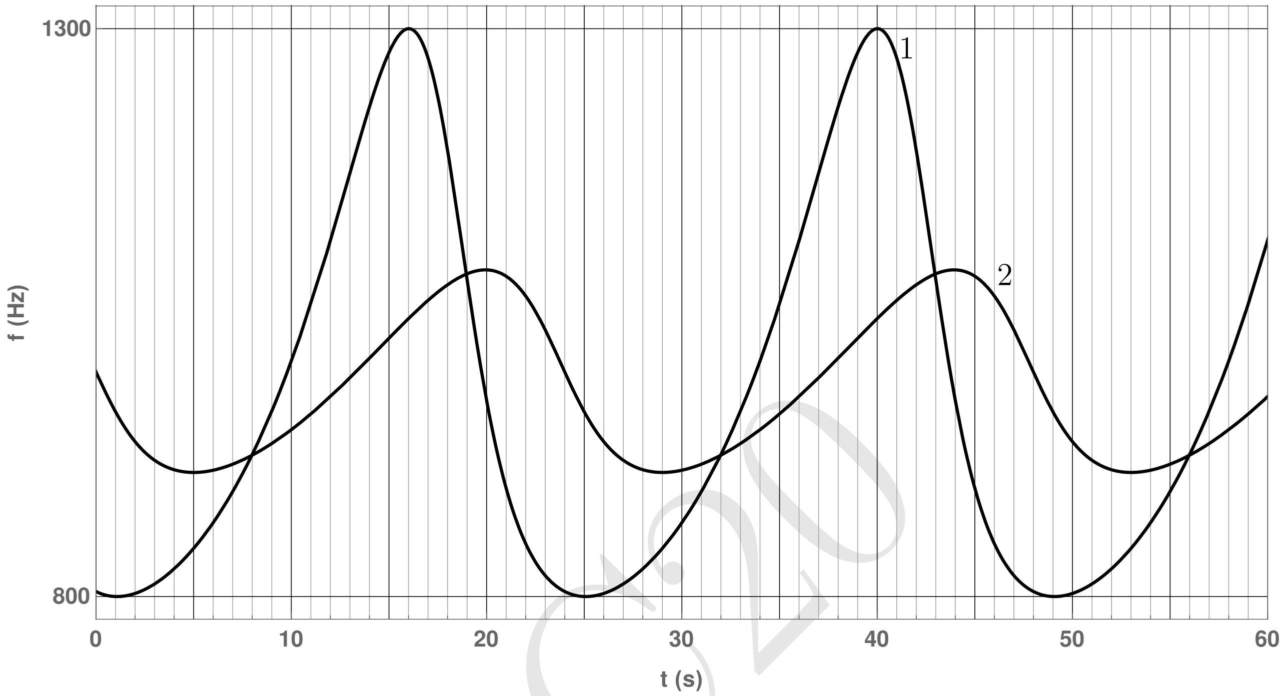
4. A sound source S is performing uniform circular motion with time period T. It is continuously emitting sound of a fixed frequency f_0. Two detectors 1 and 2 are placed somewhere in the same plane as the circular trajectory of the source. The frequency f, of the sound received by the two detectors is plotted as a function of time t as shown below (the clocks of the two detectors are synchronized).

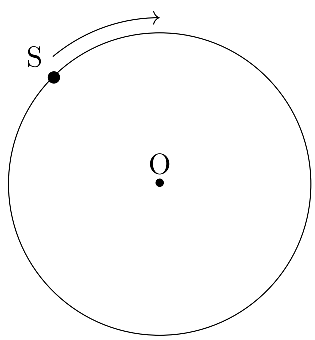


Take the speed of sound in the medium to be 330 m/s.

(a) Determine the time period T of the source. **[2]**

$T =$

(b) The figure below shows the circular trajectory of the source S. Qualitatively mark the positions of both the detectors by indicating 1 and 2. Here O denotes the centre of the trajectory. You must provide detailed justification of your answer in the detailed answer sheet. **[6]**



(c) Obtain the frequency f_0 of the source. **[3]**

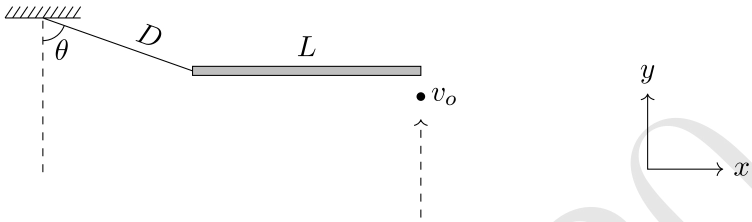
$f_0 =$

(d) Calculate the distance (D) between the detectors. **[9]**

$D =$

Detailed answers can be found on page numbers:

5. The following is the top view of an assembly kept on a smooth horizontal table. **[20]**



A massless inextensible string of length D lies with one end fixed, while the other is attached to one end of a uniform rod of length L. The system is initially at rest with the rod aligned along the x-axis and the string stretched to its natural length at an angle with the negative y-axis θ ($\cos\theta = 1/3$). At a certain instant, a bullet of the same mass m as the rod and negligible dimensions is fired horizontally along the positive y-direction. The bullet hits the rod at its right end with velocity v_o and gets lodged in it, the impact being nearly instantaneous. What is the tension (T) in the string immediately after the impact? Assume the string doesn't break.

$T =$

Detailed answers can be found on page numbers:

****** END OF THE QUESTION PAPER ******