Regional Mathematical Olympiad-2014

Time: 3 hours **December 07, 2014**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. In an acute-angled triangle ABC, $\angle ABC$ is the largest angle. The perpendicular bisectors of BC and BA intersect AC at X and Y respectively. Prove that circumcentre of triangle ABC is incentre of triangle BXY.

2. Let x, y, z be positive real numbers. Prove that
$$\frac{y^2+z^2}{x}+\frac{z^2+x^2}{y}+\frac{x^2+y^2}{z} \geq 2(x+y+z).$$

3. Find all pairs of (x, y) of positive integers such that $2x+7y$ divides $7x+2y$.

4. For any positive integer $n>1$, let $P(n)$ denote the largest prime not exceeding n. Let $N(n)$ denote the next prime larger than $P(n)$. (For example $P(10)=7$ and $N(10)=11$, while $P(11)=11$ and $N(11)=13$.) If $n+1$ is a prime number, prove that the value of the sum
$$\frac{1}{P(2)N(2)}+\frac{1}{P(3)N(3)}+\frac{1}{P(4)N(4)}+\cdots+\frac{1}{P(n)N(n)}=\frac{n-1}{2n+2}.$$

5. Let ABC be a triangle with $AB>AC$. Let P be a point on the line AB beyond A such that $AP+PC=AB$. Let M be the mid-point of BC and let Q be the point on the side AB such that $CQ \perp AM$. Prove that $BQ=2AP$.

6. Let n be an odd positive integer and suppose that each square of an $n\times n$ grid is arbitrarily filled with either by 1 or by -1. Let r_j and c_k denote the product of all numbers in j-th row and k-th column respectively, $1 \leq j, k \leq n$. Prove that
$$\sum_{j=1}^{n} r_j+\sum_{k=1}^{n} c_k \neq 0.$$

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