Model Questions and Answers for Mathematics – Part I

(All answers have been provided by a University teacher who can be reached at sssks7@gmail.com)

1. (i) Find the correct answer:- **1**

Principal, time and rate of interest – out of these three if any two remain invariant, the remaining one bears with total interest
(i) direct relation (ii) inverse relation (iii) no relation (iv) any relation

Solution: If, Principal = P, Time = T, Rate = R and Interest = I

We know, $I = \frac{PTR}{100}$

If Principal and Time are constants.

We get, $I = \frac{\text{Constant}}{100} R$

$\therefore$ It is a direct relation.

Ans: Direct relation (i)

(ii) Find the correct answer:- **1**
m, (m-1) are factors of $m^3 - m$. The remaining one will be
(i) m^2-1 (ii) 1-m (iii) m+1 (iv) $m^2 + 1$

Solution: Given, $m^3 - m$
$= m(m^2 - 1)$
$= m(m+1)(m-1)$

Since the given factors are m and (m-1), the remaining factor is (m+1)

Ans: The remaining factor is (m+1) (iii)

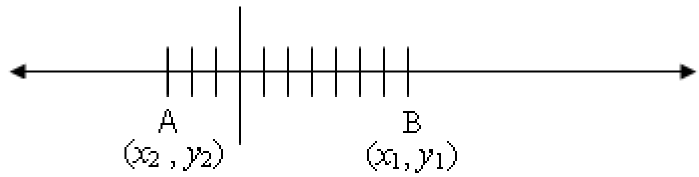
(iii) Determine the value of *a* for which the expression $(a\text{-}2)x^2 + 3x + 5 = 0$ will not be a quadratic equation.
1

Solution: The expression will not be a quadratic equation,
if $(a\text{-}2) = 0$
i.e. $a - 2 = 0$
or, $a = 2$

Ans: For $a = 2$, the given expression will not be a quadratic equation.

(iv) Find the distance between the points (-3, 0) and (7, 0). **1**

Solution:



$\therefore$ Length of AB $= \sqrt{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}$

$= \sqrt{[(7 + 3)^2 + (0 - 0)^2]}$

$= \sqrt{100} = 10$ units

Ans: The distance is 10 units.

(v) If the whole surface area and the volume of a cube are numerically equal, what is the length of its side? **1**

Solution: Let the length of one side of the cube be a units.

$\therefore$ Total surface area of the cube $= 6a^2$ units

And, volume $= a^3$ units

By the problem, $a^3 = 6a^2$

or, $a = 6$

Ans: The required length of the cube is 6 units.

(v) Find the correct answer:- **1**
If $0^o \leq \theta \leq 90^o$ and $\sin\theta = \cos\theta$ then θ will be
(i) 30^o (ii) 60^o (iii) 45^o (iv) 90^o

Solution: $\sin\theta = \cos\theta$

or, $\dfrac{\sin\theta}{\cos\theta} = 1$

or, $\tan\theta = \tan 45^o$

or, $\theta = 45^o$

Ans: $\theta = 45^o$ (iii)

2 (a) If there be a loss of 11% in selling an article at Rs. 178, at what price should it be sold to earn a profit of 11%? **2**

Solution: Let the CP of an article be C

Since there is a loss of 11%,

$\therefore$ SP $= C - \frac{11C}{100} = \frac{89C}{100}$

By the problem, $\frac{89C}{100} = 178$

or, $C = \frac{178 \times 100}{89} = 200$

$\therefore$ The cost price (CP) of the article = Rs. 200

He should earn a profit of 11 %

$\therefore$ SP of the article $= C + \frac{11C}{100}$

$= 200 + \frac{11 \times 200}{100}$

$= 222$

Ans: The sale price of the article is Rs. 222.

(b) What should be the values of *a* and *b* for which $64x^3 - 9ax^2 + 108x - b$ will be a perfect cube. **2**

Solution: $64x^3 - 9ax^2 + 108x - b$ ------------ (i)

We know, $(4x)^3 - 3(4x)^2 \times 9 + 3(4x) \times 9^2 - (9)^3$ ------------ (ii)
$= (4x - 9)^3$

Comparing equation (i) and (ii) we get,

$9ax^2 = 3 \times 16x^2 \times 9$

or, $a = 48$

and $b = (9)^3 = 729$

$\therefore$ $a = 48$ and $b = 729$

Ans: $a = 48$ and $b = 729$

(c) For which value of *r*, $rx + 2y = 5$ and $(r - 1)x + 5y = 2$ have no solution? **2**

Solution: We know, if $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ are two equations then there will no solution if,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Comparing with the given two equations, we have,

$a_1 = r,\quad b_1 = 2,\quad c_1 = 5$
and, $a_2 = r - 1,\ b_2 = 5,\quad c_2 = 2$

By the problem,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

or, $\dfrac{r}{r-1} = \dfrac{2}{5}$

or, $5r = 2r - 2$

or, $3r = -2$

or, $r = -\dfrac{2}{3}$

Ans: The value of r is $(-\dfrac{2}{3})$.

(d) If $x^2 : \dfrac{yz}{x} = y^2 : \dfrac{zx}{y} = z^2 : \dfrac{xy}{xz}$, prove with reasons that $x = y = z$ **2**

Solution:

$$x^2 : \frac{yz}{x} = y^2 : \frac{zx}{y} = z^2 : \frac{xy}{z}$$

or, $\dfrac{x^2 \times x}{yz} = \dfrac{y^2 \times y}{zx} = \dfrac{z^2 \times z}{xy}$

or, $\dfrac{x \times (x^3)}{xyz} = \dfrac{y \times (y^3)}{xyz} = \dfrac{z \times (z^3)}{xyz}$

or, $\dfrac{x^4}{xyz} = \dfrac{y^4}{xyz} = \dfrac{z^4}{xyz}$

or, $x^4 = y^4 = z^4$ [considering $xyz \neq 0$]

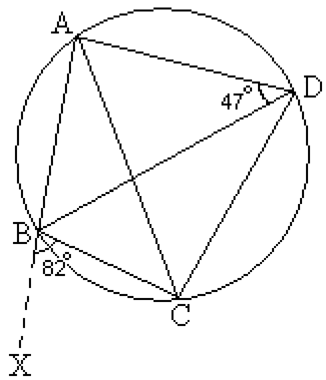
or, $x = y = z$ (Proved)

(e) State Pythagoras' Theorem. **2**

Ans: The area of the square on the hypotenuse of a right angled triangle is equal to the sum of areas of the squares on other two sides.

