Chapter Four

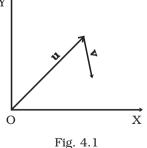
MOTION IN A PLANE



MCQI

- 4.1 The angle between $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} \hat{\mathbf{j}}$ is
 - (a) 45 (b) 90 (c) -45 (d) 180
- **4.2** Which one of the following statements is true?
 - (a) A scalar quantity is the one that is conserved in a process.
 - (b) A scalar quantity is the one that can never take negative values.
 - (c) A scalar quantity is the one that does not vary from one point to another in space.
 - (d) A scalar quantity has the same value for observers with different orientations of the axes.
- **4.3** Figure 4.1 shows the orientation of two vectors \mathbf{u} and \mathbf{v} in the XY plane.

If
$$\mathbf{u} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$$
 and $\mathbf{v} = p\hat{\mathbf{i}} + q\hat{\mathbf{j}}$



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which of the following is correct?

- (a) a and p are positive while b and q are negative.
- (b) a, p and b are positive while q is negative.
- (c) a, q and b are positive while p is negative.
- (d) a, b, p and q are all positive.
- **4.4** The component of a vector \mathbf{r} along X-axis will have maximum value if
 - (a) \mathbf{r} is along positive Y-axis
 - (b) \mathbf{r} is along positive X-axis
 - (c) r makes an angle of 45 with the X-axis
 - (d) \mathbf{r} is along negative Y-axis
- 4.5 The horizontal range of a projectile fired at an angle of $15\,$ is $50\,$ m. If it is fired with the same speed at an angle of $45\,$, its range will be
 - (a) 60 m
 - (b) 71 m
 - (c) 100 m
 - (d) 141 m
- **4.6** Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are
 - (a) Impulse, pressure and area
 - (b) Impulse and area
 - (c) Area and gravitational potential
 - (d) Impulse and pressure
- 4.7 In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then which of the following are necessarily true?
 - (a) The average velocity is not zero at any time.
 - (b) Average acceleration must always vanish.
 - (c) Displacements in equal time intervals are equal.
 - (d) Equal path lengths are traversed in equal intervals.
- **4.8** In a two dimensional motion, instantaneous speed v_o is a positive constant. Then which of the following are necessarily true?
 - (a) The acceleration of the particle is zero.
 - (b) The acceleration of the particle is bounded.
 - (c) The acceleration of the particle is necessarily in the plane of motion.
 - (d) The particle must be undergoing a uniform circular motion

Motion in a Plane

- **4.9** Three vectors **A,B** and **C** add up to zero. Find which is false.
 - (a) (A B) C is not zero unless B,C are parallel
 - (b) (A B).C is not zero unless B,C are parallel
 - (c) If A,B,C define a plane, (A B) C is in that plane
 - (d) (A B).C=|A||B||C| \rightarrow C²=A²+B²
- **4.10** It is found that |A+B| = |A|. This necessarily implies,
 - (a) B = 0
 - (b) A,B are antiparallel
 - (c) A,B are perpendicular
 - (d) $\mathbf{A} \cdot \mathbf{B} \leq 0$

MCQ II

- **4.11** Two particles are projected in air with speed v_o at angles θ_1 and θ_2 (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then tick the right choices
 - (a) angle of projection : $q_1 > q_2$
 - (b) time of flight : $T_1 > T_2$
 - (c) horizontal range : $R_1 > R_2$
 - (d) total energy: $U_1 > U_2$.
- **4.12** A particle slides down a frictionless parabolic $(y = x^2)$ track (A B C) starting from rest at point A (Fig. 4.2). Point B is at the vertex of parabola and point C is at a height less than that of point A. After C, the particle moves freely in air as a projectile. If the particle reaches highest point at P, then



- (b) height at P = height at A
- (c) total energy at P = total energy at A
- (d) time of travel from A to B = time of travel from B to P.

