Regional Mathematical Olympiad-2015

Time: 3 hours **December 06, 2015**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Let ABC be a triangle. Let B' and C' denote respectively the reflection of B and C in the internal angle bisector of $\angle A$. Show that the triangles ABC and $AB'C'$ have the same incentre.

2. Let $P(x) = x^2 + ax + b$ be a quadratic polynomial with real coefficients. Suppose there are real numbers $s \neq t$ such that $P(s) = t$ and $P(t) = s$. Prove that $b - st$ is a root of the equation $x^2 + ax + b - st = 0$.

3. Find all integers a, b, c such that
$$a^2 = bc + 1, \quad b^2 = ca + 1.$$

4. Suppose 32 objects are placed along a circle at equal distances. In how many ways can 3 objects be chosen from among them so that no two of the three chosen objects are adjacent nor diametrically opposite?

5. Two circles Γ and Σ in the plane intersect at two distinct points A and B, and the centre of Σ lies on Γ. Let points C and D be on Γ and Σ, respectively, such that C, B and D are collinear. Let point E on Σ be such that DE is parallel to AC. Show that $AE = AB$.

6. Find all real numbers a such that $4 < a < 5$ and $a(a - 3\{a\})$ is an integer. (Here $\{a\}$ denotes the fractional part of a. For example $\{1.5\} = 0.5$; $\{-3.4\} = 0.6$.)

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