(f) Side AB of a cyclic quadrilateral ABCD is produced to X. if $\angle$ XBC = 82^o and $\angle$ADB=47^o find the value of $\angle$ BAC. **2**

Solution:



$\angle XBC = 82^o$

$\therefore \angle ABC = 180^o - 82^o = 98^o$

Again, $\angle ABC + \angle ADC = 180^o$

or, $\angle ADC = 180^o - \angle ABC = 180^o - 98^o = 82^o$

Now, $\angle BDC = \angle ADC - \angle ADB$

$= 82^o - 47^o$

$= 35^o$

Since $\angle BAC = \angle BDC$ [Same segment of a circle]

i.e., $\angle BAC = 35^o$

Ans: The value of $\angle$ BAC is 35^o

(g) Show that $1^o < 1^c$ 2

Solution: We know, $180^o = \pi^c$

$$\text{or, } 180^o = \left(\frac{22}{7}\right)^c$$

$$\text{or, } 1^o = \left(\frac{22}{7 \times 180}\right)^c$$

or, $1^o < 1^c$ (Proved)

3. Answer any *two* questions:- **$5\times 2=10$**

(a) A and B started a business with capitals of Rs. 3000 and Rs. 4000 respectively. After 8 months, A invested Rs. 2500 more in the business and 7 months after this, total profit becomes Rs. 980. Find the share of profit for each.

Solution: According to the problem A invested Rs. 3000 for 8 months and after 8 months A also invested Rs. 2500 for another 7 months.

$\therefore$ In terms of 1 month, A's capital investment

$= \text{Rs. } (3000\times 8) + \text{Rs. } ((3000+2500)\times 7)$

$= \text{Rs. } 24000 + \text{Rs. } 38500$

$= \text{Rs. } 62500$

$\therefore$ In terms of 1 month, B's capital investment

$= \text{Rs. } (4000 \times 15)$

$= \text{Rs. } 60000$

$\therefore$ the ratio of capital investment of A and B is

$A:B = 62500:60000$

or, $A:B = 25:24$

$\therefore$ the share of A's profit after 15 months

$= \text{Rs. } 980 \times \frac{25}{49}$

$= \text{Rs. } 500$

$\therefore$ the share of B's profit after 15 months

$= \text{Rs. } 980 \times \frac{25}{49}$

$= \text{Rs. } 480$

Ans: A's profit is Rs. 500 and B's profit is Rs. 480.

(b) At 10% per annum, the difference of compound interest, compounded annually and simple interest on a certain sum of money for 3 years is Rs. 124. Find the sum of money.

Solution:

Rate (R) = 10%

Time (T) = 3 Years

Principal = P

$$\therefore \text{Simple Interest} = \frac{PTR}{100} = \frac{P\times 3\times 10}{100} = \frac{30P}{100}$$

$\therefore$ Compound Interest $= P\,(1 + \frac{10}{100})^n - P$

$$= P(1 + \frac{10}{100})^3 - P$$

$$= P(\frac{110}{100})^3 - P$$

$$= P(\frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}) - P$$

$$= \frac{1331P - 1000P}{1000} = \frac{331P}{1000}$$

By the problem,

$$\frac{331P}{1000} - \frac{30P}{100} = 124$$

or, $\frac{31P}{1000} = 124$

or, $P = \frac{124 \times 1000}{31} = 4000$

Ans: The sum of money is Rs. 4000.

(c) A person purchased some agricultural land at Rs. 720000. He sold $\frac{1}{3}$ of the land at 20% loss, $\frac{2}{5}$ at 25% profit. At what price should he sell the remaining land to get an overall profit of 10%.

Solution:

Total cost price of the agricultural land is Rs. 7,20,000 and the overall profit is 10%

$\therefore$ SP = Rs. 7,20,000 $\times \frac{110}{100}$ = Rs. 7,200 $\times$ 110 = Rs. 7,92,000.

CP of $\frac{1}{3}$ of land = 7,20,000 $\times \frac{1}{3}$ = Rs. 2,40,000

and there is a loss of 20%.

$\therefore$ SP of $\frac{1}{3}$ part of the land = Rs. 2,40,000 $\times \frac{80}{100}$ = Rs. 1,92,000

Now, CP of $\frac{2}{5}$ part of land = 7,20,000 × $\frac{2}{5}$ = Rs. 2,88,000
and there is a profit of 25%.

∴ SP of $\frac{2}{5}$ part of the land = Rs. 2,88,000 × $\frac{125}{100}$ = Rs. 3,60,000

∴ SP of ($\frac{1}{3}$ + $\frac{2}{5}$) part of the land = Rs. (1,92,000 + 3,60,000) = Rs. 5,52,000

∴ the SP of the remaining land
= Rs. (7,92,000 – 5,52,000)
= Rs. 2,40,000.

Ans: The person should sell the remaining land at Rs. 240000 to get an overall profit of 10%.

(d) Ratio of acid and water in one container is 2:7 and the ratio of same acid and water in another container is 2:9. At what ratio, the contents of the two containers should be mixed to have the ratio of acid and water as 1:4 in the new mixture?

Solution:

Let the contents of two containers are mixed in ratio x:y to have the ratio of acid and water 1:4.

Since ratio of acid and water in first container is 2:7 and x quantity is taken from it,

quantity of acid = $\frac{2x}{9}$ and

quantity of water = $\frac{7x}{9}$

Since ratio of acid and water in second container is 2:9 and y quantity is taken from it, the quantity of acid = $\frac{2y}{11}$ and quantity of water = $\frac{9y}{11}$.