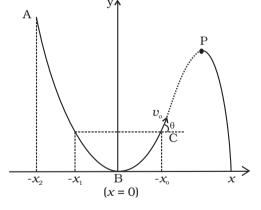


Fig. 4.2

4.13 Following are four different relations about displacement, velocity and acceleration for the motion of a particle in general. Choose the incorrect one (s):

(a)
$$\mathbf{v}_{av} = \frac{1}{2} [\mathbf{v}(t_1) + \mathbf{v}(t_2)]$$

(b)
$$\mathbf{v}_{av} = \frac{\mathbf{r}(t_2) - \mathbf{r}(t_1)}{t_2 - t_1}$$

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(c)
$$\mathbf{r} = \frac{1}{2} (\mathbf{v}(t_2) - \mathbf{v}(t_1))(t_2 - t_1)$$

(d)
$$\mathbf{a}_{av} = \frac{\mathbf{v}(t_2) - \mathbf{v}(t_1)}{t_2 - t_1}$$

- **4.14** For a particle performing uniform circular motion, choose the correct statement(s) from the following:
 - (a) Magnitude of particle velocity (speed) remains constant.
 - (b) Particle velocity remains directed perpendicular to radius vector.
 - (c) Direction of acceleration keeps changing as particle moves.
 - (d) Angular momentum is constant in magnitude but direction keeps changing.
- **4.15** For two vectors **A** and **B**, $|\mathbf{A}+\mathbf{B}|=|\mathbf{A}-\mathbf{B}|$ is always true when
 - (a) $|\mathbf{A}| = |\mathbf{B}| \neq 0$
 - (b) **A** \perp **B**
 - (c) $|\mathbf{A}| = |\mathbf{B}| \neq 0$ and \mathbf{A} and \mathbf{B} are parallel or anti parallel
 - (d) when either $|\mathbf{A}|$ or $|\mathbf{B}|$ is zero.

VSA

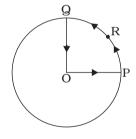


Fig. 4.3

- **4.16** A cyclist starts from centre O of a circular park of radius 1km and moves along the path OPRQO as shown Fig. 4.3. If he maintains constant speed of 10ms⁻¹, what is his acceleration at point R in magnitude and direction?
- **4.17** A particle is projected in air at some angle to the horizontal, moves along parabola as shown in Fig. 4.4, where *x* and *y* indicate horizontal and vertical directions, respectively. Show in the diagram, direction of velocity and acceleration at points A, B and C.

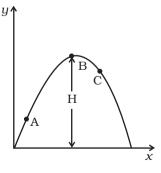


Fig. 4.4

Motion in a Plane

- **4.18** A ball is thrown from a roof top at an angle of 45 above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have
 - (a) greatest speed.
 - (b) smallest speed.
 - (c) greatest acceleration? Explain
- **4.19** A football is kicked into the air vertically upwards. What is its (a) acceleration, and (b) velocity at the highest point?
- **4.20 A, B** and **C** are three non-collinear, non co-planar vectors. What can you say about direction of **A** (B C)?

SA

- **4.21** A boy travelling in an open car moving on a levelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.
- **4.22** A boy throws a ball in air at 60 to the horizontal along a road with a speed of 10 m/s (36km/h). Another boy sitting in a passing by car observes the ball. Sketch the motion of the ball as observed by the boy in the car, if car has a speed of (18km/h). Give explanation to support your diagram.
- **4.23** In dealing with motion of projectile in air, we ignore effect of air resistance on motion. This gives trajectory as a parabola as you have studied. What would the trajectory look like if air resistance is included? Sketch such a trajectory and explain why you have drawn it that way.
- **4.24** A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 km/h. At what angle of sight (w.r.t. horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target?
- **4.25** (a) Earth can be thought of as a sphere of radius 6400 km. Any object (or a person) is performing circular motion around the axis of earth due to earth's rotation (period 1 day). What is acceleration of object on the surface of the earth (at equator) towards its centre? what is it at latitude θ ? How does these accelerations compare with $g = 9.8 \text{ m/s}^2$?

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(b) Earth also moves in circular orbit around sun once every year with on orbital radius of $1.5 \times 10^{11} \, m$. What is the acceleration of earth (or any object on the surface of the earth) towards the centre of the sun? How does this acceleration compare with $g = 9.8 \, \text{m/s}^2$?

