Regional Mathematical Olympiad-2014

Time: 3 hours **December 07, 2014**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Let ABC be an acute-angled triangle and suppose $\angle ABC$ is the largest angle of the triangle. Let R be its circumcentre. Suppose the circumcircle of triangle ARB cuts AC again in X. Prove that RX is pependicular to BC.

2. Find all real numbers x and y such that

$$x^2 + 2y^2 + \frac{1}{2} \leq x(2y + 1).$$

3. Prove that there does not exist any positive integer $n < 2310$ such that $n(2310 - n)$ is a multiple of 2310.

4. Find all positive real numbers x, y, z such that

$$2x - 2y + \frac{1}{z} = \frac{1}{2014}, \quad 2y - 2z + \frac{1}{x} = \frac{1}{2014}, \quad 2z - 2x + \frac{1}{y} = \frac{1}{2014}.$$

5. Let ABC be a triangle. Let X be on the segment BC such that $AB = AX$. Let AX meet the circumcircle Γ of triangle ABC again at D. Show that the circumcentre of $\triangle BDX$ lies on Γ.

6. For any natural number n, let $S(n)$ denote the sum of the digits of n. Find the number of all 3-digit numbers n such that $S(S(n)) = 2$.

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