1. Let Γ be a circle with centre O. Let Λ be another circle passing through O and intersecting Γ at points A and B. A diameter CD of Γ intersects Λ at a point P different from O. Prove that
$$\angle APC = \angle BPD\,.$$

2. Determine the smallest prime that does not divide any five-digit number whose digits are in a strictly increasing order.

3. Given real numbers $a, b, c, d, e > 1$ prove that
$$\frac{a^2}{c-1} + \frac{b^2}{d-1} + \frac{c^2}{e-1} + \frac{d^2}{a-1} + \frac{e^2}{b-1} \geq 20\,.$$

4. Let x be a non-zero real number such that $x^4 + \frac{1}{x^4}$ and $x^5 + \frac{1}{x^5}$ are both rational numbers. Prove that $x + \frac{1}{x}$ is a rational number.

5. In a triangle ABC, let H denote its orthocentre. Let P be the reflection of A with respect to BC. The circumcircle of triangle ABP intersects the line BH again at Q, and the circumcircle of triangle ACP intersects the line CH again at R. Prove that H is the incentre of triangle PQR.

6. Suppose that the vertices of a regular polygon of 20 sides are coloured with three colours – red, blue and green – such that there are exactly three red vertices. Prove that there are three vertices A, B, C of the polygon having the same colour such that triangle ABC is isosceles.

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