Hint: acceleration $\frac{V^2}{R} = \frac{4\pi^2 R}{T^2}$

4.26 Given below in column I are the relations between vectors a, b and c and in column II are the orientations of a, b and c in the XY plane. Match the relation in column I to correct orientations in column II.

Column I Column II

(a)
$$\mathbf{a} + \mathbf{b} = \mathbf{c}$$
 (i)

(b) $\mathbf{a} - \mathbf{c} = \mathbf{b}$ (ii)

(c) $\mathbf{b} - \mathbf{a} = \mathbf{c}$ (iii)

(d) $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$ (iv)

4.27 If $|\mathbf{A}| = 2$ and $|\mathbf{B}| = 4$, then match the relations in column I with the angle θ between **A** and **B** in column II.

Column

Column II

- (a) A.B = 0
- (i) $\theta = 0$
- (b) A.B = +8
- (ii) $\theta = 90^{\circ}$
- (c) A.B = 4
- (iii) $\theta = 180^{\circ}$
- (d) A.B = -8
- (iv) $\theta = 60^{\circ}$
- **4.28** If $|\mathbf{A}| = 2$ and $|\mathbf{B}| = 4$, then match the relations in column I with the angle θ between A and B in column II

Column I

Column II

- (a) $|\mathbf{A} \times \mathbf{B}| = 0$
- (i) $\theta = 30^{\circ}$
- (b) $| \mathbf{A} \times \mathbf{B} | = 8$
- (ii) $\theta = 45^{\circ}$
- (c) $|\mathbf{A} \times \mathbf{B}| = 4$
- (iii) $\theta = 90^{\circ}$
- (d) $| \mathbf{A} \times \mathbf{B} | = 4\sqrt{2}$ (iv) $\theta = 0^{\circ}$

LA

- 4.29 A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800m from the foot of hill and can be moved on the ground at a speed of 2 m/s; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take $g=10 \text{ m/s}^2$.
- 4.30 A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $R = \frac{v_0^2}{r}$.

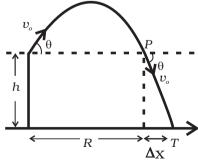


Fig 4.5

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If a target farther away by distance Δx (beyond R) has to be hit with the same gun (Fig 4.5), show that it could be achieved by raising the gun to a height at least

$$h = \Delta x \left[1 + \frac{\Delta x}{R} \right]$$

(*Hint*: This problem can be approached in two different ways:

- (i) Refer to the diagram: target T is at horizontal distance $x = R + \Delta x$ and below point of projection y = -h.
- (ii) From point P in the diagram: Projection at speed v_o at an angle θ below horizontal with height h and horizontal range Δx .)
- **4.31** A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal (Fig. 4.6).
 - (a) Find an expression of range on the plane surface (distance on the plane from the point of projection at which particle will hit the surface).
 - (b) Time of flight.
 - (c) β at which range will be maximum.

Fig. 4.6

(*Hint*: This problem can be solved in two different ways:

- (i) Point P at which particle hits the plane can be seen as intersection of its trajectory (parobola) and straight line. Remember particle is projected at an angle $(\alpha + \beta)$ w.r.t. horizontal.
- (ii) We can take x-direction along the plane and y-direction perpendicular to the plane. In that case resolve g (acceleration due to gravity) in two different components, g_x along the plane and g_y perpendicular to the plane. Now the problem can be solved as two independent motions in x and y directions respectively with time as a common parameter.)

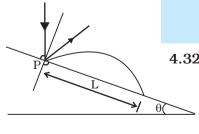


Fig 4.7

4.32 A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed v_o and rebounds elastically (Fig 4.7). Find the distance along the plane where if will hit second time.

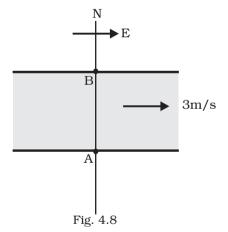
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(*Hint:* (i) After rebound, particle still has speed V_0 to start.

