1. Find the number of eight-digit numbers the sum of whose digits is 4.

2. Find all 4-tuples (a, b, c, d) of natural numbers with $a \leq b \leq c$ and $a! + b! + c! = 3^d$.

3. In an acute-angled triangle ABC with $AB < AC$, the circle Γ touches AB at B and passes through C intersecting AC again at D. Prove that the orthocentre of triangle ABD lies on Γ if and only if it lies on the perpendicular bisector of BC.

4. A polynomial is called a *Fermat polynomial* if it can be written as the sum of the squares of two polynomials with integer coefficients. Suppose that $f(x)$ is a Fermat polynomial such that $f(0) = 1000$. Prove that $f(x) + 2x$ is not a Fermat polynomial.

5. Let ABC be a triangle which it not right-angled. Define a sequence of triangles $A_iB_iC_i$, with $i \geq 0$, as follows: $A_0B_0C_0$ is the triangle ABC; and, for $i \geq 0$, $A_{i+1}, B_{i+1}, C_{i+1}$ are the reflections of the orthocentre of triangle $A_iB_iC_i$ in the sides B_iC_i, C_iA_i, A_iB_i, respectively. Assume that $\angle A_m = \angle A_n$ for some distinct natural numbers m, n. Prove that $\angle A = 60°$.

6. Let $n \geq 4$ be a natural number. Let $A_1A_2 \cdots A_n$ be a regular polygon and $X = \{1, 2, \ldots, n\}$. A subset $\{i_1, i_2, \ldots, i_k\}$ of X, with $k \geq 3$ and $i_1 < i_2 < \cdots < i_k$, is called a *good subset* if the angles of the polygon $A_{i_1}A_{i_2} \cdots A_{i_k}$, when arranged in the increasing order, are in an arithmetic progression. If n is a prime, show that a **proper** good subset of X contains exactly four elements.

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