∴ The total quantity of acid = $\frac{2x}{9} + \frac{2y}{11} = \frac{22x+18y}{99}$

∴ The total quantity of water = $\frac{7x}{9} + \frac{9y}{11} = \frac{77x+81y}{99}$

By the problem,

$$\frac{\frac{22x+18y}{99}}{\frac{77x+81y}{99}} = \frac{1}{4}$$

or, $\frac{(22x+18y)}{(77x+81y)} = \frac{1}{4}$

or, $88x + 72y = 77x + 81y$

or, $88x - 77x = 81y - 72y$

or, $11x = 9y$

or, $\frac{x}{y} = \frac{9}{11}$

$\therefore$ $x{:}y = 9{:}11$

Ans: <u>At 9:11 ratio, the contents of two containers should be mixed to have the ratio of acid and water as 1:4 in the new mixture.</u>

4. **Resolve into factor:-**

$\boldsymbol{a^2 + \frac{1}{a^2} + 2 - 2a - \frac{2}{a}}$ **4**

Solution: $a^2 + \frac{1}{a^2} + 2 - 2a - \frac{2}{a}$

$= a^2 + 2 + \frac{1}{a^2} - 2(a + \frac{1}{a})$

$= (a + \frac{1}{a})^2 - 2(1 + \frac{1}{a})$

$= (a + \frac{1}{a})(a + \frac{1}{a} - 2)$

Ans: $\underline{(a + \frac{1}{a})(a + \frac{1}{a} - 2)}$

Or,

Find the HCF of:-

$\boldsymbol{x^3 - 16x, 2x^3 + 9x^2 + 4x, 2x^3 + x^2 - 28x}$

Solution:

1st expression: $x^3 - 16x = x(x^2 - 16)$
$= x(x + 4)(x - 4)$

2nd expression: $2x^3 + 9x^2 + 4x$
$= x(2x^2 + 9x + 4)$
$= x(2x^2 + 8x + x + 4)$
$= 2[2x(x + 4) + 1(x + 4)]$
$= x(x + 4)(2x + 1)$

3rd expression: $2x^3 + x^2 - 28x$
$= x(2x^2 + x - 28)$
$= x(2x^2 + 8x - 7x - 28)$
$= x[2x(x + 4) + 7(x + 4)]$
$= x(2x - 7)(x + 4)$

$\therefore$ HCF $= x(x+4)$

Ans: HCF is $x(x+4)$.

5. Solve (any *one*):-

(a) $\dfrac{x}{2} + \dfrac{y}{3} = 1$ **and** $\dfrac{x}{3} + \dfrac{y}{2} = 1$ **3**

Solution:

$$\frac{x}{2} + \frac{y}{3} = 1 \qquad \text{(i)}$$

$$\text{or, } \frac{(3x + 2y)}{6} = 1$$

$$\text{or, } 3x + 2y = 6 \qquad \text{(iii)}$$

$$\frac{x}{3} + \frac{y}{2} = 1 \qquad \text{(ii)}$$

$$\text{or, } \frac{(2x + 3y)}{6} = 1$$

$$\text{or, } 2x + 3y = 6 \qquad \text{(iv)}$$

Multiplying equation (iii) with 2 and equation (iv) with 3, we get

$6x + 4y = 12$
$6x + 9y = 18$
(-) (-)

$5y = 6$

or, $y = \frac{6}{5}$

Putting the value of y in equation (iii) we get,

$3x + 2y = 6$

or, $3x + 2(\frac{6}{5}) = 6$

or, $3x + \frac{12}{5} = 6$

or, $3x = 6 - \frac{12}{5}$

or, $3x = \frac{(30-12)}{5} = \frac{18}{5}$

or, $x = \frac{6}{5}$

$\therefore\ x = \frac{6}{5}$ and $y = \frac{6}{5}$

Ans: $x = \frac{6}{5}$ and $y = \frac{6}{5}$

(b) Solve: $\mathbf{(\frac{x+3}{x+1})^2 - 7(\frac{x+3}{x+1}) + 12 = 0}$

Solution:

Let $\frac{x+3}{x+1} = a$

$\therefore\ a^2 - 7a + 12 = 0$

or, $a^2 - 3a - 4a + 12 = 0$

or, $a(a-3) - 4(a-3) = 0$

or, $a(a-4)(a-3) = 0$

Either $a - 4 = 0$ or $a - 3 = 0$

Taking, $a - 4 = 0$

$$\frac{x+3}{x+1} = 4$$

or, $x + 3 = 4x + 4$

or, $3x = -1$

or, $x = -\frac{1}{3}$

Taking, a-3 = 0

$$\frac{x+3}{x+1} = 3$$

or, $x + 3 = 3x + 12$

or, $2x = 3 - 12 = -9$

or, $x = -\frac{9}{2}$

Ans: $x = -\frac{1}{3}$ and $x = -\frac{9}{2}$

6. Answer any *one*:- **4**

(a) After traveling 108 km, a cyclist observed that he would have required 3 hr less if he could have travelled at a speed 3 km/hr more. At what speed did he travel? (use algebraic method)

Solution:

Let the speed be x km / hr.

Since distance is 108 km, time = $\frac{108}{x}$ hrs

When speed is increased by 3 km/hr, speed is = $(x + 3)$ km/hr

$\therefore$ The required time = $\frac{108}{(x+3)}$ hrs

By the problem, $\frac{108}{x} - \frac{108}{(x+3)} = 3$

or, $\dfrac{108(x+3)-108x}{x(x+3)}=3$

or, $\dfrac{108x+324-108x}{x^2+3x}=3$

or, $324 = 3x^2 + 9x$

or, $324 - 3x^2 - 9x = 0$

or, $-3x^2 - 9x + 324 = 0$

or, $-3(x^2 + 3x - 108) = 0$

or, $x^2 + 3x - 108 = 0$

or, $x^2 + 12x - 9x - 108 = 0$

or, $x(x + 12) - 9(x + 12) = 0$

or, $(x - 9)(x + 12) = 0$

So either, $x - 9 = 0$ or, $x + 12 = 0$

If $x - 9 = 0$ then $x = 9$

If $x + 12 = 0$ then $x = -12$

$\therefore$ the speed = 9 km / hr [Since velocity $x \neq -12$]

Ans: He traveled at a speed of 9 km/hr.

