Regional Mathematical Olympiad-2014

Time: 3 hours **December 07, 2014**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Let $ABCD$ be an isosceles trapezium having an incircle; let AB and CD be the parallel sides and let CE be the perpendicular from C on to AB. Prove that CE is equal to the geometric mean of AB and CD.

2. If x and y are positive real numbers, prove that

$$4x^4 + 4y^3 + 5x^2 + y + 1 \geq 12xy.$$

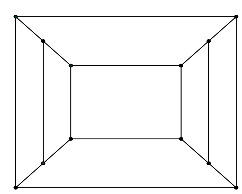
3. Determine all pairs $m > n$ of positive integers such that

$$1 = \gcd(n+1, m+1) = \gcd(n+2, m+2) = \cdots = \gcd(m, 2m-n).$$

4. What is the minimal area of a right-angled triangle whose inradius is 1 unit?

5. Let ABC be an acute-angled triangle and let I be its incentre. Let the incircle of triangle ABC touch BC in D. The incircle of the triangle ABD touches AB in E; the incircle of the triangle ACD touches BC in F. Prove that B, E, I, F are concyclic.

6. In the adjacent figure, can the numbers $1, 2, 3, 4, \cdots, 18$ be placed, one on each line segment, such that the sum of the numbers on the three line segments meeting at each point is divisible by 3?



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