1. Prove that there do not exist natural numbers x and y, with $x > 1$, such that

$$\frac{x^7 - 1}{x - 1} = y^5 + 1 \,.$$

2. In a triangle ABC, AD is the altitude from A, and H is the orthocentre. Let K be the centre of the circle passing through D and tangent to BH at H. Prove that the line DK bisects AC.

3. Consider the expression

$$2013^2 + 2014^2 + 2015^2 + \cdots + n^2 \,.$$

Prove that there exists a natural number $n > 2013$ for which one can change a suitable number of plus signs to minus signs in the above expression to make the resulting expression equal 9999.

4. Let ABC be a triangle with $\angle A = 90°$ and $AB = AC$. Let D and E be points on the segment BC such that $BD : DE : EC = 1 : 2 : \sqrt{3}$. Prove that $\angle DAE = 45°$.

5. Let $n \geq 3$ be a natural number and let P be a polygon with n sides. Let $a_1, a_2, \ldots, a_n$ be the lengths of the sides of P and let p be its perimeter. Prove that

$$\frac{a_1}{p - a_1} + \frac{a_2}{p - a_2} + \cdots + \frac{a_n}{p - a_n} < 2 \,.$$

6. For a natural number n, let $T(n)$ denote the number of ways we can place n objects of weights $1, 2, \ldots, n$ on a balance such that the sum of the weights in each pan is the same. Prove that $T(100) > T(99)$.

——— ⋆⋆ ———