(b) Price of 3 tables and 5 chairs is Rs. 2000. Again, price of 5 tables and 7 chairs is Rs. 3200. What is the price of 1 table and 1 chair. (use algebraic method)

Solution:

Let the price of 1 chair be x and 1 table be y.

From the 1st condition, $3x + 5y = 2000$
From the 2nd condition, $5x + 7y = 3200$

i.e. $3x + 5y = 2000$ ------- (i)
$5x + 7y = 3200$ ------- (ii)

Multiplying equation-(i) by 5 and equation-(ii) by 3, we get,

$$15x + 25y = 10000$$
$$15x + 21y = \ \ 9600$$
$$(-) \quad (-)$$
$$\overline{\qquad\qquad\qquad\qquad}$$
$$4y = 400$$
$$\text{or,} \quad y = 100$$

Putting the value of y in equation (i)

$$3x + 5(100) = 2000$$
$$\text{or,} \quad 3x = 2000 - 500 = 1500$$
$$\text{or,} \quad x = 500.$$

$\therefore$ The price of 1 chair is Rs. 500 and 1 table is Rs. 100.

Ans: <u>The price of 1 chair is Rs. 500 and 1 table is Rs. 100.</u>

8. If x=2, y=3 and z=6, what is the value of: $\dfrac{3\sqrt{x}}{\sqrt{y}+\sqrt{z}} - \dfrac{4\sqrt{y}}{\sqrt{z}+\sqrt{x}} + \dfrac{\sqrt{z}}{\sqrt{x}+\sqrt{y}}$? **3**

Solution: Putting the values of x, y and z

$$= \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$$

$$= \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$$

1st expression:

$$\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}$$
$$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})} = \frac{3(\sqrt{12}-\sqrt{6})}{6-3} = \sqrt{12}-\sqrt{16}$$

2nd expression;

$$\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$$
$$= \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} = \frac{4(\sqrt{18}-\sqrt{6})}{6-2} = \sqrt{18}-\sqrt{6}$$

3rd expression:

$$\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{(\sqrt{18}-\sqrt{12})}{3-2} = \sqrt{18}-\sqrt{12}$$

So, 1st expression - 2nd expression + 3rd expression

$$= \sqrt{12}-\sqrt{16}-\sqrt{18}+\sqrt{6}+\sqrt{18}-\sqrt{12}$$
$$= 0$$

Ans: The value of $\frac{3\sqrt{x}}{\sqrt{y}+\sqrt{z}}$ - $\frac{4\sqrt{y}}{\sqrt{z}+\sqrt{x}}+\frac{\sqrt{z}}{\sqrt{x}+\sqrt{y}}$ is 0 when $x = 2$, $y = 3$ and $z = 6$

Or,

Simplify:

$$\frac{\dfrac{a^2}{x-a}+\dfrac{b^2}{x-b}+\dfrac{c^2}{x-c}+a+b+c}{\dfrac{a}{x-a}+\dfrac{b}{x-b}+\dfrac{c}{x-c}}$$

Solution:

$$\frac{\dfrac{a^2}{x-a}+\dfrac{b^2}{x-b}+\dfrac{c^2}{x-c}+a+b+c}{\dfrac{a}{x-a}+\dfrac{b}{x-b}+\dfrac{c}{x-c}}$$

$$= \frac{\dfrac{a^2}{x-a}+a+\dfrac{b^2}{x-b}+b+\dfrac{c^2}{x-c}+c}{\dfrac{a}{x-a}+\dfrac{b}{x-b}+\dfrac{c}{x-c}}..$$

$$= \frac{\dfrac{a^2+ax-a^2}{x-a}+\dfrac{b^2+bx-b^2}{x-b}+\dfrac{c^2+cx-c^2}{x-c}}{\dfrac{a}{x-a}+\dfrac{b}{x-b}+\dfrac{c}{x-c}}$$

$$= \frac{x[\frac{a}{x-a} + \frac{b}{x-b} + \frac{c}{x-c}]}{\frac{a}{x-a} + \frac{b}{x-b} + \frac{c}{x-c}}$$

$$= x$$

Ans: x

9. **If $\frac{a}{q-r} = \frac{b}{r-p} = \frac{c}{p-q}$ then show that, $a + b + c = pa + qb + rc$** **3**

Solution:

$$\frac{a}{q-r} = \frac{b}{r-p} = \frac{c}{p-q} = \text{k} \quad \text{(say)}$$

$$\therefore \ a = \text{k}(q-r)$$
$$b = \text{k}(r-p)$$
$$c = \text{k}(p-q)$$

L.H.S. $= a + b + c$
$= \text{k}(q-r) + \text{k}(r-p) + \text{k}(p-q)$
$= \text{k}(q - r + r - p + p - q)$
$= \text{k} \,.\, 0$
$= 0$

R.H.S. $= pa + qb + rc$
$= p[\text{k}(q-r)] + q[\text{k}(r-p)] + r[\text{k}(p-q)]$
$= p(\text{k}q - \text{k}r) + q(\text{k}r - \text{k}p) + r(\text{k}p - \text{k}q)$
$= p\text{k}q - p\text{k}r - q\text{k}r - p\text{k}q - p\text{k}r - q\text{k}r$
$= 0$

Therefore, $a + b + c$ = pa + qb + rc = 0. [Hence proved]

Or,

If $x^2 \propto yz$, $y^2 \propto zx$, $z^2 \propto xy$, show that the product of the three constants of variations=1

Solution:

$x^2 \ \alpha \ yz$

$\therefore \ x^2 = \text{k}_1 \times yz$ where k_1 =constant.

