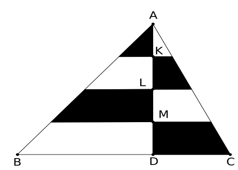
Regional Mathematical Olympiad-2014

Time: 3 hours **December 07, 2014**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

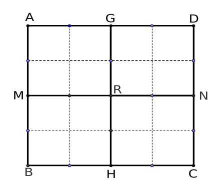
1. Let ABC be a triangle and let AD be the perpendicular from A on to BC. Let K, L, M be points on AD such that $AK = KL = LM = MD$. If the sum of the areas of the shaded regions is equal to the sum of the areas of the unshaded regions, prove that $BD = DC$.



2. Let $a_1, a_2, \ldots, a_{2n}$ be an arithmetic progression of positive real numbers with common difference d. Let
(i) $a_1^2 + a_3^2 + \cdots + a_{2n-1}^2 = x$, (ii) $a_2^2 + a_4^2 + \cdots + a_{2n}^2 = y$, and
(iii) $a_n + a_{n+1} = z$.
Express d in terms of x, y, z, n.

3. Suppose for some positive integers r and s, the digits of 2^r is obtained by permuting the digits of 2^s in decimal expansion. Prove that $r = s$.

4. Is it possible to write the numbers $17, 18, 19, \ldots, 32$ in a 4×4 grid of unit squares, with one number in each square, such that the product of the numbers in each 2×2 sub-grids $AMRG$, $GRND$, $MBHR$ and $RHCN$ is **divisible** by 16?



5. Let ABC be an acute-angled triangle and let H be its ortho-centre. For any point P on the circum-circle of triangle ABC, let Q be the point of intersection of the line BH with the line AP. Show that there is a unique point X on the circum-circle of ABC such that for every point $P \neq A, B$, the circum-circle of HQP pass through X.

6. Let $x_1, x_2, \ldots, x_{2014}$ be positive real numbers such that $\sum_{j=1}^{2014} x_j = 1$. Determine with proof the smallest constant K such that

$$K \sum_{j=1}^{2014} \frac{x_j^2}{1 - x_j} \geq 1.$$

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