- (ii) Work out angle particle speed has with horizontal after it rebounds.
- (iii) Rest is similar to if particle is projected up the incline.)
- 4.33 A girl riding a bicycle with a speed of 5 m/s towards north direction, observes rain falling vertically down. If she increases her speed to 10 m/s, rain appears to meet her at 45 to the vertical. What is the speed of the rain? In what direction does rain fall as observed by a ground based observer?

(*Hint:* Assume north to be $\hat{\mathbf{i}}$ direction and vertically downward to be $-\hat{\mathbf{j}}$. Let the rain velocity \mathbf{v}_r be $a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$. The velocity of rain as observed by the girl is always $\mathbf{v}_r - \mathbf{v}_{girl}$. Draw the vector diagram/s for the information given and find a and b. You may draw all vectors in the reference frame of ground based observer.)

- **4.34** A river is flowing due east with a speed 3m/s. A swimmer can swim in still water at a speed of 4 m/s (Fig. 4.8).
 - (a) If swimmer starts swimming due north, what will be his resultant velocity (magnitude and direction)?
 - (b) If he wants to start from point A on south bank and reach opposite point B on north bank,
 - (a) which direction should he swim?
 - (b) what will be his resultant speed?
 - (c) From two different cases as mentioned in (a) and (b) above, in which case will he reach opposite bank in shorter time?



- **4.35** A cricket fielder can throw the cricket ball with a speed v_o . If he throws the ball while running with speed u at an angle θ to the horizontal, find
 - (a) the effective angle to the horizontal at which the ball is projected in air as seen by a spectator.
 - (b) what will be time of flight?
 - (c) what is the distance (horizontal range) from the point of projection at which the ball will land?

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- (d) find θ at which he should throw the ball that would maximise the horizontal range as found in (iii).
- (e) how does θ for maximum range change if $u > v_0$, $u = v_0$, $u < v_0$?
- (f) how does θ in (v) compare with that for u = 0 (i.e. 45°)?
- 4.36 Motion in two dimensions, in a plane can be studied by expressing position, velocity and acceleration as vectors in Cartesian co-ordinates $\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$ where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vector along x and y directions, respectively and A_x and A_y are corresponding components of A (Fig. 4.9). Motion can also be studied by expressing vectors in circular polar co-ordinates as $\mathbf{A} = A_r \hat{\mathbf{r}} + A_\theta \hat{\mathbf{\theta}}$

where $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \cos\theta \,\hat{\mathbf{i}} + \sin\theta \,\hat{\mathbf{j}}$ and $\hat{\mathbf{\theta}} = -\sin\theta \,\hat{\mathbf{i}} + \cos\theta \,\hat{\mathbf{j}}$ are unit vectors along direction in which 'r' and ' θ ' are increasing.

- (a) Express $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ in terms of $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$.
- (b) Show that both $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors and are perpendicular to each other.
- (c) Show that $\frac{d}{dt}(\hat{\mathbf{r}}) = \omega \hat{\boldsymbol{\theta}}$ where

$$\omega = \frac{d\theta}{dt}$$
 and $\frac{d}{dt}(\hat{\theta}) = -\omega \hat{\mathbf{r}}$

- (d) For a particle moving along a spiral given by $\mathbf{r} = a\theta \hat{\mathbf{r}}$, where a = 1 (unit), find dimensions of 'a'.
- (e) Find velocity and acceleration in polar vector represention for particle moving along spiral described in (d) above.

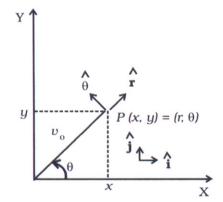


Fig. 4.9

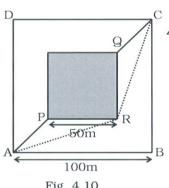


Fig. 4.10

4.37 A man wants to reach from A to the opposite corner of the square C (Fig. 4.10). The sides of the square are 100 m. A central square of 50m 50m is filled with sand. Outside this square, he can walk at a speed 1 m/s. In the central square, he can walk only at a speed of v m/s (v < I). What is smallest value of v for which he can reach faster via a straight path through the sand than any path in the square outside the sand?