$y^2 \ \alpha \ zx$

$\therefore\ y^2 = k_2 \times zx$ where k_2 =constant.

$z^2 \ \alpha \ xy$

$\therefore\ z^2 = k_3 \times xy$ where k_3 =constant.

$$x^2 \times y^2 \times z^2 = k_1 \times yz \times k_2 \times zx \times k_3 \times xy$$
$$= k_1 \times k_2 \times k_3 \times x^2 \times y^2 \times z^2$$

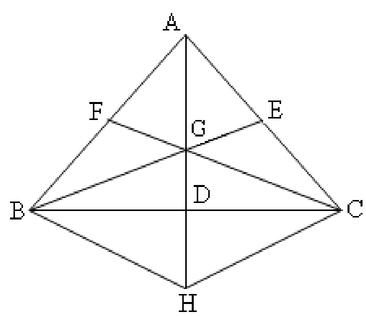
or, $k_1 \times k_2 \times k_3 = 1$ = constant (Proved)

$\therefore$ The product of three constants of variations = 1 (Proved)

10. Answer (a) or (b) and (c) or (d):-

(a) Prove that the medians of a triangle are concurrent **5**

Solution:



Given: Let ABC be a triangle in which F and E are the mid-points of the side AB and AC respectively. BE and CF intersects at point G. AG is joined and produced which intersect BC at the point D.

R.T.P: BD = DC; AD is the third median
Therefore the medians of a triangle are concurrent.

Construction: AD is produced to point H in such a way that GH = AG. BH and CH are joined.

Proof: F and G are the mid-points of the sides AB and AH of the ΔABH.

$\therefore$ FG || BH
i.e. GC || BH ----------------(i)
$\therefore$ E and G are the mid-points of the sides AC and AH of the ΔACH
$\therefore$ EG || CH
i.e., BG || CH ----------------(ii)
$\therefore$ BGCH is a parallelogram

Since GH and BC are the diagonals of the parallelogram and bisects each other.

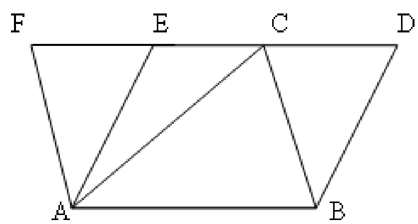
$\therefore$ D is a point of BC.

$\therefore$ AD is the third median.

$\therefore$ <u>The medians of the Δ are concurrent. (Proved)</u>

(b) If a triangle and a parallelogram are on the same base and between the same parallels, prove that the area of the triangle is half of the parallelogram.

Solution:



Given: Let ΔABC and parallelogram ABDE be on the same base AB and between the same parallels AB and ED.

R.T.P: $\Delta ABC = \frac{1}{2}$ parallelogram ABDE

Construction: The straight line through the point A, drawn parallel to BC, intersects DC produced at F.

Proof: By construction ABCF is a parallelogram and AC is one of its diagonal

$\therefore \Delta ABC = \frac{1}{2}$ parallelogram ABCF

$\therefore \Delta ABC = \frac{1}{2}$ parallelogram ABDE

$\therefore \Delta ABC = \frac{1}{2}$ of the parallelogram (Proved).

(c) Prove: The angle which on arc of a circle subtends at the centre is twice the angle subtended by the same at any point in the remaining part of the circle. 5

Solution:

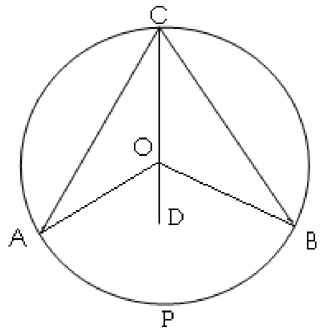


Fig 1

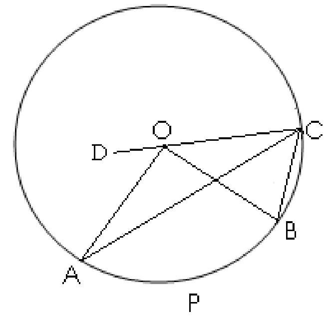


Fig 2

Given: Let angle AOB be the angle at the centre standing on the arc APB of the circle with centre O and angle ACB is the angle at any point C in the remaining part of the circle, standing on the same arc.

R.T.P: $\angle AOB = 2\angle ACB$

Construction: C and O are joined and CO is produced to any point D.

Proof: In ΔAOC,
OA = OC (radii of the same circle)
$\therefore \angle OCA = \angle OAC$
Again, since side CO of ΔAOC is produced to point D
$\therefore$ Exterior $\angle AOD = \angle OAC + \angle OCA = 2\angle OCA$

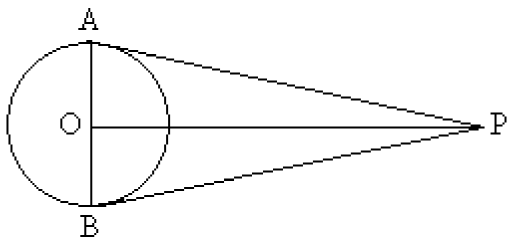
Similarly from ΔBOC, we get
$\angle BOD = 2\angle OCB$

$\therefore$ From fig. we get,
$\angle AOB = \angle AOD + \angle BOD$
$= 2(\angle OCA + \angle OCB)$
$= 2\angle ACB$

$\therefore \angle AOB = 2\angle ACB$ (Proved).

(d) If two tangents be drawn to a circle from a point outside it, then the line-segments joining the points of contact and the exterior point are equal and they subtend equal angles at the centre.

Solution:



Given: Let P be a point outside the circle with centre O. From the point P, two tangents PA and PB are drawn, whose points of contact are A and B respectively. OA; OB; OP are joined. Due to this, PA and PB subtend angle POA and angle POB respectively at the point O.

