Regional Mathematical Olympiad-2015

Time: 3 hours **December 06, 2015**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Let ABC be a triangle. Let B' denote the reflection of B in the internal angle bisector ℓ of $\angle A$. Show that the circumcentre of the triangle $CB'I$ lies on the line ℓ, where I is the incentre of ABC.

2. Let $P(x) = x^2 + ax + b$ be a quadratic polynomial where a is real and b is rational. Suppose $P(0)^2$, $P(1)^2$, $P(2)^2$ are integers. Prove that a and b are integers.

3. Find all integers a, b, c such that
$$a^2 = bc + 4, \quad b^2 = ca + 4.$$

4. Suppose 40 objects are placed along a circle at equal distances. In how many ways can 3 objects be chosen from among them so that no two of the three chosen objects are adjacent nor diametrically opposite?

5. Two circles Γ and Σ intersect at two distinct points A and B. A line through B intersects Γ and Σ again at C and D, respectively. Suppose that $CA = CD$. Show that the centre of Σ lies on Γ.

6. How many integers m satisfy both the following properties:
$$\text{(i) } 1 \le m \le 5000; \text{ (ii) } \left[\sqrt{m}\right] = \left[\sqrt{m+125}\right]?$$
(Here $[x]$ denotes the largest integer not exceeding x, for any real number x.)

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