R.T.P i) PA = PB
ii) $\angle POA = \angle POB$

Proof: PA and PB are tangents and OA, OB are the radii through the points of contact

$\therefore$ OA is perpendicular to PA and OB is perpendicular to PB.

$\therefore$ ΔPAO and ΔPBO are right angled triangles.

In ΔPAO and ΔPBO,

(i) Hypotenuse PO is common

(ii) OA = OB (radii of the same circle)

(iii) $\angle PAO = \angle PBO$ $(=90^o)$

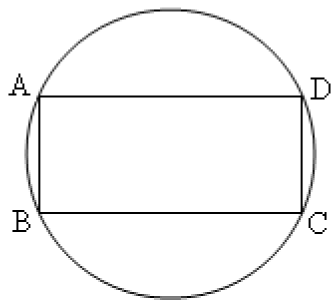
$\therefore$ $\Delta PAO \approxeq \Delta PBO$ [by S-A-S congruency]

$\therefore$ PA = PB (corresponding sides) [Hence proved (i)]

$\therefore$ $\angle POA = \angle POB$ [corresponding angles] (Proved (ii)).

11. Prove that a cyclic parallelogram must be a rectangle. **3**

Solution:



Given: Let ABCD be a cyclic parallelogram.

R.T.P.: Quadrilateral ABCD is a rectangle.

Proof: Since ABCD is a parallelogram

$\therefore \angle ABC = \angle\ ADC$

Since ABCD is a cyclic parallelogram

$\therefore\ \angle ABC + \angle\ ADC = 180^o$

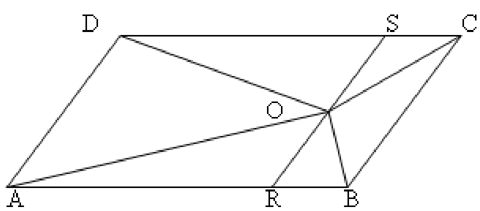
$\therefore\ \angle ABC = 90^o$

$\therefore$ Quadrilateral ABCD is a rectangle. (Proved).

Or,

ABCD is a parallelogram and O is a point inside the parallelogram. Prove that ΔAOD + ΔBOC = ½ × parallelogram ABCD.

Solution:



Given: ABCD is a parallelogram and O is a point inside the parallelogram.

R.T.P. $\Delta AOD + \Delta BOD = \frac{1}{2}$ of parallelogram ABCD

Construction: Through the point O, a straight line is drawn parallel to BC to intersect the sides AB and DC at the points R and S respectively.

Proof: $\Delta AOD = \frac{1}{2}$ of Parallelogram ARSD [since they have the same base AD and lie between the same parallels AD and RS]

Similarly $\Delta BOC = \frac{1}{2}$ of parallelogram BRSC.

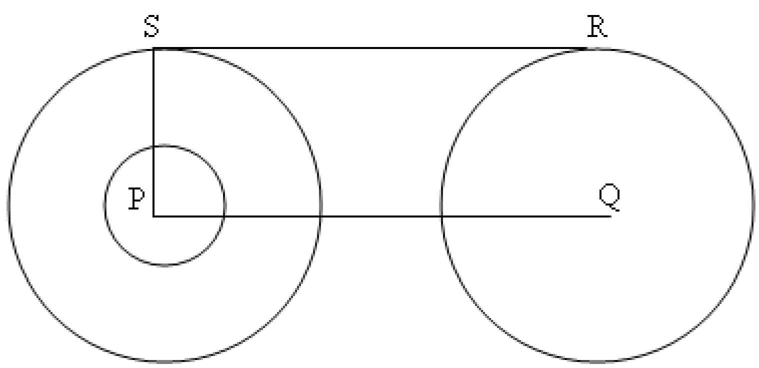
Therefore $\Delta AOD - \Delta BOD = \frac{1}{2}$ of [parallelogram ARSD + parallelogram BRSC]

Therefore $\Delta AOD + \Delta BOC = \frac{1}{2}$ of parallelogram ABCD (Proved).

---

12. Draw two circles each of radius 3.5cm; such that the distance between their centers is 7.5cm. Draw a direct common tangent to the two circles. [Only traces of construction are needed] **6**

Construction:



The length of PQ is 7.5cm.

Taking the same radii of the length 3.5cm two circles are drawn with the centers P and Q.

Perpendicular is drawn to PQ at the point P to meet the circle with the center P at the point S.

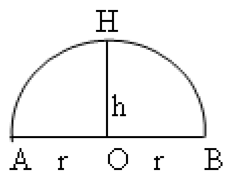
An arc of a circle is drawn with center S and radius equal to PQ to meet the circle with center Q at the point R. SR are joined.

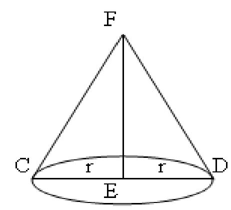
$\therefore$ SR is the direct common tangent of the two circles

13. Answer any two questions:- **$4 \times 2 = 8$**

(a) A hemisphere and a right circular cone on equal bases are of equal height. Find the ratio of their volumes and ratio of their curved surface area.

Solution:





By the problem, AB = CD = 2r and OH = EF = h

$\therefore$ For the hemisphere and cone, Height = Radius

$$\text{or,} \quad h = r$$

For the hemisphere, volume $= V_1 = \frac{1}{2} \times \frac{4}{3} \pi r^3$

$$= \frac{2}{3} \pi r^3$$

For the cone, volume $= V_2 = \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \pi r^2 \times r \quad (\text{since } h = r)$$

$$= \frac{1}{3} \pi r^3$$

By the problem, $V_1 : V_2 = \frac{2}{3} \pi r^3 : \frac{1}{3} \pi r^3$

$$= \frac{2}{3} : \frac{1}{3}$$

$$= 2:1$$

$\therefore$ the ratio of the volumes of hemisphere and cone is 2:1

Curved surface area of the hemisphere = S1 = $2r^2$

Curved surface area of the cone = S2 = πrl [l is the slant height]

For the cone: $l^2 = h^2 + r^2$

or, $l^2 = r^2 + r^2$

or, $l^2 = 2r^2$

or, $l = \sqrt{2}\,r$

By the problem,

$$S1 : S2 = 2\pi r^2 : \pi r l$$

$$= 2\pi r^2 : \pi r \times \sqrt{2}\,r$$

$$= 2\pi r^2 : \sqrt{2}\,\pi r^2$$

$$= 2 : \sqrt{2}\,.$$

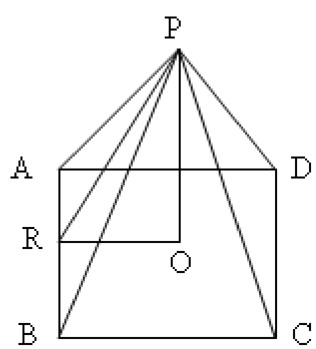
$$= \sqrt{2} : 1$$

Ans: The ratio of their volumes is 2:1

The ratio of their curved surface area is $\sqrt{2} : 1$

(b) Base of a pyramid is a square of side 24 cm. If the height of the pyramid be 16 cm, find its slant height and the whole surface area.

Solution:



The base of a pyramid is the square of side 24 cm. Height (PO) is 16 cm.

$\therefore$ the area of the square ABCD = $(24)^2$ sq. cm = 576 sq. cm

$\therefore$ the perimeter of the square ABCD $= 4 \times 24$ metre = 96 metre.

$\therefore$ slant height (PR) = $\sqrt{OR^2 + OP^2}$

$$= \sqrt{\left(\frac{BC}{2}\right)^2 + OP^2}$$

$$= \sqrt{12^2 + 16^2}$$

$$= \sqrt{144 + 256}$$

$= \sqrt{400}$

$= 20$ cm

Therefore surface area $= \frac{1}{2} \times \text{perimeter} \times \text{slant height} + \text{area of the square}$

$= \frac{1}{2} \times 96 \times 20 + 576$

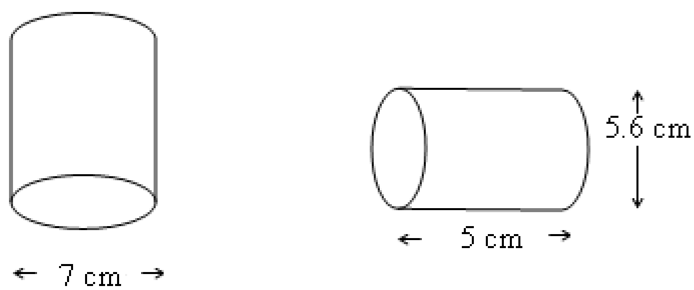
$= 960 + 576$

$= 1536$ sq cm

Ans: The slant height is 20 cm and surface area is 1536 Sq cm.

(c) There is some water in a long upright gas jar of diameter 7 cm. If a solid right circular cylindrical piece of iron of length 5 cm and diameter 5.6 cm be immersed completely in that water, how much the level of water will rise?

Solution:



Volume of solid right circular cylinder $= \pi r^2 h$

$= \pi \times (\frac{5.6}{2})^2 \times 5$

$= \pi \times 2.8 \times 2.8 \times 5$

Let on complete immersion of the solid right circular cylinder in jar, the level of water be raised by d cm.
By the problem,
Volume of displaced water = volume of solid cylinder

or, $\pi \times (\frac{7}{2})^2 \times d = \pi \times 2.8 \times 2.8 \times 5$

or, $d = 2.8 \times 2.8 \times 5 \times \frac{2}{7} \times \frac{2}{7}$

$= 0.16 \times 20$

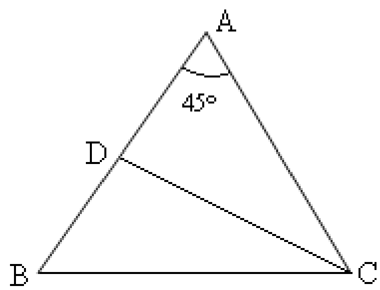
$= 3.2$

$\therefore$ The water will be raised by a level of 3.2 cm in the jar.

Ans: The water will be raised by a level of 3.2 cm in the jar

(d) Length of each equal side of an isosceles triangle is 10 cm and the included angle between those two sides is 45°. Find the area of the triangle.

Solution:



In triangle ABC AB = AC = 10 cm and $\angle BAC = 45^{o}$

CD is perpendicular to AB.

We have taken AB as the base of the triangle; then its altitude is CD.

By the problem,

$\angle ACD + \angle CAD = 90^{o}$

or, $\angle ACD = 90^{o} - \angle CAD = 90^{o} - 45^{o} = 45^{o}$

$\therefore$ AD = CD

In Δ ADC,

$CD^2 + AD^2 = AC^2 = (10)^2$ sq cm. = 100 sq. cm

$\therefore\ 2CD^2 = \sqrt{50}$ sq. cm

or, CD $= 5\sqrt{2}$ cm.

$\therefore$ Area of the triangle $= \frac{1}{2} \times AB \times CD$

$= \frac{1}{2} \times 10 \times 5\sqrt{2}$

$= 25\sqrt{2}$ sq cm

Ans: Area of triangle ABC is $25\sqrt{2}$ sq cm

14 Answer any *two* questions:- **3+3**

(a) If $\cot\theta = \frac{x}{y}$, then prove that $\frac{x\cos\theta - y\sin\theta}{x\cos\theta + y\sin\theta} = \frac{x^2 - y^2}{x^2 + y^2}$

Solution:

$$\text{L.H.S.} = \frac{x\cos\theta - y\sin\theta}{x\cos\theta + y\sin\theta}$$

$$= \frac{\frac{x\cos\theta}{\sin\theta} - y}{\frac{x\cos\theta}{\sin\theta} + y} \text{ [Dividing numerator and denominator by } \sin\theta]$$

$$= \frac{x\cot\theta - y}{x\cot\theta + y} = \frac{x\frac{x}{y} - y}{x\frac{x}{y} + y} = \frac{x^2 - y^2}{x^2 + y^2}$$

Therefore, $\frac{x\cos\theta - y\sin\theta}{x\cos\theta + y\sin\theta} = \frac{x^2 - y^2}{x^2 + y^2}$ (Proved)

(b) If $x \sin60^o \cos^2 30^o = \frac{\tan^2 45° \sec 60°}{\operatorname{cosec} 60°}$, What is the value of x?

Solution:

$$x \times \sin60^o \times \cos^2 30^o = \frac{\tan^2 45° \sec 60°}{\operatorname{cosec} 60°}$$

$$\text{or, } x \times \frac{\sqrt{3}}{2} \times (\frac{\sqrt{3}}{2})^2 = \frac{(1)^2 \times 2}{\frac{2}{\sqrt{3}}}$$

$$\text{or, } x \times \frac{\sqrt{3}}{2} \times \frac{3}{4} = \sqrt{3}$$

$$\text{or, } x = \sqrt{3} \times \frac{2}{\sqrt{3}} \times \frac{4}{3} = \frac{8}{3}$$

$$\text{or, } x = \frac{8}{3}$$

Ans: $x = \frac{8}{3}$

(c) Show that $\text{cosec}^2 22^o \cot^2 68^o = \sin^2 22^o + \sin 68^o + \cot^2 68^o$

Solution:

L.H.S.:

$$\text{cosec}^2 22^o \times \cot^2 68^o$$
$$= \text{cosec}^2 22^o \times \cot^2(90^o - 22^o)$$
$$= \text{cosec}^2 22^o \times \tan^2 22^o$$
$$= \frac{1}{\sin^2 22°} \times \frac{\sin^2 22°}{\cos^2 22°}$$
$$= \frac{1}{\cos^2 22°}$$
$$= \sec^2 22^o$$

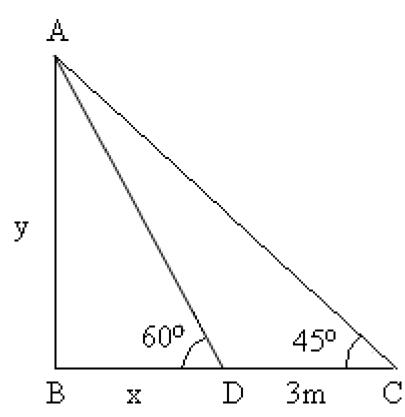
R.H.S.:

$$\sin^2 22^o + \sin 68^o + \cot^2 68^o$$
$$= \sin^2 22^o + \cos^2(90^o - 22^o) + \cot^2(90^o - 22^o)$$
$$= \sin^2 22^o + \cos^2 22^o + \tan^2 22^o$$
$$= 1 + \tan^2 22^o$$
$$= \sec^2 22^o$$

$\therefore \text{cosec}^2 22^o \times \cot^2 68^o = \sin^2 22^o + \sin 68^o + \cot^2 68^o$ (Proved)

15. Length of shadow of a post decreases by 3 m when the altitude of the Sun increases from 45° to 60°. Find the height of the post. ($\sqrt{3}$ =1.732) **5**

Solution:



BC is the shadow of the post.

When the sun's altitude increases from 45° to 60°; BD is diminished by 3 meters

Let the height of the post (AB) be *y*

$\therefore$ $\angle$ABC and $\angle$ABD be the two right angled triangles.

In Δ ABC, tan $45^o = 1$

or, $1 = \frac{AB}{BC}$

or, $1 = \frac{y}{x+3}$

or, $y = x + 3$ ------------------------------------(i)

In ΔABD, tan $60^o = \sqrt{3}$

or, $\sqrt{3} = \frac{AB}{BD}$

or, $\sqrt{3} = \frac{y}{x}$

or, $y = x\sqrt{3}$ --(ii)

Comparing equation-(i) and equation-(ii)

$x\sqrt{3} = x + 3$

or, $x\sqrt{3} - x = 3$

or, $x(\sqrt{3} - 1) = 3$

or, $x = \frac{3}{\sqrt{3}-1}$

or ,$x = \frac{3(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{3\times(1.732+1)}{2}$

$= \frac{8.196}{2}$

$= 4.098$

$\therefore$ BD = 4.098 m

Now putting the value of x in equation (ii)

$y = (4.098 + 3)$ m

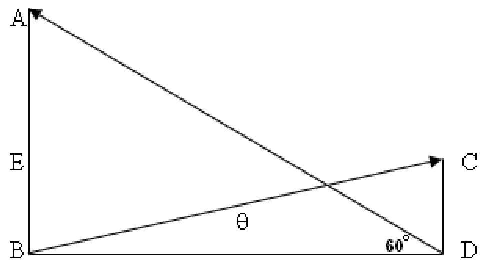
$= 7.098$ m

Ans: The height of the post is 7.098 m.

Or,

Two pillars of height 180 m and 60 m. Angle of elevation of the top of the first post from the bottom of the second post is 60°. What will be the angle of elevation of the top of the second post from the bottom of the first?

Solution:



Let AB and CD are two posts of heights 180 m and 60 m respectively.
By the problem, the angle of elevation BDA is 60^{o}.
Let the angle of elevation of the top of second post from the bottom of first post be θ

$\therefore \angle DBC = \theta$

Now from the right angled triangle ABD,

$$\tan 60^{o} = \frac{AB}{BD}$$

$$\text{or, } BD = \frac{AB}{\tan 60^{0}}$$

$$= \frac{180}{\sqrt{3}}$$

Now from the right angled triangle BDC,

$$\tan \theta = \frac{CD}{BD}$$

$$= \frac{60}{\frac{180}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{3}$$

$$= \sqrt{3}$$

$$= \tan 30^{o}$$

$$\text{or, } \theta = 30^{o}$$

$\therefore$ The angle of elevation is 30^{o}

Ans: The angle of elevation of the top of the second post from the bottom of the first post is 30^{o}